Turbo-Codes and Turbo-Coded Modulation in CDMA Mobile Radio Systems for Short Frame Transmission

Volker Kühn
University of Bremen (Germany)
Department of Telecommunications
Kufsteiner Straße NW 1
D-28359 Bremen
e-mail: kuehn@comm.uni-bremen.de

Abstract— ***Recently, binary parallel concatenated recursive convolutional codes termed Turbo-Codes received great attention due to their amazing error correcting capabilities. Nevertheless, reaching near Shannon limit performance is only possible with very large interleavers prohibiting an application in speech-transmission. This paper presents simulation results for a short-frame data transmission system based on DS-CDMA (Direct-Sequence Code Division Multiple Access) employing Turbo-Codes. Furthermore, the performance of a turbo-coded modulation scheme is investigated with respect to the typical conditions of DS-CDMA systems. As a result, it is shown that Turbo-Codes are no longer superior to classical convolutional codes if the same decoding costs are assumed. Additionally, simulation results illustrate that coded modulation schemes are not well suited for the application in CDMA systems. Finally, it is proved that an accurate channel estimation is necessary for allowing of high coding gains.***

I. Introduction

In order to ensure an appropriate performance in digital mobile radio systems, a sophisticated channel coding concept is necessary. In 1993, a novel class of binary parallel concatenated recursive convolutional codes termed Turbo-Codes received great attention because they allow near Shannon limit performance [1]. Furthermore, their combination with bandwidth-efficient modulation schemes termed Turbo-Coded Modulation [2] outperforms the classical trellis coded modulation (TCM) of Ungerboeck [3].

Unfortunately, the amazing results can only be achieved with very large interleaver sizes causing tremendous transmission delays and prohibiting an application in speech-transmission. In [1], a gap of only 0.7 dB to Shannon's limit remains using a random interleaver of 256 rows and 256 columns $L = 65536$ causing a transmission delay of $\tau = 13.65$ s at a data rate of $R_d = 9.6$ kbit/s. Nevertheless, the general concept of parallel concatenated codes in combination with an iterative decoding scheme involves a lot of possibilities to improve the performance of modern communication systems. Therefore, it is an important task to investigate whether Turbo-Codes and Turbo-Coded Modulation are still superior to conventional coding concepts under the constraint of moderate interleaver sizes.

In order to reach this goal, Turbo-Codes and Turbo-Coded Modulation were investigated for an interleaver size of $L = 400$ bits causing a transmission delay of about $\tau = 83,3$ ms at a data rate of $R_d = 9.6$ kbit/s. With this restriction, a Direct-Sequence CDMA system is used to compare the performance of Turbo-Codes with that of classical convolutional codes taking into account the computational costs of the specific decoding procedures. Additionally, the influence of an imperfect channel estimation on the coding gain is illustrated.

This paper is organized as follows: Section 2 briefly reviews the general concept of Turbo-Codes and defines some specific parameters concerning generator polynomials and decoding algorithms. Section 3 describes a Turbo-Coded Modulation scheme [2] and explains details for an optimal mapping strategy. Additionally, it presents a low cost demodulation algorithm supplying soft-output values. Section 4 treats the simulation model and section 5 illustrates the received results.

II. Turbo-Codes

A. Turbo-Encoder

This section briefly describes the concept of turbo-encoding and -decoding as it is used in this article. The principle structure of the encoder is shown in Figure 1 [1]. Turbo-Codes are systematic codes consisting of at least two parallel concatenated constituent codes C_1 and C_2 connected by an interleaver IL. The interleaver approximately ensures statistical independence of the parity bits $\mathbf{c}_1(k)$ and $\mathbf{c}_2(k)$ at the output of the constituent encoders. Assuming identical constituent codes with rates $R_{c1} = R_{c2} = 1/N$, the overall code rate of the unpunctured Turbo-Code is $R_c = 1/(2N-1)$. A larger overall code rate R_c can be realized by puncturing $\mathbf{c}_1(k)$ and $\mathbf{c}_2(k)$ with respect to a matrix $\mathbf{P}$ given in Table I. Each column of $\mathbf{P}$ contains $2(N-1)$ elements associated with the same number of parity bits of the vector $\mathbf{c}'(k) = [\mathbf{c}_1(k) \;\; \mathbf{c}_2(k)]^T$. A zero indicates that the corresponding bit of $\mathbf{c}'(k)$ is not transmitted. The columns are read periodically one after another so that puncturing is performed with a period identical to the number of columns of $\mathbf{P}$.

In this article, Turbo-Codes with only two constituent codes are considered. As shown in Table I, the overall code rates $R_c = 1/2$ and $R_c = 2/3$ are obtained with constituent codes of rate $R_{c1} = R_{c2} = 1/2$ and appropriate puncturing. The Turbo-Code with $R_c = 1/4$ employs

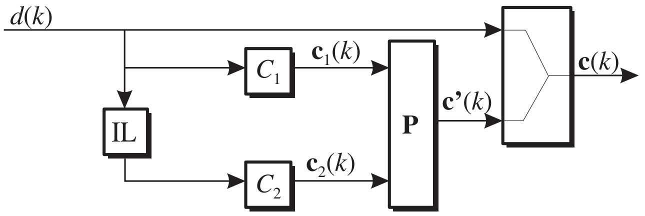

Fig. 1. Structure of the turbo-encoder

constituent codes of rate $R_{c1} = R_{c2} = 1/3$. As derived in [4], systematic and recursive convolutional codes have to be employed in order to take advantage of the interleaver. Furthermore, it is shown in [5] that the effective free distance

$$d_{eff} = 2 + 2 \cdot z_{min} \tag{1}$$

is a critical parameter to be optimized. In (1) z_{min} represents the minimum weight of the redundancy bits of a constituent code for an input weight of $w = 2$. In order to maximize d_{eff}, constituent codes with primitive feedback polynomials were chosen because they maximize the minimal length of output sequences for an input weight of $w = 2$. The generator polynomials and their corresponding puncturing matrices used in this article are given in Table I. The superscript r indicates the recursive polynomial. One aim of this paper is the comparison between Turbo-Codes and convolutional codes assuming the same computational decoding costs. For six decoding iterations a convolutional code with a constraint length of $L_c = 9$ is required. Here, it has to be mentioned that the data rate is slightly reduced in case of convolutional coding because more tail bits are needed to determine the trellis. Nevertheless, this disadvantage can be compensated by slightly puncturing the convolutional code with negligible performance loss.

TABLE I

GENERATOR POLYNOMIALS FOR DIFFERENT CODE RATES

Overall code rate	Convolutional codes $L_c = 9$	Turbo Codes $L_c = 5$	Puncturing matrix $\mathbf{P}$
$R_c = \frac{1}{2}$	$\mathbf{g}_1 = 561_8$ $\mathbf{g}_2 = 753_8$	$\mathbf{g}_1^r = 23_8$ $\mathbf{g}_2 = 35_8$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$R_c = \frac{1}{4}$	$\mathbf{g}_1 = 463_8$ $\mathbf{g}_2 = 535_8$ $\mathbf{g}_3 = 733_8$ $\mathbf{g}_4 = 745_8$	$\mathbf{g}_1^r = 23_8$ $\mathbf{g}_2 = 35_8$ $\mathbf{g}_3 = 25_8$	$\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$
$R_c = \frac{2}{3}$		$\mathbf{g}_1^r = 23_8$ $\mathbf{g}_2 = 35_8$	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$

Concerning the application of Turbo-Codes for speech transmission, the tolerable transmission delay has to be restricted to some 10 ms. For this reason, an interleaver size of $L = 400$ bits is chosen, limiting the performance of turbo-codes dramatically. In this work, a a simple block interleaver is used. The performance degradation due to the missing optimization of the interleaver structure can be neglected for the mentioned interleaver size and bit error rate above $P_b \geq 10^{-4}$ [6].

B. Turbo-Decoder

Figure 2 shows the structure of the turbo-decoder. The two decoders D_1 and D_2 corresponding to the constituent codes C_1 and C_2, respectively, are serially arranged and connected by an interleaver. The extrinsic information $Z_1(k)$ of the first decoder D_1, i.e. the information obtained only from the redundancy bits of C_1 [1], can be used as a-priori-information for D_2 and vice versa. As a consequence, after de-interleaving (IL^{-1}) the extrinsic information of D_2, $Z_2(k)$ is passed back to D_1 and an iterative decoding procedure arises. As a decoding algorithm the Max-Log-MAP-algorithm is applied [7], [8]. Investigations have shown that this is an appropriate compromise between performance, robustness and computational effort [9]. As already mentioned, six decoding iterations are used in the simulations.

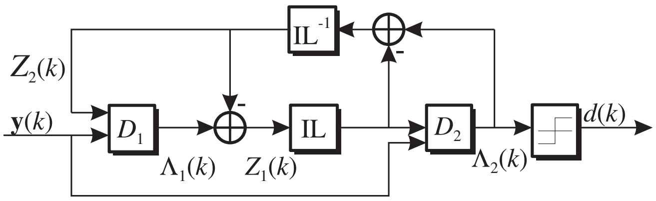

Fig. 2. Structure of the turbo-decoder

III. Turbo-coded Modulation

In many applications, the bandwidth expansion introduced by the coder's redundancy cannot be tolerated. In these cases it is necessary to use a bandwidth-efficient modulation scheme. A first approach combining Turbo-Codes with a multilevel modulation scheme is presented in [2]. The structure of this approach is depicted in Figure 3. Due to the desired high spectral efficiency only constituent codes of rate $R_{c1} = R_{c2} = 1/2$ are used. Here, n unpunctured code words $\mathbf{c}(k)$ of the turbo-encoder in Figure 1 consisting of 3 bits are arranged in a parallel manner separating the systematic information bits $\mathbf{d}$ and the redundancy bits $\mathbf{c}_1$ and $\mathbf{c}_2$ of the constituent codes C_1 and C_2, respectively. After puncturing the $2n$ redundancy bits, the resulting $(m-n)$-tuple $\mathbf{c}'$ is combined with n information bits to the m-tuple $\mathbf{u}(l)$ selecting one of $M = 2^m$ possible symbols.

Concerning the mapping of $\mathbf{u}(l)$ to a specific symbol $s(l)$, Figure 5 illustrates for the coded 8-PSK that Gray coding offers a gain of more than 2 dB over Ungerboeck's Set Partitioning [3]. The latter mapping rule is not suited for this approach because it neglects the code structure [9]. It is rather advantageous to perform Gray encoding ensuring that the binary representations of neighboring symbols differ only in 1 bit (s. Figure 4). This results in

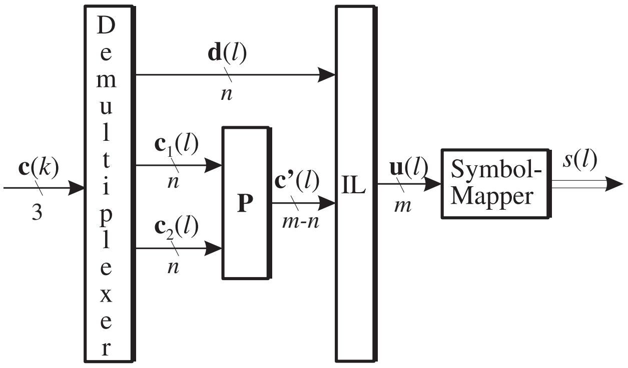

Fig. 3. Combination of turbo-encoder and modulator

a higher average signal-to-noise-ratio after demodulation and consequently in a lower error rate after decoding [9].

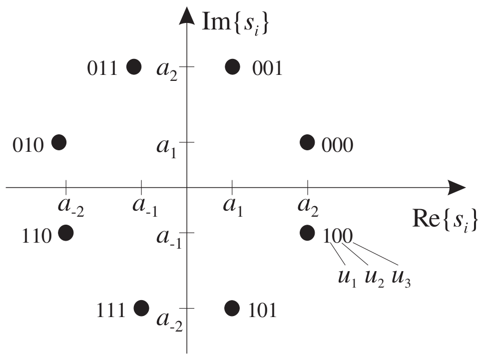

Fig. 4. Signal constellation for 8-PSK and Gray-coding

Another question using Gray-coding is how to map specific bits c_i of a code word $\mathbf{c}$ to bits u_j of a symbol $s(l)$. Obviously, u_1 and u_2 possess a lower error probability than u_3. Simulation results indicate that the information bits $\mathbf{d}$ and the redundancy bits $\mathbf{c}_1$ corresponding to the first encoder C_1 (if possible) should be assigned to bits u_1 and u_2 [9]. Using this strategy, the error rate after the first constituent decoder is lower leading to a lower error rate after the whole decoding procedure. Although the Turbo-Code's performance itself is independent of the specific mapping, it is advantageous to offer the most reliable bits to the first decoder. This behaviour is caused by the iterative decoding process and it is expected that it does not occur in the case of maximum likelihood decoding of the entire code. In the case of a rate 2/3 coded 8-PSK the two information bits per symbol should be assigned to u_1 and u_2 (labeled as System 1 in Figure 5) leading to a gain of 0.3 dB over the mapping to u_2 and u_3 (labeled as System 2) as depicted in Figure 5.

The separation of encoder and modulator at the transmitter site results in separate demodulation and decoding procedures as shown in Figure 6. Therefore, soft-output demodulation is required to avoid information loss after demodulation. A very simple way supplying reliability information is the exploitation of the symmetric arrangement of symbols in the signal space as depicted in Figure 4 [9]. Then, a pragmatic approach using only the inphase component x' and the quadrature component x'' of the received symbol $x(l)$ leads to the following results for 8-PSK

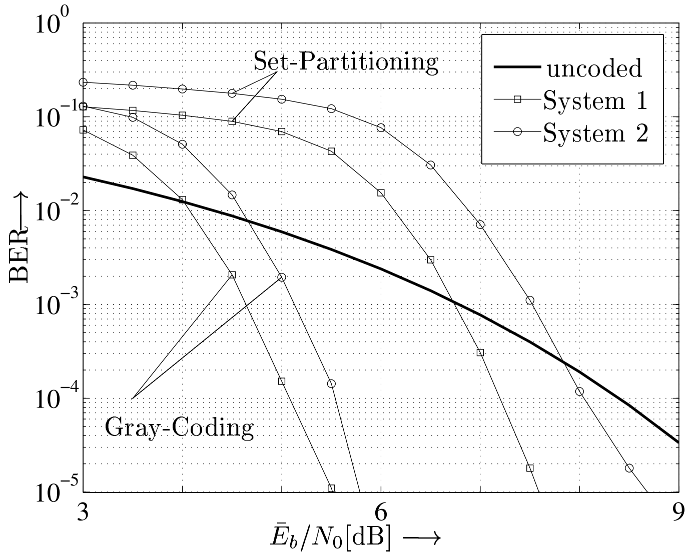

Fig. 5. Bit error rate for 8-PSK and different mapping

modulation [9]:

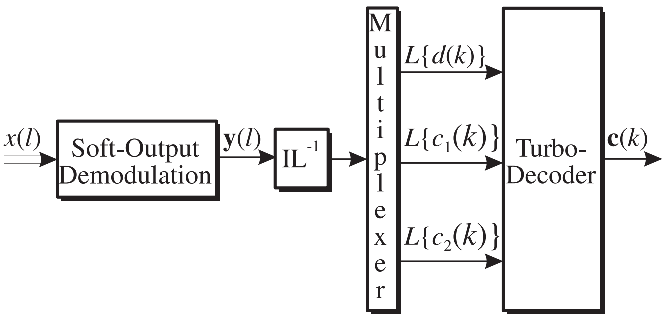

Fig. 6. Combination of demodulator and turbo-decoder

$$L\{u_j\} = \frac{1}{\sigma^2} \cdot \begin{cases} -2a_1 \cdot x'' & j = 1 \\ -2a_1 \cdot x' & j = 2 \\ (a_2 - a_1) \cdot (|x''| - |x'|) & j = 3 \end{cases} \qquad (2)$$

where σ^2 describes the noise variance. This expression can be easily extended for higher order modulation. Investigations indicate only slight differences between the optimal MAP-demodulation [9], [2] and pragmatic approach of (2) for 8-PSK modulation. A further simplification was achieved by replacing the estimated noise variance σ^2 by a constant value of $\sigma^2 \equiv 0.1$ leading only to a minor performance loss. Using this approach, nearly no additional costs are required to supply soft-output information.

Additionally, the interpretation of $L\{u_j\}$ as a log-likelihood-ratio reduces the computational costs of the decoding algorithms. Following the concept in [7], no estimation of the noise variance is necessary.

IV. CDMA Simulation Model

Figure 7 shows the structure of the simulation model. The data stream $d(k)$ with a rate of R_d =9.6 kbit/s is first encoded by the channel encoder using conventional convolutional codes (CC) or Turbo-Codes (TC) as described above. After interleaving the code words

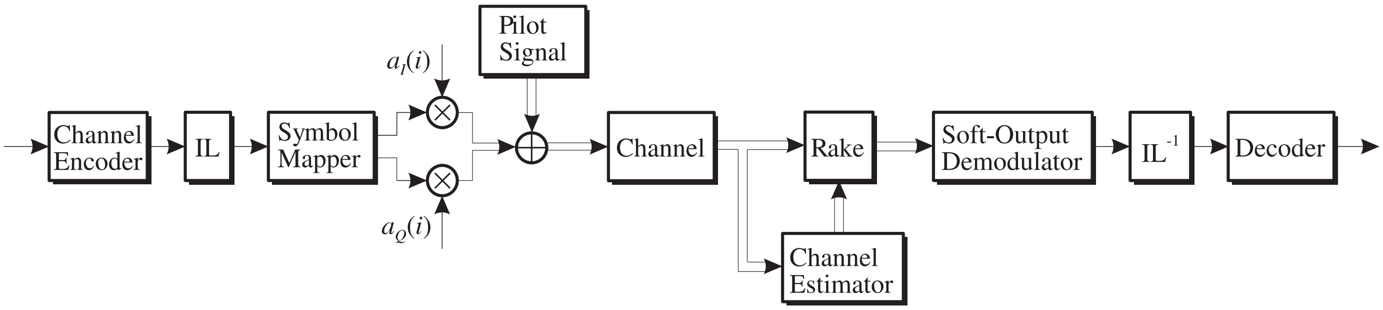

Fig. 7. Structure of CDMA system model

$\mathbf{c}(k)$, they are mapped to PSK-symbols $s(l)$ and Direct-Sequence Spreading is performed as shown in Table II. In the case of an uncoded transmission the bandwidth is increased by a factor 127. When channel coding is used, the Direct-Sequence bandwidth expansion is reduced by a factor equal to the code rate. As a consequence, all system configurations are based on approximately the same processing gain of $G_P \approx 63$. Direct-Sequence spreading is carried out separately for both inphase and quadrature component. Gold Codes with periods of 31, 63 and 127 are employed.

TABLE II
Parameter configuration

Modulation	Interleaver	DS-Spreading	Processing gain
QPSK	uncoded	127	63,5
QPSK $R_c = 1/2$	CC: 18x45 TC: 10x80	63	63
QPSK $R_c = 1/4$	CC: 36x45 TC: 20x80	31	62
8-PSK $R_c = \frac{2}{3}$	TC: 10x60	31	62

The mobile radio channel is modeled as a four-path Rayleigh fading channel with a maximum Doppler frequency shift of $f_{d\text{max}} = 200$ Hz. Therefore, channel interleaver as shown in Table II are used. The number of rows depends on the code rate and the constraint length of the actual code. The Turbo-Code with its inherent interleaver of 20 rows and 20 columns causes a transmission delay of 83 ms. This delay is not increased by the second external channel interleaver with $n = L_c/R_c = 10$ rows and 80 columns. For the convolutional code with $L_c = 9$, $n = L_c/R_c$ rows is a good choice. The number of columns is determined in a way to obtain the same total delay for all configurations. This choice results in a time spacing between two successive bits of $\Delta t = 2.34$ ms that suffices for a coherence time of the channel of $t_c = 2.5$ ms.

Furthermore, an unmodulated pilot signal, transmitted with a power being 6 dB higher than the power of the data channels, delivers the phase reference [10] and is used for the channel estimation [9].

V. Simulation Results

This paper pursues two goals: First, Turbo-Codes are to be compared with convolutional codes under the constraint of the same computational decoding effort. Second, a compromise between a high coding gain (low code rate) and long spreading sequences should be worked out. A long spreading sequence possesses better correlation properties and thus yields a better suppression of interference due to multi-path propagation and multi-user interference. This advantage is attached with a high code rate and consequently a small coding gain.

Figure 8 illustrates the bit error rates for convolutional codes, Turbo-Codes and Turbo-Coded Modulation for different code rates. Concerning the convolutional codes, no differences for 1/2-rate and 1/4-rate coding can be observed. The Turbo-Code with rate 1/4 performs as well as the latter ones, whereas the rate 1/2-Turbo-Code has a lower performance. The turbo-coded 8-PSK loses up to 1 dB over the other configurations caused by the higher signal density in combination with a higher code rate. These disadvantages cannot be compensated by better correlation properties of long spreading sequences [9].

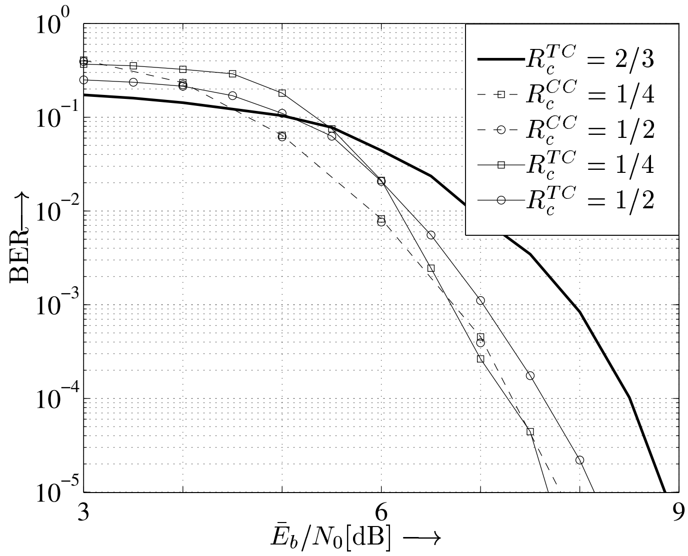

Fig. 8. Bit error rate for different coding scenarios

Surprisingly, it can be recognized that there occurs only a small difference between code rates of 1/4 and 1/2. This observation can be explained with the inaccurate channel estimation. Although the pilot signal is transmitted with a higher power in comparison with the data signals, the signal-to-noise-ratio is still too low. In order to reduce the effect of the background noise ten estimated coefficients were averaged leading to more accurate estimates of the channel coefficients [9].

Figure 9 shows the bit error rates for perfectly known channel coefficients, conventionally estimated coefficients and the approach by averaging 10 estimated coefficients. Obviously, the accuracy of the channel estimation determines strongly the performance of the channel coding. Averaging over ten estimated values leads to a gain of 3 dB over the estimation without averaging, there remains only a loss of 1 dB over perfectly known coefficients.

Additionally, a code rate of 1/4 with a spreading sequence of length 31 now outperforms the code with rate 1/2 and a spreading sequence with period 63 by 1 dB. These results lead to the conclusion that the advantages of powerful channel codes with low code rate can only be exploited with an accurate estimation of the channel state. The difference between perfectly known coefficients and conventional estimation for the coded 8-PSK modulation is only 1 dB, so additional averaging will not result in large gains and is therefore not considered here.

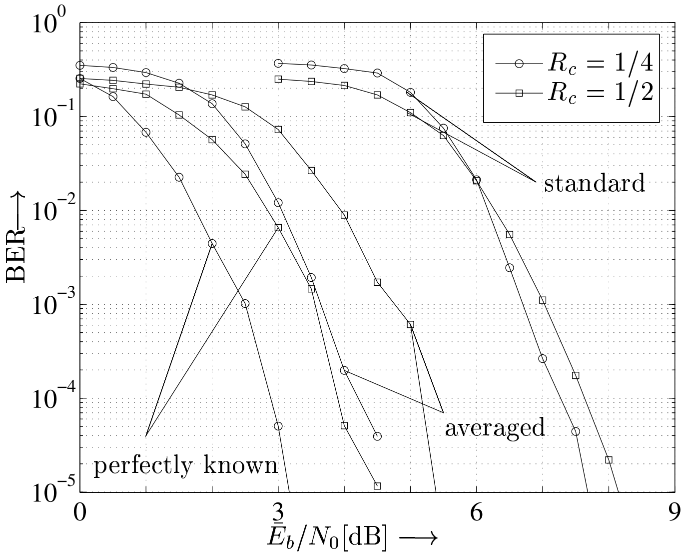

Fig. 9. Bit error rate for perfectly known and estimated channel coefficients and Turbo-Codes

Finally, the performance is checked under the consideration of ten additional users causing mulit-user interference. As can be seen in Figure 10, the difference between rate 1/4 and rate 1/2 coding is reduced to only 0.5 dB. The longer Gold-sequence of length 63 (s. Table II) ensures a better estimation of the channel and a better suppression of additional interfering user signals. Concerning the coded 8-PSK the unsatisfying performance is obvious.

VI. Conclusion

It is shown that Turbo-Codes are no longer superior to convolutional codes if the interleaver size is restricted

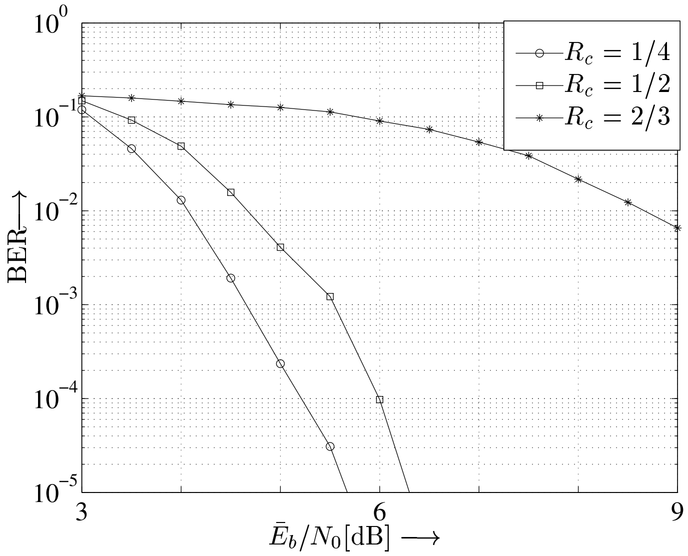

Fig. 10. Bit error rate for additional users and Turbo-Codes

to some hundred bits and the decoding effort is approximately the same. Furthermore, a coded modulation scheme is not attractive in a CDMA-system. It is rather advantageous to decrease the code rate and to shorten the spreading sequence. In order to take advantage of the more powerful code, an accurate channel estimation is necessary. This can be realized by averaging 10 estimated channel coefficients leading to a performance gain of up to 3 dB.

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