COMPLEX VALUED BLOCK CODES FOR OFDM-CDMA APPLICATION

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Abstract This paper introduces the extension of Walsh-Hadamard block codes in order to obtain complex valued block codes. These complex valued block codes lead to an improved spectral efficiency with additional lower error rate. Applied for low rate concatenated coding schemes required for CDMA uplink transmission, the gain in spectral efficiency due to the new codes can be used for strengthening other codes participating. We observe a gain of about 1.7 dB for BER=10^{-3} for OFDM-CDMA transmission.

1. INTRODUCTION

In the asynchronous uplink of CDMA based systems inherent *multiple access interference* (MAI) requires efficient low rate channel coding under the constraint of low decoding complexity. In order to meet these requirements serial concatenated coding schemes using a $(M, \log_2(M), M/2)$ *Walsh-Hadamard* (WH) code as inner block code followed by a repetition code can be applied [1; 2; 3]. With WH codes, the decoding can be carried out by the *Fast*

*Compressed postscript files of our publications are also available from our WWW server `http://www.comm.uni-bremen.de`.

Hadamard Transform (FHT).
Nevertheless, conventional coding schemes, e.g. with WH codes as inner block codes, represent real valued signal processing systems without exploitation of the complex signal space.

In this paper, we present an extension of block codes by an additional M-PSK modulation in order to obtain *complex valued block codes* (CBC), where the extension is based on multiplication of code words with M-PSK symbols. In case of a WH block code as constituent block code, the entire code represents a *complex valued WH block code* (CWC). This scheme was presented for the first time as *hybrid modulation* in [4], where non-coherent detection and no *maximum-likelihood detection* (MLD) in a single-carrier CDMA system was performed. The new aspects of this paper are *(i)* a complete new analysis based on the interpretation as CWC, *(ii)* MLD of the CWC, and *(iii)* the application in an OFDM-CDMA system [5; 6].
The paper is structured as follows: In section 2, we describe the construction and MLD of the CWCs. Section 3 presents analytical derivations for fully interleaved Rayleigh fading channels. The application in OFDM-CDMA will be discussed in section 4, and section 5 concludes the paper.

2. CONSTRUCTION AND MLD FOR CWC

Fig. 1 illustrates the construction of CWCs. The data bits, $b \in \{0,1\}$ each of duration T_b are serial-to-parallel converted to groups of $(\log_2(M_1)+\log_2(M_2))$ data bits each. Each group is divided into parts of $\log_2(M_1)$ and $\log_2(M_2)$ data bits, respectively. A $(M_1, \log_2(M_1), M_1/2)$ *Walsh-Hadamard* (WH) block code with additional antipodal modulation maps the $\log_2(M_1)$ data bits to vector $\boldsymbol{w}^{m_1} = [w_0^{m_1}, w_1^{m_1}, \ldots, w_{M_1-1}^{m_1}]^T$. Vector $\boldsymbol{w}^{m_1}$ can be interpreted as a code word in the Euclidean signal space including M_1 code symbols $w_\mu^{m_1} \in \{\pm 1\}$, $\mu = 0, \ldots, M_1 - 1$. This mapping is also well-known as M_1-ary Walsh modulation [3].
Afterwards, each complete codeword $\boldsymbol{w}^{m_1}$ is multiplied by an M_2-PSK symbol $e^{j\varphi_{m_2}}$, $m_2 \in \{0, \ldots, M_2 - 1\}$, determined by the group of $\log_2(M_2)$ bits. This multiplication results in code words $\boldsymbol{u}^{m_1,m_2} = \boldsymbol{w}^{m_1} e^{j\varphi_{m_2}}$ with code symbols $u_\mu^{m_1,m_2} = w_\mu^{m_1} e^{j\varphi_{m_2}}$. In order to simplify notation, we can denote the code words by $\boldsymbol{u}^i = \boldsymbol{w}^i e^{j\varphi_i}$, where subscript i indicates the i-th word with $i \in \{0, \ldots, M_1 M_2 - 1\}$.

Thus, the proposed scheme is to multiply the complete codeword of a WH block code with an M_2-PSK symbol in order to construct a new entire block code with rate

$$R_c^{cwc} = \frac{\log_2(M_1 \cdot M_2)}{M_1}. \tag{1}$$

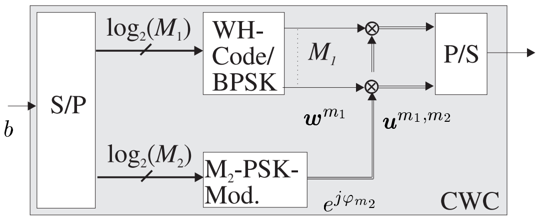

Figure 1 Construction of a complex valued block code based on a WH code

In case of M_2-PSK modulation with $M_2 \geq 4$, this new block code represents a *complex* valued WH block code. Therefore, the analysis in terms of error correction capability should be performed in the Euclidean space instead of the Galois-Field GF(2).

Generally, the quadratic Euclidean distances of an arbitrary CBC can be expressed by

$$e_w^2(i,j) = \sum_{\mu=0}^{M_1-1} e_\mu^2(i,j) = 2M_1 - 2\cos(\varphi_i - \varphi_j) \sum_{\mu=0}^{M_1-1} w_\mu^i w_\mu^j, \qquad (2)$$

where $e_w^2(i,j)$, $e_\mu^2(i,j) = |u_\mu^i - u_\mu^j|^2$ are the distances between the two code words $\boldsymbol{u}^i$, $\boldsymbol{u}^j$ and the two symbols u_μ^i and u_μ^j of these code words, respectively. Note that the distances (2) are increased in comparison with the distances $e_w^2(i,j) = 2M_1 - 2\sum_{\mu=0}^{M_1-1} w_\mu^i w_\mu^j$ of an arbitrary real valued block code, and this increase depends on the used M_2-PSK modulation. As an example, we will confine our discussion on QPSK ($M_2 = 4, \Delta\varphi = (\varphi_i - \varphi_j) \in \{0, \pm\pi/2, \pi\}$) combined with WH codes ($\sum_{\mu=0}^{M_1-1} w_\mu^i w_\mu^j = 0,\ \forall i \neq j$) throughout the paper.

In this case, the quadratic Euclidean distance for different code words ($i \neq j$) lead to

$$e_w^2(i,j) = \begin{cases} 4M_1,\ \boldsymbol{w}^i = \boldsymbol{w}^j\ \wedge \Delta\varphi = \pi \\ 2M_1,\ \text{else}, \end{cases} \qquad (3)$$

where the quadratic Euclidean distance between two code symbols determined by the QPSK results in

$$e_\mu^2(i,j) = \begin{cases} 0,\ \Delta\varphi = 0 \\ 2,\ \Delta\varphi = \pm\pi/2 \\ 4,\ \Delta\varphi = \pi. \end{cases} \qquad (4)$$

In summary, the proposed combination of WH codes with QPSK modulation benefits from a higher code rate (1), and, hence, from an improved spectral efficiency. This is done without loss of error correction capability. We can also observe by (1) that CWCs gain more in terms of spectral efficiency than lower the code rate of the WH code.

Fig. 2 shows the MLD of the CWC that can be performed by the *Fast Hadamard Transform* (FHT) followed by a multiplication with all possible alternatives of transmitted conjugate complex QPSK symbols. Finally, the ML decoder is to take the real part with consecutive maximum decision and decoding of the decided codeword to $\log_2(M_1 M_2)$ data bits. Since the used WH block code and QPSK are combined in a way that a *complete* WH code word is multiplied with a QPSK symbol, it is possible to interchange the FHT and the QPSK detection in the receiver. This leads to complex valued input signals for the FHT. Nevertheless, the conventional FHT can further be applied since the complex symbol acts in a constant factor manner.

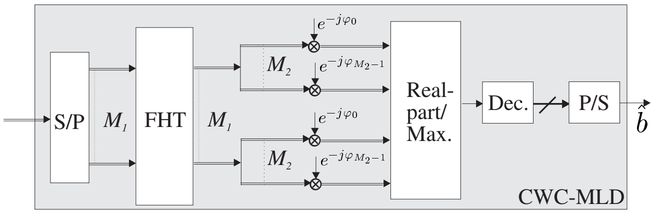

Figure 2 Maximum-Likelihood-Detection of CWC

3. ANALYTICAL PERFORMANCE

With regard to the application of CWC in OFDM-CDMA systems, we will briefly derive the bit error probability P_b by using the well known union bound for 1-Path fully interleaved Rayleigh fading. We assume perfect channel state information, equalization with the conjugate complex channel coefficients and a non-dissipative channel. Since the distances given by (3) are independent of the chosen reference word, we can use $\boldsymbol{u}^0 = [1, \ldots, 1]$ as reference.
Let us start with the code word specific pairwise error probability. First, we define $L(0, j)$ as the effective length of error events in code symbols which means that code words differ in $L(0, j)$ positions. At these positions, we obtain identical distances $e^2(0, j) = e_\mu^2(0, j)$ given by (4) (except zero) of the corresponding codeword $\boldsymbol{u}^j$. With (2), the lengths $L(0, j)$ are directly related to the distances,

$$e_w^2(0, j) = \sum_{\mu=0}^{M_1-1} e_\mu^2(0, j) = L(0, j) e^2(0, j) = \begin{cases} 4M_1, \; \boldsymbol{w}^i = \boldsymbol{w}^j \wedge \Delta\varphi = \pi \\ 2M_1, \text{ else.} \end{cases} \tag{5}$$

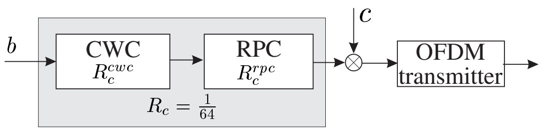

Figure 3 OFDM transmitter with serial concatenated coding scheme

Thus, the codeword specific pairwise error probability is given by

$$P(e^2(0,j)) = \left(\frac{1-\nu}{2}\right)^{L(0,j)} \sum_{l=0}^{L(0,j)-1} \binom{L(0,j)-1+l}{l} \left(\frac{1+\nu}{2}\right)^{l} \quad (6)$$

with

$$\nu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}, \quad \bar{\gamma} = \frac{1}{4} R_c^{cwc} \frac{E_b}{N_0} \, e^2(0,j). \quad (7)$$

In order to accomplish the derivation, we sum all code words with identical input weight w as well as distance $e^2 \equiv e^2(0,j)$, and, hence, with identical $P(e^2(0,j))$ up in sets. Denoting the number of code words in the sets by the *input-output weight enumerating function* (IOWEF) coefficient A_{w,e^2}, we finally obtain with (6) the upper bounded bit error probability

$$P_b \leq \sum_{w} \frac{w}{k} \sum_{e^2} A_{w,e^2} P(e^2), \quad k = \log_2(M_1 M_2). \quad (8)$$

From the analysis we note for fully-interleaved Rayleigh fading the absence of the codeword distance e_w^2 in (6) which is used to play the main role for the correction capability of codes in case of AWGN [7]. Instead, the distance between code symbols e^2 has to be taken into account due to (6) and (7). Moreover, it turns out by (5) that the higher the code symbol distance e^2 the lower the effective length L associated with code diversity. This attribute is similar to trellis-coded modulation [8], and it is contrary to the well-known knowledge that coding schemes with subsequent antipodal modulation maximizing the Euclidean distance ($\equiv$ Hamming distance) also have the largest built-in degree of diversity [7; 9]. Finally, it is remarkable that QPSK reveals to exploit the highest degree of diversity (M_1) in case of Rayleigh fading (4), (5).

4. APPLICATION TO OFDM-CDMA

In this section the introduced CWC will be analyzed in combination with OFDM-CDMA. Fig. 3 shows the transmitter with the CWC of rate R_c^{cwc} (1) followed by a *repetition code* (RPC). The repetition encoder has the code rate $R_c^{rpc} = \frac{1}{N_p}$ ensuring a constant entire code rate

$$R_c = R_c^{cwc} \cdot R_c^{rpc} \overset{!}{=} 1/64 \quad (9)$$

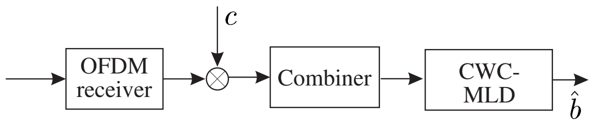

Figure 4 OFDM receiver with serial concatenated decoding scheme

of the serial concatenated coding scheme. Note that the whole spreading is incorporated in the serial coding scheme. After encoding, the sequence is scrambled by a user specific code c. Due to asynchronous transmission in the uplink, we use *pseudo-noise* (PN) codes for scrambling. The resulting signal is fed to the OFDM transmitter consisting of a serial-parallel converter, a *perfect frequency interleaver*, an IFFT, and a parallel-serial converter. Furthermore, it also includes the insertion of the guard interval.
The number of carriers for OFDM transmission is chosen to be $N_M = M_1\, N_p$. This choice leads to a scenario where in the average $\log_2(M_1 M_2)$ information bits are mapped to one OFDM symbol, and it will result in the advantage of less mismatching if the product $M_1\, N_p$, i.e. the number of subcarriers, is raised (the guard time is assumed to remain unchanged).

For one user, the coherent OFDM receiver with subsequent decoding is shown in Fig. 4. We assume conventional OFDM reception [3], *maximum-ratio-combining* (MRC) [6], and perfectly known channel state information. After OFDM reception and descrambling by the user specific code c, the signal enters the combiner that correlates N_p successive samples (decoding the RPC). Finally, the above described MLD of the applied CWC is performed. In case of fully interleaved Rayleigh fading, the coding gain of the RPC in terms of diversity, the number of active users as well as the mismatching caused by the guard time have to be taken into account in the following way:
With (6), (7), and (9) the new average signal-to-noise ratio and diversity are

$$\bar{\gamma} = \frac{1}{4} \cdot \frac{R_c S/N}{1 + (J-1) R_c S/N} \cdot e^2(0,j), \qquad L'(0,j) = L(0,j) \cdot N_p, \quad (10)$$

where J is the number of simultaneously active users[1], and S/N the signal-to-noise ratio with mismatching [3; 6].

4.1 RESULTS

Monte-Carlo simulation as well as union bounded analytical results for P_b with $J = 8$ active users and $M_1 = 16$ are shown in Fig. 5. Besides the *bit error rate*

[1] We assumed delay times of the other active users that equals an integer multiple of the OFDM sampling time (worst case). This is in contrast to the well-known formula [5] with user interference $\frac{1}{3}(J-1)$.

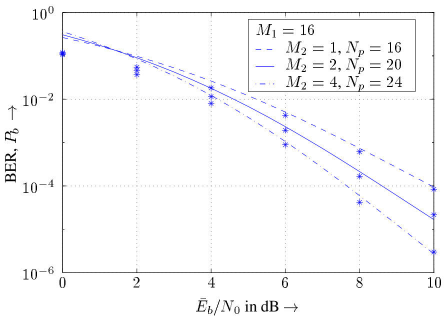

Figure 5 Comparison of WH code with CWC for $M_2 = 2, 4$ and $M_1 = 16$ in OFDM-CDMA with $J = 8$ active users

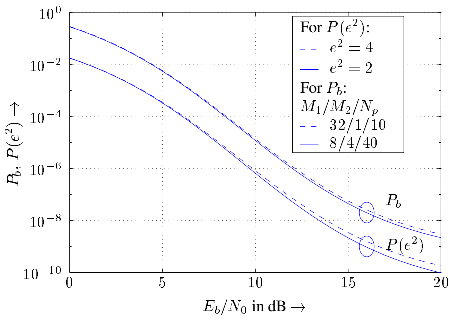

Figure 6 Pairwise error and bit error probability for $M_1/M_2/N_p = 32/1/10$ and $8/4/40$ with $J = 8$ active users

(BER) of the WH block code ($M_2 = 1$) and the CWC with QPSK ($M_2 = 4$), we also plotted results for a "CWC" with BPSK ($M_2 = 2$) instead of QPSK[2]. First, Fig. 5 indicates conformity and the expected divergence between simulation (marked by $*$) and analytical results for high and low $\bar{E}_b/N_0$, respectively. We also observe the best performance for the CWC with QPSK. Note a gain of about 1.7 dB at BER=10^{-3}. Thus, raising M_2 in conjunction with an unchanged M_1 results in a higher code rate R_c^{cwc} (improved spectral efficiency) for the CWC, and, hence, allows lower rate repetition coding (lower R_c^{rpc}). Due to the latter fact, the entire systems degree of diversity $L'(0, j)$ being decisive in case of perfectly interleaved Rayleigh fading is also raised.

In order to analyse diversity aspects, Fig. 6 shows P_b as well as $P(e^2)$ for $M_1/M_2/N_p$ parameter constellations $32/1/10$ and $8/4/40$ with a mismatching of 0.17 dB in both cases. For the WH block code with RPC we have $e^2 = 4$ and $L' = 160$, where for CWC with RPC we obtain $e^2 = 4$, $L' = 160$ as well as $e^2 = 2$, $L' = 320$, and one code word with $e^2 = 4$, $L' = 320$ which can be neglected. Thus, there exists an additional slightly lower $P(e^2)$ for the CWC which results with (8) in a slightly improved P_b.

5. CONCLUSION

The main objective of this paper has been the extension of block codes by M-PSK modulation in order to perform *complex valued block coding*. In this context, we have focused on *Walsh-Hadamard* block codes with QPSK modulation. In general, the extension of WH codes to CWCs results in an improved

[2]The mismatching is about 0.26 dB, 0.21 dB and 0.18 dB for $M_2 = 1, 2$ and 4, respectively. Thus, the influence of different mismatching can be neglected.

spectral efficiency with additional lower bit error rate. Thus, the spectral efficiency gained can be used for strengthening codes participating in a concatenated coding scheme if a constant entire code rate is assumed. Here, we have considered RPC, where future analysis has to be carried out concerning more powerful codes. Moreover, it would be an interesting task to investigate the extension of arbitrary block codes to CBCs.

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