Iterative Decoding with M-ary Orthogonal Walsh Modulation in OFDM-CDMA Systems

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ABSTRACT

Iterative decoding of a serial concatenated coding scheme consisting of an outer convolutional code and M-ary orthogonal Walsh modulation as inner Walsh-Hadamard block code in an OFDM-CDMA system is applied. New aspects arise because OFDM enables the iterative decoding structure to exploit time as well as frequency diversity of a mobile channel. Moreover, the paper points out the low complex symbol-by-symbol maximum a posteriori (SS-MAP) decoding of Walsh-Hadamard block codes. Besides the derivation of the log-likelihood-ratio used for SS-MAP Walsh-Hadamard decoding, simulation results are shown for a differently interleaved indoor channel. They confirm the iterative decoding property to benefit from selectivity in time as well as frequency direction.

I. INTRODUCTION

One interesting candidate for future mobile communication systems is the combination of *Orthogonal Frequency Division Multiplexing* (OFDM) with *code division multiple access* (CDMA) [1, 2] leading to diversity with additional user separation. In recent publications, we proposed the application of *M-ary orthogonal Walsh modulation* [3] for an indoor OFDM-CDMA uplink transmission. M-ary orthogonal Walsh modulation can also be viewed as a systematic linear *Walsh-Hadamard* (WH) block code with code rate $\log_2(M)/M$. This interpretation opens a completely new point of view especially with additional *convolutional coding* (CC) [4]. In this sense, combining CC and Walsh modulation represents a *serial concatenated coding scheme* (SCCS): the CC as an outer code and the Walsh modulation as an inner block code. Hence, the system proposed is one possibility of low rate coding which is required in CDMA uplink systems with inherent high *multiple access interference* (MAI)[1]. Other well-known schemes are, e.g., the QUALCOMM system (that is based on single carrier *Direct Sequence* (DS)-CDMA and also uses M-ary orthogonal Walsh modulation) or systems with one single powerful low rate channel code [5].

In contrast to these systems, the proposed one *(i)* incorporates a serial concatenation of codes, and *(ii)* it is embedded in OFDM-CDMA. Due to these differences, some important new aspects arise. First, the decoding of the Walsh-Hadamard block code merely works in frequency direction, whereas the decoding of the outer CC is mainly arranged in time direction. Therefore, in case of mobile channels, the proposed SCCS benefits from the selectivity of the channel in frequency as well as time domain (e.g. large interleaver sizes can be avoided in case of low Doppler frequencies). Moreover, the system performance for a SCCS can be improved by iterative decoding. Results for the application in the above mentioned single carrier DS-CDMA based QUALCOMM system show promising performance gains [6]. Finally, in order to carry out *symbol by symbol maximum a-posteriori* (SS-MAP) decoding of WH block codes, we can use the *Fast Hadamard Transform* (FHT), i.e. MAP decoding of these block codes can be implemented with very low complexity.

The main objective of this paper is to examine iterative decoding, arranged in frequency and time domain, for a SCCS of CC and M-ary orthogonal Walsh modulation in an OFDM-CDMA based transmission. In particular, we will derive the *log-likelihood-ratio* (LLR) that is necessary for SS-MAP Walsh-Hadamard decoding. In the recent few years, a lot of work has been done to compare conventional *Non-Systematic convolutional codes* (NSC) with *Recursive Systematic convolutional codes* (RSC). This gives us rise for analyzing RSC and NSC codes in the proposed iterative decoding scheme. Finally, in order to show the two dimensional exploitation of the iterative decoding structure, the examination is accomplished by Monte-Carlo simulation results for a differently interleaved indoor channel. For comparison, we will also present results for a standard OFDM-CDMA system with BPSK modulation [2] instead of M-ary orthogonal modulation viewed as WH block code.

The paper is organized as follows: In section II, the SCCS embedded in the OFDM-CDMA system is presented. The derivation of the input LLR, and the description of the iterative decoding scheme are given in section III. Section IV deals with the Walsh-Hadamard SS-MAP decoding. In section V, simulation results are presented, and section VI concludes the paper.

II. SCCS WITH AN OFDM-CDMA TRANSMITTER

An OFDM-CDMA transmitter with channel encoding and Walsh modulation is illustrated in fig. 1. For simplicity, one of J active users is shown and corresponding subscripts are omitted.

The data bits $d \in \{0, 1\}$, each of duration T_d, are convolutionally encoded with code rate $R_c = K/N$. The input of the encoder is a sequence $\boldsymbol{d}$ of K subsequent data bits and the output is the encoded bit sequence $\tilde{\boldsymbol{b}}$ of N bits, each with duration $T_b = R_c T_d$. In order to analyze the proposed system, we apply familiar NSC [7] or RSC codes [8]. After block inter-

[1] Due to asynchronous received user signals, CDMA uplink transmission systems uses non-optimized *Pseudo-Noise* (PN)-sequences for user separation.

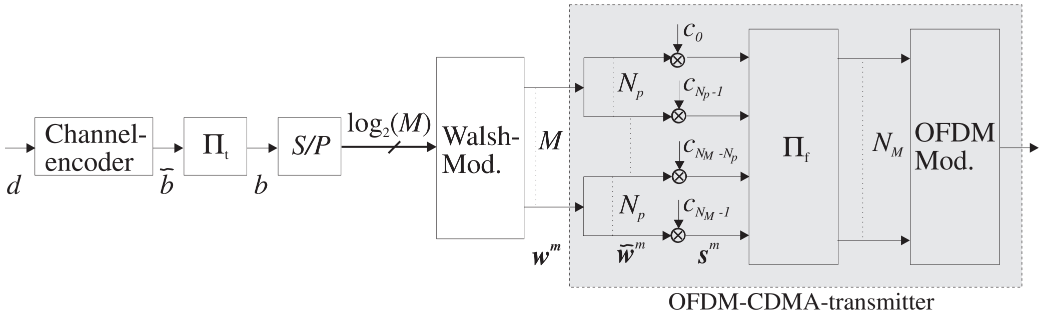

Figure 1: SCCS and OFDM-CDMA transmitter

leaving (Π_t) in time domain the encoded bits are serial/parallel converted to groups of $\log_2(M)$ bits each. The Walsh modulation maps the $\log_2(M)$ encoded bits to one corresponding Walsh symbol (vector) $\boldsymbol{w}^m = [w_0^m, w_1^m, \dots, w_{M-1}^m]^T$, $m \in \{0, \dots, M-1\}$ including M Walsh chips $w_\mu^m \in \{\pm 1\}$, $\mu = 0, \dots, M-1$. Each of the M parallel Walsh chips has a duration of $T = \log_2(M)\, T_b$.

In contrast to the application of the Walsh modulation in DS-CDMA based systems [6, 9], be aware that for the considered OFDM-CDMA system each Walsh symbol is arranged in frequency direction. Furthermore, the Walsh modulation can also be interpreted as a systematic block code (Walsh-Hadamard block code) of rate $\log_2(M)/M$ and Hamming-distance $M/2$ [5]. Each Walsh symbol (Walsh-Hadamard codeword) contains $\log_2(M)$ systematic bits b_κ at positions

$$sys(\kappa) = \frac{1}{2^{\kappa+1}} M, \ \kappa = 0, \dots, \log_2(M) - 1, \tag{1}$$

with Walsh chips (Walsh-Hadamard code symbols)

$$w_{sys(\kappa)}^m = (-1)^{b_\kappa}, \quad b_\kappa \in \{0, 1\}. \tag{2}$$

In order to obtain the symbols to be transmitted, the WH code symbols are replicated into N_p copies. Each branch of the parallel stream is then multiplied with one chip of the user specific PN-code $c_i \in \{\pm 1/\sqrt{N_M}\}$, $i = 0, \dots, N_M - 1$, and N_M indicates the number of used subcarriers.

For the j-th user, we obtain the vector $\boldsymbol{s}^{m,j}$

$$\boldsymbol{s}^{m,j} = [s_0^{m,j}, \dots, s_{N_M-1}^{m,j}]^T, \ s_i^{m,j} = \tilde{w}_i^{m,j} \cdot c_i^j \tag{3}$$

with $\tilde{w}_i^{m,j} = w_\mu^{m,j}$, $\forall\ \mu = \lfloor i/N_p \rfloor, m, j$. The interleaver ($\Pi_f$) scrambles $\boldsymbol{s}^{m,j}$ and passes it onto the OFDM modulator. Finally, OFDM modulation includes the IDFT and inserts the guard interval between adjacent OFDM symbols.

Note that for the proposed transmission, $\log_2(M)$ systematic bits b_κ are always mapped to one corresponding OFDM symbol. In this context, $N_M = M \cdot N_p$, i.e. the number of subcarriers N_M will be increased if the product $M \cdot N_p$ will be raised. Moreover, for an unchanged entire bandwidth, this proposed mapping on an OFDM symbol will result in reduced mismatching if M is raised.

Due to the insertion of the guard interval, each subcarrier is affected by only one channel transfer coefficient [2]. For an uplink transmission scenario, the i-th coefficient for each user j is given by H_i^j, where H_i^j is complex valued Gaussian distributed. For the remainder of the paper, we assume coherent reception with perfectly known channel coefficients H_i^0 for all subcarriers i of user $j = 0$. The other active users are received asynchronously[2].

III. COHERENT OFDM-CDMA RECEPTION WITH ITERATIVE DECODING

The receiver with iterative decoding is shown in fig. 2. The OFDM-CDMA receiver (shadowed block) includes the OFDM demodulation, deinterleaving (Π_f^{-1}), multiplication with the user specific code, one tap equalization, and correlation of N_p subcarriers [2]. Finally, since the Walsh-Hadamard code symbols are real-valued, we take the real part.

Reception for the user $j = 0$ is assumed. With (3) the components of vector $\boldsymbol{v}$ can be expressed by

$$v_\mu = \sum_{i=\mu N_p}^{(\mu+1)N_p-1} \mathrm{Re}\left\{ c_i^0 s_i^{m,0} G_i^0 + \sum_{j=1}^{J-1} c_i^0 s_i^{m,j} G_i^j + c_i^0 E_i n_i \right\}$$

$$= \quad w_\mu^{m,0} \underbrace{\sum_i \frac{\mathrm{Re}\left\{G_i^0\right\}}{N_M}}_{\alpha} + \underbrace{\sum_{j=1}^{J-1} w_\mu^{m,j} \sum_i \frac{\mathrm{Re}\left\{\pm G_i^j\right\}}{N_M}}_{\beta}$$

$$+ \underbrace{\sum_i \frac{\mathrm{Re}\left\{E_i n_i\right\}}{\sqrt{N_M}}}_{\eta}, \tag{4}$$

where E_i indicates the equalization coefficient of the i-th subcarrier, $G_i^j = E_i\, H_i^j$ the equalized channel coefficient and $\mu, m \in \{0, \dots, M-1\}$. If we assume independent adjacent fading subcarriers ensured by perfect frequency interleaving (Π_f) as well as independent user signals, the interference terms β and η can both be considered as real-valued zero mean Gaussian noise with variance σ_β^2 and σ_η^2 by applying the central limit theorem. Consequently, the total relevant Gaussian noise has a variance equal the sum of σ_β^2 and σ_η^2.

The MAP decoder for the Walsh-Hadamard block code applied here uses LLR's as input signals. In order to derive the LLR L_μ we benefit from the knowledge that signal ($w_\mu^{m,0}$) suffers from fading (α) and disturbance by real-valued AWGN (β, η) [2, 10] which is given by

$$L_\mu \ = \ \ln \frac{P(v_\mu | w_\mu^{m,0} = +1)}{P(v_\mu | w_\mu^{m,0} = -1)} \ \approx \ 2\, \frac{\alpha}{\sigma_\beta^2 + \sigma_\eta^2}\, v_\mu. \tag{5}$$

[2] We assumed delay times of the other active users that equals an integer multiple of the OFDM sampling time. Hence, the reception can also be interpreted as quasi-synchronous.

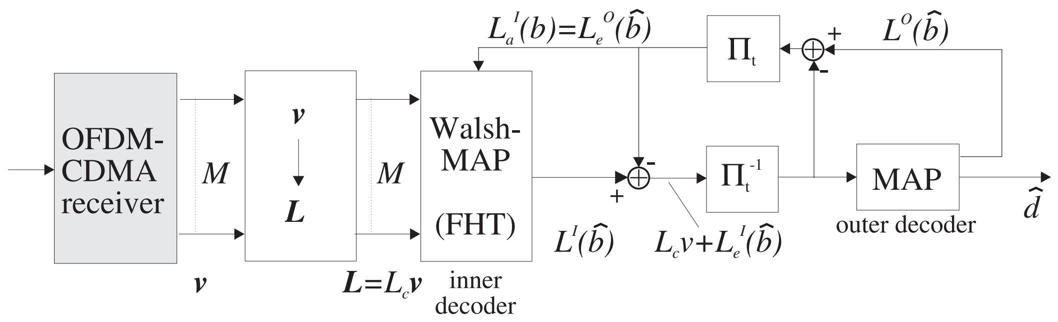

Figure 2: Coherent OFDM-CDMA receiver and iterative decoding scheme

We assume a non-dissipative channel ($E\{|H_i^j|^2\} = 1, \forall\, i, j$, where E indicates the expected value). With (4) and (5) the LLR results in

$$L_\mu \approx 4\,\frac{E_s}{N_0}\frac{1}{1+\frac{J-1}{N_M}\frac{E_s}{N_0}}\;\frac{\frac{1}{N_M}\sum_i \mathrm{Re}\left\{E_i\,H_i^0\right\}}{\frac{1}{N_M}\sum_i |E_i|^2}\,v_\mu, \tag{6}$$

where E_s is the energy of a WH code word in the bandpass, and $N_0/2$ indicates the two-sided noise spectral density. Moreover, we take into account the well-known equalization scheme *maximum ratio combining* (MRC) [2]. In order to estimate the coefficients, pilot based estimation in one or two dimensions [2] is possible. Due to the orthogonality of the Walsh-Hadamard code words, the first code symbol always equals $+1$, and dependent on the amount of spreading N_p, this known information can here be used for pilot tones[3]. In this paper we regard MRC with perfectly known coefficients, thus, $E_i = (H_i^0)^*$. The derivation of the LLR is then accomplished by the expression

$$L_\mu \approx 4\,\underbrace{\frac{E_s}{N_0}\frac{1}{1+\frac{J-1}{N_M}\frac{E_s}{N_0}}}_{L_c}\,v_\mu, \tag{7}$$

where L_c is called reliability of the channel.
The iterative decoding structure is a serial decoding scheme [6] being similar to the scheme introduced in [8], and is shown in detail in fig. 2. It consists of two *symbol-by-symbol* MAP (SS-MAP) decoders, where the inner one represents a special SS-MAP decoder for the systematic Walsh-Hadamard block code. This means, in order to carry out the decoding process, we can use the efficient *Fast Hadamard Transform* FHT being a major advantage because of low decoding complexity. The outer SS-MAP decoder is based on the MAP decoding algorithm introduced in [12] and in contrast to the modified algorithm in [8] it delivers soft information for the information bits as well as for the code bits. Both decoders are working in the logarithmic domain and therefore require input LLR's. From [10] we know that the LLR's given in (1) consists of three parts:

$$\begin{aligned} L^I(\hat{b}_\kappa) &= L_c v_{sys(\kappa)} + L_a^I(b_\kappa) + L_e^I(\hat{b}_\kappa) \\ L^O(\hat{b}_\kappa) &= L_c v_{sys(\kappa)} + L_a^O(b_\kappa) + L_e^O(\hat{b}_\kappa). \end{aligned} \tag{8}$$

In the first part, L_c indicates the channel reliability. The second one is the a-priori information $L_a^I(b_\kappa)$ for the decoded bit of the inner code, and $L_a^O(b_\kappa)$ for the coded bit of the outer code. Finally, we obtain the extrinsic LLR's, $L_e^I(\hat{b}_\kappa)$ and $L_e^O(\hat{b}_\kappa)$, gleaned from the decoding process for the decoded/coded bit, respectively. In the first iteration, no a-priori information is available, whereas for the iteration steps following the extrinsic information delivered by the inner decoder is used as a-priori information for the outer decoder, and vice versa. Thus, we obtain

$$\begin{aligned} L_a^I(b_\kappa) &\overset{!}{=} L_e^O(\hat{b}_\kappa) \\ L_a^O(b_\kappa) &\overset{!}{=} L_e^I(\hat{b}_\kappa). \end{aligned} \tag{9}$$

IV. SS-MAP DECODING OF WALSH-HADAMARD BLOCK CODES

SS-MAP decoding of block codes with low decoding complexity is still an open problem, and there exist different decoding algorithm or implementations. Some significant examples are the trellis implementation, the dual code method, and the direct implementation [10]. Since we use WH block codes, it is possible to meet the requirement of low decoding complexity by applying the FHT embedded in the MAP-decoding structure.
Furthermore, besides the foregoing mentioned general decoding implementations, we can use special Walsh-Hadamard-MAP decoding structures first introduced in [13], enhanced in [6] for iterative decoding in DS-CDMA systems, and introduced in [14] for coherent OFDM-CDMA transmission with Viterbi decoding. All these algorithms have in common that the evaluation of the output LLR proceed from the knowledge of the inherent probability density functions, and uses the direct signal v_μ instead of the corresponding LLR as input.

In this paper, we focus on the direct implementation of systematic block codes [6, 10], where especially for Walsh-Hadamard codes the SS-MAP decoding rule can be expressed by[4]

$$\begin{aligned} L^I(\hat{b}_\kappa) &= \ln \frac{\sum\limits_{\mathrm{W}^m \varepsilon C^I,\, b_\kappa=+1} P(\boldsymbol{w}|\boldsymbol{v})}{\sum\limits_{\mathrm{W}^m \varepsilon C^I,\, b_\kappa=-1} P(\boldsymbol{w}|\boldsymbol{v})} \\ &= \ln \frac{\sum\limits_{\mathrm{W}^m \varepsilon C^I,\, b_\kappa=+1} \exp(\frac{1}{2}\mathrm{FHT}\{L(w_\mu; v_\mu)\})}{\sum\limits_{\mathrm{W}^m \varepsilon C^I,\, b_\kappa=-1} \exp(\frac{1}{2}\mathrm{FHT}\{L(w_\mu; v_\mu)\})}, \end{aligned} \tag{10}$$

with

$$L(w_\mu; v_\mu) = \begin{cases} L_c v_\mu + L_a^I(b_\kappa) & \text{for} \quad \mu = sys(\kappa), \\ L_c v_\mu & \text{else.} \end{cases} \tag{11}$$

[3]The application of WH block codes also offers the possibility of channel phase estimation based on decision-directed estimation without required pilot tones. This method is investigated in [11].

[4]In comparison to (2), the coded bits b_κ can have values $\{-1, +1\}$ by mapping $0 \rightarrow 1$ and $1 \rightarrow -1$.

Eq. (10) shows the inherent application of the FHT in the MAP-decoding implementation, where it is used to carry out the correlation operation.

The approximation $\ln(e^{x_1}+e^{x_2}) \approx \max(x_1, x_2)$ can be used in order to further reduce the complexity of the MAP decoding algorithm in (10). Then we obtain the simple expression [6]

$$L^I(\hat{b}_\kappa) \simeq \frac{1}{2}\max_{\substack{\mathrm{W}^m \varepsilon C^I,\\ b_\kappa=+1}} \mathrm{FHT}\{\cdot\} - \frac{1}{2}\max_{\substack{\mathrm{W}^m \varepsilon C^I,\\ b_\kappa=-1}} \mathrm{FHT}\{\cdot\}. \qquad (12)$$

V. SIMULATION RESULTS

Detailed results concerning the trade-off between convolutional coding of rate R_c, WH block coding, and simple replication N_p were presented in [14] when straight-forward Viterbi decoding is applied. For the analysis of the iterative decoding scheme, we choose $R_c = 1/2$, $M = 64$, and $N_p = 4$ throughout the presented simulations. Termination of the convolutional code is performed for $18 \cdot 32 = 576$ code bits, and, hence, the influence on the code rate of the tail bits can be neglected. Moreover, in order to avoid high complexity, we apply the MAX approximation (12) for the decoding of the WH block code and the MAX-LOG-MAP decoding algorithm for the convolutional codes [10].

For all cases, $J = 8$ active users are taken into account and perfect channel estimation as well as perfect synchronization is assumed for the user concerned.

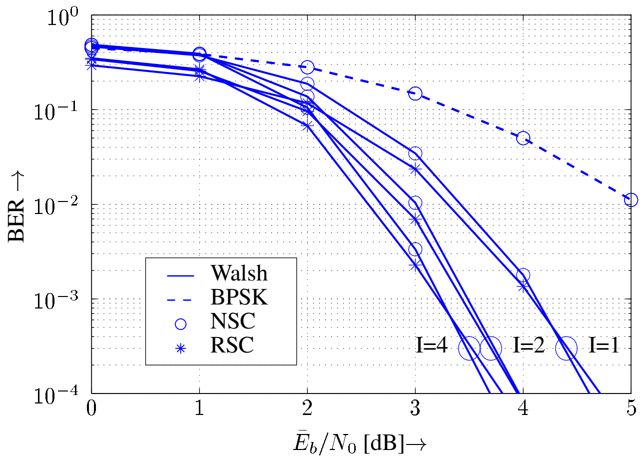

Figure 3: Perfect interleaving: Concatenation of RSC/NSC codes with WH block code and iterative decoding. $I = 1, 2, 4$ iterations compared with concatenating an NSC code with BPSK

Results for an RSC code as well as an NSC code are plotted in fig. 3 when perfect interleaving in time (Π_t) as well as in frequency (Π_f) direction is considered. The results indicate that RSC codes perform better at small $\bar{E}_b/N_0$, and at large $\bar{E}_b/N_0$ it is the other way around [8] (be aware that $\bar{E}_b$ denotes the bit energy at the output of the outer decoder).

Furthermore, independent of the code used, note the gain of about 0.7 dB for the second and approximately 1.0 dB for the fourth iteration at a BER of 10^{-3}. For comparison, a concatenation of an NSC code with $R_c = 1/2$, BPSK modulation and simple replication ($N_p = 24$), i.e., the standard OFDM-CDMA system with convolutional coding [2], leads to an unacceptable BER.

For an uplink indoor channel differently interleaved, fig. 4 shows the BER using an RSC code and iterative decoding

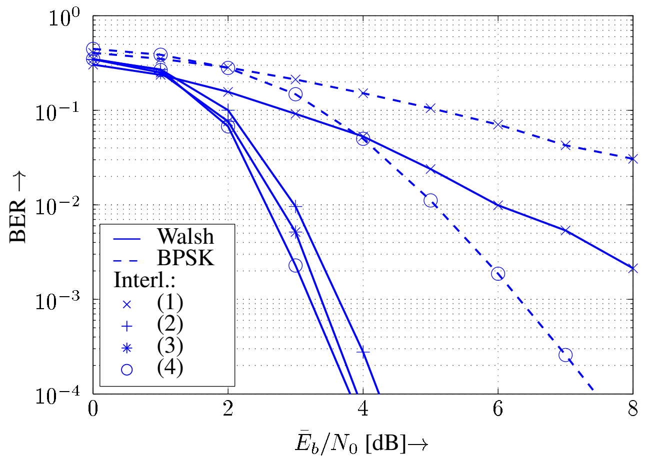

Figure 4: Differently interleaved channel: Concatenation of an RSC code with WH block code and iterative decoding $I = 4$ compared with concatenating an NSC code with BPSK. (1) random frequency (Π_f) and a 18x32 block time interleaver (Π_t), (2) merely sufficient in time (3) merely sufficient in frequency, and (4) sufficient in time as well as frequency

with four iterations. The indoor channel is related to the European HIPERLAN/2 standardization. In particular, we assumed a very low Doppler frequency of 9 Hz, a coherence bandwidth of approx. 1.8 MHz, and a total bandwidth of 25 MHz. Therefore, the fading on each subcarrier as well as the fading between the subcarriers is highly correlated, resulting in burst error structures. Four cases of interleaving are considered: (1) random frequency (Π_f) and a 18x32 block time interleaver (Π_t), (2) merely sufficient in time, (3) merely sufficient in frequency, and (4) sufficient in time as well as frequency.

Note, although we use a powerful outer CC in time direction, and a less powerful inner block code in frequency direction, the system sufficiently interleaved in frequency direction outperforms the one sufficiently interleaved in time domain. Thus, the inner system, and, hence, the achievable diversity gain in frequency direction mainly determines the overall system performance. There exists only a small loss for the sufficiently frequency interleaved system in comparison with the two domain perfectly interleaved one.

The results for the random frequency and block time interleaver confirm the extreme sensitivity of existing channel decoding algorithm to burst error structures caused by the indoor scenario (Not shown in the figure, only less gain can be achieved by iterative decoding in this case).

Fig. 4 also shows the results for BPSK modulation, and again, they emphasizes the tremendous loss compared to the application of M-ary orthogonal Walsh modulation viewed as WH block code.

VI. CONCLUSIONS

In this paper, we examined iterative decoding of serial concatenating an outer convolutional code and M-ary orthogonal Walsh modulation as an inner linear block code in an OFDM-CDMA system. Since the outer code mainly works in time direction and the Walsh-Hadamard block code is exclusively arranged in frequency direction, the new aspect arises that the iterative decoding scheme exploits the time as well as the frequency domain. In this context, we gave a derivation of the input LLR for SS-MAP decoding of Walsh-Hadamard block codes.

Simulation results for RSC as well as NSC codes show a gain of about 1 dB in $\bar{E}_b/N_0$ (BER=10^{-3}) for the fourth iteration assuming a perfect interleaved Rayleigh fading channel. The analysis for a different interleaved indoor mobile channel indicate the robustness of the proposed two dimensional coding scheme against very slowly fading or non-frequency selective channels, and, thus, enhance the system property to benefit from time as well as frequency selectivity.

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