On the Bit Error Behaviour of Coded DS-CDMA with Various Modulation Techniques

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ABSTRACT

In order to combat transmission errors in digital communication systems, forward error protection based on convolutional codes and interleaving in conjunction with soft decision maximum-likelihood decoding through the Viterbi algorithm is often applied. The expected bit error rate for a given signal-to-noise ratio is generally difficult to foresee for a certain system. In this paper, we introduce a method which vividly treats the error behaviour on particular stages of the receiver, hereby allowing a coarse estimate of the overall bit error rate. Although no close mathematical results are given, several astonishing effects can easily be explained thus providing a better understanding of concatenated systems.

I. INTRODUCTION

IN SOME CASES when channel coding is used in conjunction with digital modulation, there is an uncertainty about the expected bit error rate (BER) for a specific system. Channel encoding means adding redundancy to the data prior to transmission and should ensure a reduced BER. Normally – and opposed to a later section of this paper – the following holds: the higher the redundancy added to the data (i.e. the more powerful the coding, or, the lower the code rate), the lower the BER. However, as a first restriction, if convolutional codes are applied as channel codes in practical implementations, the reduction in BER is achievable only if the signal-to-noise ratio (SNR) on the channel is above a certain threshold.

For the concatenation of convolutional encoding and binary phase shift keying (BPSK), i.e. antipodal signalling, system behaviour is such as widely expected beyond the threshold. That is, if the data is coherently demodulated after transmission over an additive white Gaussian noise (AWGN) channel, the BER falls towards zero more rapidly than in the uncoded case. However, there are several influences that might change system performance in a (more or less) unexpected manner, e.g.

- the modulation technique applied to the system, e.g. M-ary PSK like QPSK or M-ary orthogonal modulation, each with a wide range of M,
- the change from coherent to noncoherent demodulation,
- mobile radio channels with multipath propagation, where the order of diversity has to be considered,
- the necessity of interleaving in conjunction with the above topic,
- a loss due to nonperfect channel estimation [1],
- a change in code rate R_C, and
- hard bit versus soft bit decoding.

In this paper, we consider the use of coded signals for bandwidth-constrained channels in a direct-sequence code division multiple access system (DS-CDMA). Each combination of various coding and modulation schemes is assigned the same bandwidth, which is quite large relative to the source data rate $1/T_b$ with spread spectrum. The relation between the data rate $1/T_b$ and the "chip rate" $1/T_c$, which is the reciprocal of the duration of the channel symbols, is given by

$$1/T_c \;=\; P_G \cdot 1/T_b\,. \tag{1}$$

Here, P_G is called "processing gain" and describes the overall bandwidth expansion from the source data to the bandwidth required for transmission. Assuming a constant P_G to enable fair comparisons, the ratio between the channel SNR and the SNR per bit, E_b/N_0, keeps constant for all system configurations, too. Both the encoder and the modulator generally change the data rate and may increase the signal bandwidth. Usually, at least a part of P_G is used for coding and modulation in practical transmission systems while the DS-spreading is carried out with the remaining portion of P_G.

The modulation techniques under consideration in this paper comprise both the commonly used BPSK and a 64-ary orthogonal modulation by means of Walsh functions which has found commercial application in the IS-95 system [2], also known as QUALCOMM-CDMA. The obtained results are therefore quite specific. However, the considerations hold for any modulation form and can be generalized.

The paper is organized as follows: In Section II, the model of the transmission system used in the sequel is discussed. In Section III, a method is given providing us with a good understanding for the examples following hereafter in Section IV, where one introductory and two further examples are treated. The latter show quite surprising effects, but can be explained in the same manner, thus demonstrating the significance of the preparing considerations. A brief conclusion is given in Section V.

II. MODEL OF TRANSMISSION SYSTEM

A model of the digital communications system for which various error rate performances will be evaluated is shown in **Fig. 1**. The

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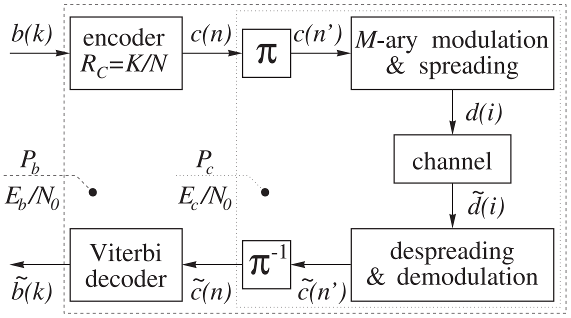

Fig. 1: Model of DS-CDMA transmission system.

channel code generated by the encoder will be a binary convolutional code throughout the paper. The performance improvement obtained with the encoding is achieved by adding redundancy to the data $b(k)$. This expands the signal bandwidth by a factor equal to the reciprocal of the code rate, $1/R_C$. However, since we consider the use of coded signals for *DS-CDMA systems*, the required redundancy is free due to a naturally large spreading factor, introduced as processing gain P_G in (1). Although the coding could be carried to an extreme by fully exploiting P_G for the code rate, i.e. $R_C = 1/P_G$, only a portion of P_G (typically $2 \ldots 4$ corresp. to $R_C = 1/2 \ldots 1/4$) is used for the coding in practical systems for mobile communications.

When transmitting the convolutionally encoded data $c(n)$ over mobile radio channels, it is mandatory to perform sufficient interleaving (block "π" in Fig. 1) because the Viterbi decoder at the receiver end is sensitive to burst errors, mainly caused by deep fades. Since there is a restricted round-trip delay acceptable for e.g. speech traffic channels, only a limited number of information bits can be grouped for each interleaving block. However, for the simulation results in this paper, we applied "perfect" (i.e. sufficiently large) interleaving to clearly expose the effects under investigation. If adequate interleaving is applied, the inner system in Fig. 1 consisting of the interleaver, modulator, (mobile radio) channel, demodulator, and deinterleaver can be modelled as a memoryless discrete binary symmetric channel (BSC) in case of hard bit decisions.

Whether or not the subsequent M-ary modulator further increases the transmission rate depends on the modulation technique. If M-ary phase-shift keying (MPSK) is used, the symbol rate is even lower by $\log_2(M)$ after modulation because $\log_2(M)$ coded bits $c(n')$ at a time are assigned to *one* of M possible symbols in the complex plane. In contrast, if an M-ary modulation in time direction is used, e.g. by means of orthogonal Walsh functions, the transmission rate is increased by $M/\log_2(M)$, for example $64/6 \approx 10.67$. This implies an increase in the required bandwidth in comparison to MPSK. However, as already mentioned above, the processing gain in practical spread spectrum systems is usually high enough to accomodate this further expansion [3].

Throughout the discussion in this paper we only compare various system configurations on the basis of a constant processing gain P_G at a given data rate T_b. Under these conditions, the required total bandwidth is kept constant in any case. In the sequel we assume a data rate of $9.6\ \mathrm{kbit/s}$ that is suitable for both speech and low data rate transmission, and a processing gain of $P_G = 128$, which are the parameters of the IS-95 system [2]. With this, a bandwidth of approximately $1.25\ \mathrm{MHz}$ is allocated. To achieve this bandwidth in a specific case, it is necessary to adjust the factor of DS spreading (N_p) in such a way that the requirements are met. As an example, we refer to the IS-95 system, where $N_p = 4$:

$$P_G = \frac{M}{\log_2(M) \cdot R_C} \cdot N_p = \frac{64}{6 \cdot 1/3} \cdot 4 = 128\,. \qquad (2)$$

Note that the mere process of DS spreading does not enhance system performance in terms of E_b/N_0, that is the signal-to-noise ratio (SNR) per source data bit $b(k)$. The amount of spreading only determines the SNR on the channel at a given E_b/N_0. The spreading itself is achieved by impressing a pseudo-noise (PN) binary-valued sequence with rate $1/T_c$, i.e. "the code" in CDMA, on the modulation symbols.

After transmission, the PN sequence is removed from the received signal at the demodulator. The demodulator shall deliver a *soft value* $\tilde{c}(n')$ as an estimate of the original data $c(n')$, if needed for the subsequent Viterbi decoding. To be able to determine the bit error rate P_c of the un(de)coded bits $\tilde{c}(n)$, a hard decision has to be made at this point to allow the comparison with $c(n)$.

Finally, maximum-likelihood (ML) decoding through the Viterbi algorithm with either hard or soft decisions at the input is applied. In contrast to P_c, the system's bit error rate *including* the channel coding is denoted P_b, see Fig. 1.

III. ASSESSMENT OF THE CODED BER BASED ON THE UNCODED BER

Just two dependences have to be known to explain the effects resulting from the concatenation of coding and modulation, or – more precisely – *de*modulation and *de*coding. These can either be expressed as mathematical functions in an analytical manner or as look-up tables which contain information gained by simulations.

The first dependence is nearly always well-known, since it is the bit error probability P_b in terms of E_b/N_0[1] for a certain type of *(de)modulation*. Referring to Fig. 1, we use E_c/N_0, that means the SNR per *undecoded bit*, instead of E_b/N_0 in our notation in this case. This is to indicate the distinction between the uncoded and the coded system, corresponding to the inner and the outer block in Fig. 1. The corresponding BERs are denoted P_c and P_b, respectively. For example, the most common expression for P_c is that for binary coherent PSK in AWGN [4]:

$$P_c = Q\left(\sqrt{\frac{2E_c}{N_0}}\right) = \frac{1}{2}\mathrm{erfc}\sqrt{E_c/N_0} \quad , \qquad (3)$$

where $N_0/2$ represents the (two-sided) spectral density of the AWGN. Nearly all kinds of modulation are described in a similar way in terms of E_c/N_0 – the corresponding equations can be found abundantly in the literature (e.g. [4]).

Secondly, the *decoding* has to be carried out which is performed most efficiently by the Viterbi algorithm[2] (VA) [4] in case of convolutional encoding. Although error bounds on P_b are known for the case of the concatenation of convolutional encoding and binary signalling, we rather rely on simulations since the bounds are too loose for our purposes in some case. Furthermore, the

[1] In case of a time varying channel transfer function, E_b/N_0 is further marked with a bar to emphasize a *mean* value: $\bar{E}_b/N_0$.

[2] We restrict ourselves to only consider a Viterbi decoder with an Euclidian metric and a (nearly) unquantized input signal in case of soft bit decoding.

Table 1: Maximum free distance convolutional codes with constraint length $L_C = 9$ [4].

R_C	generators (in octal)				d_{free}
1/2			561	753	12
1/3		557	663	711	18
1/4	463	535	733	745	24

possibility to choose code rate R_C, constraint length L_C, and generator functions widely independent of each other results in an abundant variety of available codes. **Table 1** lists the parameters of three codes which we will further use throughout the paper. The simulation results in **Fig. 2** exhibit that the threshold above which coding is beneficial compared to the uncoded case (see curve labeled with "uncoded BPSK" in Fig. 2) is obviously in the vicinity of 0 dB in case of soft bit decoding.

For our purpose, the decoding is just a nonlinear conversion of an input BER into the corresponding output BER: $P_b = f(P_c)$. Provided that the code symbol errors at the input of the Viterbi decoder are statistically independent (where appropriate, sufficient interleaving has to be applied), it is of no interest how the errors arose when *hard bit decoding* is considered – just their probability P_c is of further interest in that case. That means, P_c can be treated independently of the underlying modulation type or channel. Once the rule how P_c is converted into P_b by the VA is known for a specific code, all other cases besides BPSK with AWGN can be treated equivalently. In contrast, when *soft bit decoding* is applied, the nonlinear conversion would normally have to be treated differently for various conditions. However, for our purpose it is sufficient to treat all scenarios in the same manner for soft bit decoding, too. This method does not allow exact calculations of P_b, but is nonetheless well suitable to vividly explain several effects.

The remaining principal item is to ally the above topics. For that purpose we bring to mind that any coding of rate R_C causes a loss in SNR which is described in our notation (in dB) as

$$\frac{E_c}{N_0} = \frac{E_b}{N_0} + 10 \cdot \log_{10}(R_C) , \tag{4}$$

e.g. E_c/N_0 is 3 dB worse compared to E_b/N_0 with $R_C = 1/2$. Furthermore, $E_c/N_0 \equiv E_b/N_0$ when no channel coding is applied. The meaning of this loss is also illustrated in Fig. 2. In the

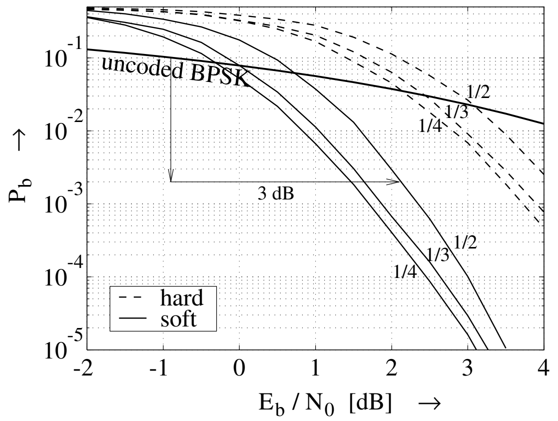

Fig. 2: Convolutional (de)coding in AWGN using the codes referenced in Table 1.

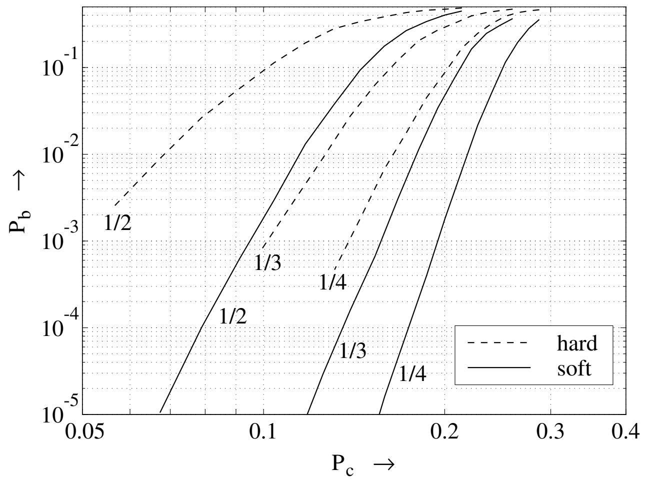

Fig. 3: Conversion of BER by means of convolutional coding ($L_C = 9$) in AWGN with antipodal signals.

example, the BER of the uncoded BPSK at $E_c/N_0 = -0.9$ dB, i.e. $P_c = 10^{-1}$, is transferred into $P_b = 2 \cdot 10^{-3}$ at $E_b/N_0 = 2.1$ dB after soft decoding. This consideration leads to the alternative representation depicted in **Fig. 3**. The curves are directly derived from Fig. 2 and simply show the mapping from P_c to P_b. Lastly, a more convenient representation of the uncoded BERs is desired which already takes into account *both* the coding according to (4) – i.e. the loss in SNR due to the coding – and the embedded modulation type. Such a depiction is shown in **Fig. 4** for the cases of coherent BPSK with AWGN, coherent BPSK with 1-path and noncoherent 64-ary orthogonal Walsh modulation with 4-path Rayleigh fading, at each case $R_C = 1$ (i.e. uncoded), $R_C = 1/2$, $1/3$, and $1/4$, respectively.

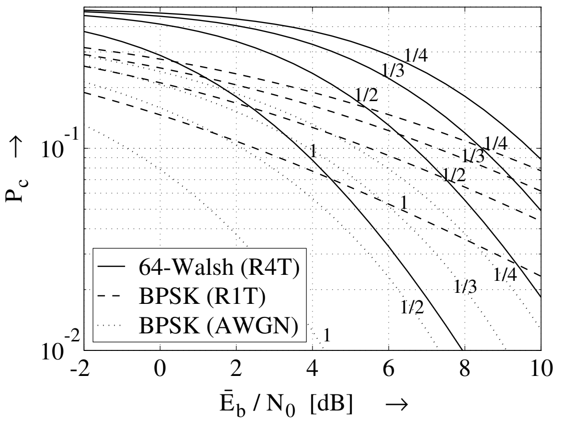

Fig. 4: Several BER curves including the E_c/N_0-loss according to (4) (analytical results).

To summarize the explanations given above, please refer to the following algorithmic description:

1. SELECT the desired E_b/N_0 for which you want to know the BER after decoding, i.e. P_b, for a given code rate R_C, modulation form under consideration, and channel.
2. DETERMINE the corresponding *uncoded BER* (P_c), e.g. in Fig. 4, taking into account the subsequent coding with rate R_C.
3. TRANSFER P_c into the corresponding *coded BER* (P_b) by means of Fig. 3.

IV. SELECTED TOPICS OF CODING & MODULATION

A. BPSK Modulation with 1-Path Rayleigh Fading Channel

Consider the $R_C = 1/3$ convolutional encoding with BPSK modulation as an introductory example. Which BER will appear when the signal is perturbed by a single-path Rayleigh fading channel and AWGN?

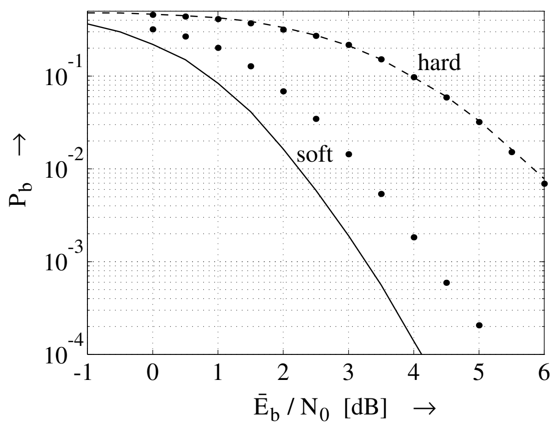

Fig. 5: Coherent BPSK using code rate $R_C = 1/3$ with perfect interleaving in case of 1-path Rayleigh fading with both hard and soft input for VA (• graphically projected / simulation results).

The corresponding curves constructed with the help of Figs. 3 and 4 are marked with bullets in **Fig. 5**. With hard bit decoding (upper group), the graphically projected curve matches the simulation results well. In case of soft bit decoding, the simulation results show a better performance compared to the constructed case. This is due to a better conversion from P_c to P_b in case of Rayleigh fading compared to the pure AWGN channel. Although this is obviously of considerable influence on the absolute values of P_b, it does not change the general validity of the considerations in Section III. The following examples show that several effects can be easily explained with the given method, provided that only one parameter in a given system is varied (e.g. the code rate) and the others are kept constant.

B. Why Powerful Coding is Not Always Good

We directly start with a phenomenon observable when channel coding is applied in concatenation with a specific sort of modulation. The simulation results in **Fig. 6** show that *the weakest coding* ($R_C = 1/2$) *offers the best performance* in this particular case (4-path Rayleigh fading channel, 64-ary orthogonal modulation, noncoherent detection with square-law combining (SLC), usage of ML soft bit metric [5], perfect interleaving). This is contrary to the well-known behaviour in Fig. 2, where it is clearly apparent that the stronger the coding is, the lower the BER becomes. Although this effect has been observed by several authors under similar conditions, e.g. [1, 6], no evident explanation could be given yet.

The crucial difference that ultimately produces the (perhaps) unexpected reversal is the underlying modulation scheme. It is apparent from Fig. 3 that uncoded error rates in the range of approximately $P_c = 0.1 \ldots 0.2$ are mapped onto coded rates of $P_b \approx 10^{-3}$ under investigation. Looking at Fig. 4, we observe that in case of BPSK in combination with an AWGN channel

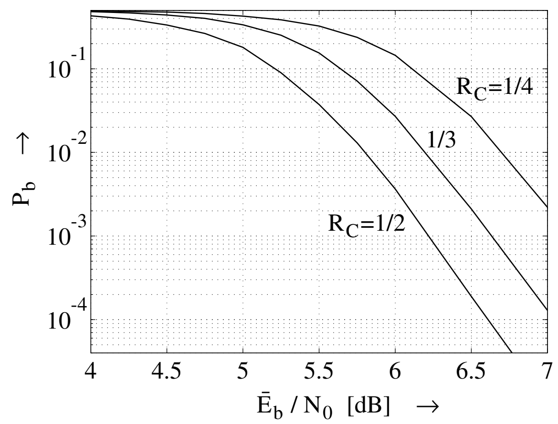

Fig. 6: Coded transmission of 64-ary orthogonal symbols with perfect interleaving and code rates $R_C = 1/2$, $1/3$, and $1/4$ in case of 4-path Rayleigh fading (SLC), soft bit decoding.

(dotted lines), the curves for all three code rates are quite close together in the range of $1 \ldots 2 \cdot 10^{-1}$, i.e. 0 dB. That means, a stronger coding with a higher coding gain is well able to cope with the slightly enhanced error rate due to a higher loss in SNR, finally yielding the results shown in Fig. 2. In contrast, when 64-ary modulation with a Rayleigh fading channel is considered, the BER-curves are somewhat more apart at different code rates[3], see solid lines in Fig. 4 at e.g. 7 dB. With this, the coding gain is too low compared to the increase in error rate P_c of the uncoded signal.

The example may appear a little bit constructed. But, several papers have been published investigating exactly this modulation form in combination with varying code rates under mobile radio conditions, e.g. [6, 7, 8]. This is mainly because the constellation with $R_C = 1/3$ is the one used in the uplink of the widely considered IS-95 system [2].

C. Always a Diversity Gain?

In this example, the code rate $R_C = 1/3$ is kept constant while the number of paths of the underlying mobile radio channel model varies. Thus, in contrast to the above example, no different losses in SNR occur. Nonetheless, a similar effect is observed in this case, also.

It is well-known that a diversity gain is achievable when a signal bearing the same information is transmitted over several independently fading paths [4]. This is possible even with noncoherent detection which is illustrated at the top of **Fig. 7** for the case of 64-ary orthogonal modulation noncoherently detected by means of square-law combining (SLC) in a Rayleigh fading environment. Each channel consists of L fading paths with equal average energy. In each case, the number of fingers in the RAKE receiver exactly matches the number of paths on the channel.

However, the diversity gain is achieved for sufficiently high values of $\bar{E}_c/N_0$ only. Based on (4) and the above considerations, we know that the magnified $\bar{E}_c/N_0$-range at the bottom of Fig. 7 becomes the relevant range to be taken into account when coding will be applied subsequently. At low SNRs, the noncoherent SLC detection scheme is not able to efficiently suppress the noise.

[3]The main cause is that the uncoded curves are nearly horizontal in the BPSK case, whereas they are clearly falling off for 64-ary modulation.

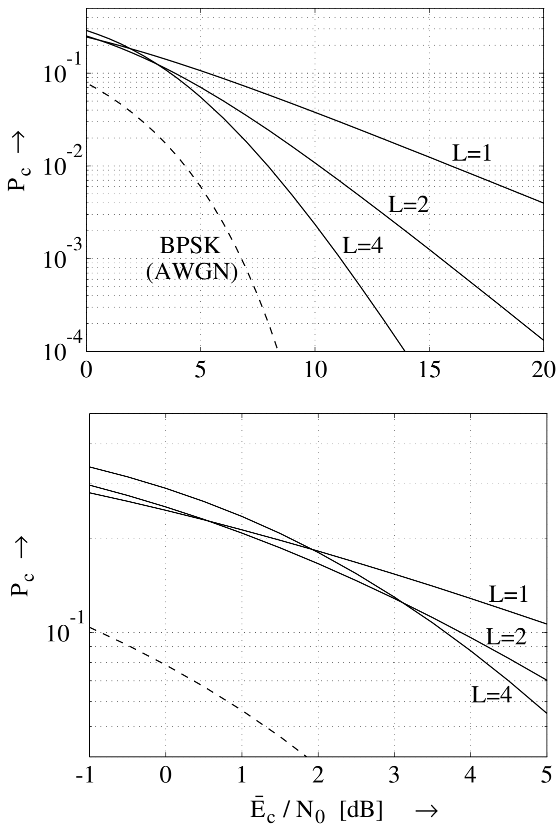

Fig. 7: Top: Uncoded transmission of 64-ary orthogonal symbols in case of $L = 1$-, 2-, and 4-path Rayleigh fading (SLC, analytical results).
Bottom: Magnified depiction of the crucial range.

When the signals of several paths are combined, the undesired increase in noise exceeds the potential diversity gain leading to a poorer performance in this range compared to the single-path scenario.

Treating the corresponding BER curves *including the coding* with $R_C = 1/3$ in **Fig. 8**, it consistently emerges that with coding a diversity gain is no longer achieved (the interleaving is sufficient enough and shall pass for perfect[4]). Furthermore, it turns out that the curves even reverse their order for values of $P_b \approx 10^{-3}$, which is suitable for speech transmission. However, one rather remembers the tendency to diversity gain shown in the upper figure of Fig. 7 (or perhaps the diversity gain achieved with coherent detection) which may lead to rash conclusions when coding is involved.

V. CONCLUSIONS

The intention of this paper was to explain the evolution of the bit error rate in a digital transmission system when a specific modulation scheme is protected from bit errors by means of channel coding. Since *(i)* the concatenation of coding and modulation is of broad interest in modern telecommunications and *(ii)* a prediction of the expected error rate is a difficult task for any desired system configuration, we provide a helpful method to further understand the mechanisms in such systems. For this purpose, the overall bit error rates are derived step-by-step from the error rates of the uncoded inner system. The gained insights are used to explain two significant effects of general interest: Firstly, it is not always good to perform as much coding as possible, and secondly, a diversity gain is not necessarily obtained in a coded system. These specific examples emphasize the common relevancy of the introduced method, for only a good understanding of concatenated systems enables the proper interpretation of unexpected effects. The application to other modulation and coding techniques as treated examplarily in this paper is straightforward.

[4] When block interleaving for speech applications is applied, e.g. with a tolerable delay of 20 ms, it is interesting to note that a diversity gain can be achieved again even with SLC. Since the influence of interleaving is not the topic of this paper, the corresponding simulation results are omitted here.

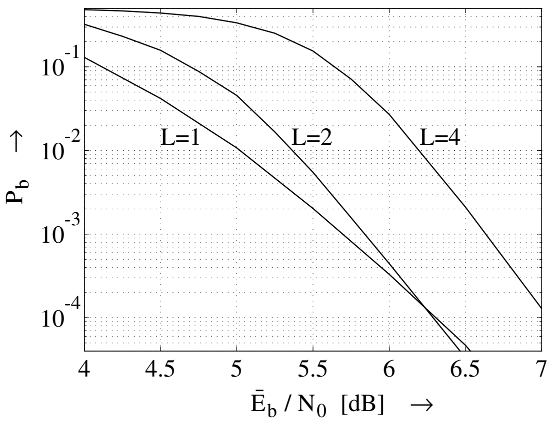

Fig. 8: Coded transmission of 64-ary orthogonal symbols with perfect interleaving and code rate $R_C = 1/3$, soft bit decoding, in case of 1-, 2-, and 4-path Rayleigh fading (SLC, simulation results).

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