Maximum-Likelihood-Decodierung von höherstufig orthogonal modulierten Signalen für OFDM-CDMA Systeme

–Maximum Likelihood Decoding of M-ary Orthogonal Modulated Signals for OFDM-CDMA Systems–

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1. INTRODUCTION

In the recent few years, most publications dealing with OFDM-CDMA [1, 2, 3] have been focused on standard modulation schemes such as M-PSK, and in combination with *forward error correction* (FEC) several *maximum likelihood* (ML) decoding strategies were proposed [4]. Instead of using standard modulation schemes, CDMA techniques offer the possibility to apply M-ary orthogonal modulation [5, 6]. With this modulation, the inner spreading of the OFDM-CDMA system is optimized by spreading in two successive steps: (*i*) spreading by the modulation itself and (*ii*) additional simple replication of the transmitted information into parallel subcarriers to perform CDMA. Investigations without FEC [7] indicate a performance gain by applying M-ary orthogonal Walsh modulation in contrast to BPSK.

If we employ FEC, the combination of the channel code, e.g. a convolutional code, with M-ary orthogonal Walsh modulation describes a concatenated coding scheme. The channel code represents the outer one and the M-ary Walsh modulation can be interpreted as an inner (block-) code [6, 8].

The intention of this paper is to examine the concatenation of ML detected M-ary orthogonal modulated signals with ML Viterbi channel decoding for an OFDM-CDMA transmission. We evaluate the (bit specific) log-likelihood ratio of the outer decoding scheme and discuss the trade-off between channel coding, M-ary orthogonal modulation and simple spreading based on Monte-Carlo simulations in case of sufficient and insufficient frequency interleaving, respectively. Moreover, the results are always contrasted with BPSK performance. The system operates in a frequency-selective and slowly Rayleigh fading indoor channel in the uplink, and all simulations are related to the European Hiperlan/2 standardization whereas the results are generally valid.

2. TRANSMISSION SYSTEM

2.1. Transmitter

For CDMA systems, the spreading codes implemented mainly determine the *multiple access interference* (MAI). For an uplink transmission scenario, every user-specific codeword is affected by different channel transfer functions. This fact leads to an asynchronous situation of the received user signals. To avoid high synchronization complexity, we suggest to apply *Pseudo-Noise* (PN)-codes for CDMA spreading. The drawback of this choice is the inherently high MAI.

An OFDM-CDMA transmitter with FEC and Walsh modulation is illustrated in Fig. 1. For simplicity, one of J active users is shown (subscripts are omitted).

The data bits $d \in \{0, 1\}$, each of duration T_d, are convolutionally encoded with code rate $R_c = K/N$. The input of the encoder is a sequence $\boldsymbol{d}$ of K subsequent data bits and the output is the encoded bit sequence $\tilde{\boldsymbol{b}}$ of N bits, each with duration $T_b = R_c\,T_d$. After block interleaving (Π_t) the encoded bits are serial/parallel converted to collect $\log_2(M)$ bits to one group. The Walsh modulation maps the group to one corresponding Walsh symbol (vector) $\boldsymbol{w}^m = [w_0^m, w_1^m, \ldots, w_{M-1}^m]^T$, $m \in \{0, \ldots, M-1\}$ including M Walsh chips $w_\mu^m \in \{\pm 1\}$, $\mu = 0, \ldots, M-1$. Each of the M parallel Walsh chips has a duration of $T = \log_2(M)\,T_b$. This modulation can also be interpreted as a (linear-) block code with rate $\log_2(M)/M$ and Hamming distance $M/2$. The set of M orthogonal Walsh symbols describes the $M \times M$ *Hadamard-Matrix* [6, 8], and the Euclidian distance is identical for all possible pairs of symbols and equals

$$\varepsilon = \sum_{\mu=0}^{M-1} |w_\mu^m - w_\mu^n|^2 = 2M \quad \forall \quad m \neq n. \tag{1}$$

Furthermore, it is easily verified that the symbol energy is the same for all symbols. Hence, the latter property together with the raising Euclidian distance leads to a favorable signal space and makes plain the idea to apply this modulation scheme as an optimization of the inner spreading for a CDMA system.

To obtain the transmitted symbols, the Walsh chips are replicated into N_p parallel copies where each branch of the parallel stream is multiplied with one chip of the user specific PN-code $c_i \in \{\pm 1/\sqrt{N_M}\}$, $i = 0, \ldots, N_M - 1$. Since the bandwidth for each of the M parallel subchannels decreases for larger values of M ($T = \log_2(M)\,T_b$), the number of subcarriers $N_M = M \cdot N_p$ increases for an unchanged overall bandwidth. For the j-th user, this yields the vector $\boldsymbol{s}^{m,j}$

$$\boldsymbol{s}^{m,j} = [s_0^{m,j}, \ldots, s_{N_M-1}^{m,j}]^T, \quad s_i^{m,j} = \tilde{w}_i^{m,j} \cdot c_i^j \tag{2}$$

with $\tilde{w}_i^{m,j} = w_\mu^{m,j}$, $\forall\ \mu = \lfloor i/N_p \rfloor$, m, j in front of the frequency interleaver (Π_f). The interleaver scrambles the outputs and passes them onto the OFDM modulator. Finally, OFDM modulation includes the IDFT and inserts the guard interval between adjacent OFDM symbols.

For an OFDM-CDMA uplink transmission scenario, one can

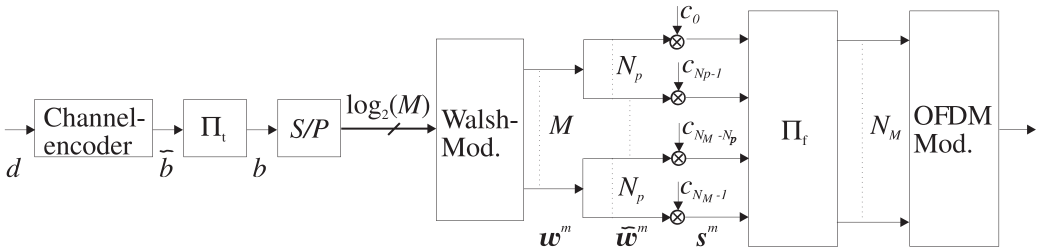

Figure 1: OFDM-CDMA transmitter with FEC and M-ary orthogonal Walsh modulation

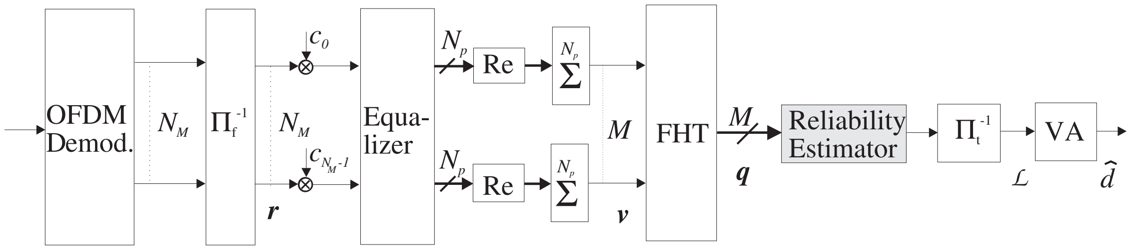

Figure 2: Ideal coherent OFDM-CDMA receiver for encoded M-ary orthogonal modulated signals

describe the channel for each user by an $N_M \times N_M$ diagonal matrix $\boldsymbol{H}^j$, where the diagonal elements H_i^j represent Rayleigh fading [4].

Applying CDMA technique, the *bit-error-rate* (BER) performance can be improved by an optimized compound design of channel coding and spreading. Here, the product between the overall bandwidth B and bit duration T_d is given by

$$B\,T_d = \underbrace{\frac{M}{\log_2(M)} \cdot \frac{N_p}{R_c}}_{P_G} \cdot \frac{T}{T-T_g}. \tag{3}$$

The first term of (3) describes the spreading of the Walsh modulation, the second one the ratio of simple spreading (N_p) over the code rate R_c of the channel code, whereas the third term takes into account the guard time T_g. For our comparisons, the product $B\,T_d$ remains unchanged due to the exploitation of the inherently overall spreading factor[1] P_G. Decreasing the BER can be achieved by sharing P_G between channel coding, M-ary Walsh modulation and simple spreading. The data rate $R = 1/T_d$ relates to binary modulation without channel coding ($R_c = 1$), and the bandwidth B is generally given.

2.2. Ideal coherent receiver

The coherent OFDM-CDMA receiver with perfectly known channel coefficients, called ideal coherent receiver, for encoded M-ary orthogonal Walsh modulated signals is shown in Fig. 2. Paying attention to J active users, the received signal after OFDM demodulation and deinterleaving (Π_f^{-1}) can be written as a sum of vectors $\boldsymbol{r}^{m,j}$,

$$\boldsymbol{r} = \sum_{j=0}^{J-1} \boldsymbol{r}^{m,j} + \boldsymbol{n} = \sum_{j=0}^{J-1} \boldsymbol{H}^j \boldsymbol{s}^{m,j} + \boldsymbol{n}, \tag{4}$$

with elements $r_i^{m,j} = s_i^{m,j} \cdot H_i^j, \quad i = 0, \ldots, N_M - 1$ and $m \in \{0, \ldots, M-1\}$. The vector $\boldsymbol{n}$ represents AWGN. After multiplication with the user specific code and equalization, the first part of despreading is obtained by subcorrelating N_p subcarriers. *Maximum likelihood detection* (MLD) of the Walsh symbols results in *maximum ratio combining* (MRC) equalization [7] (perfectly known channel coefficients are assumed) and is performed by the *Fast Hadamard Transform* (FHT) representing a low complexity ML decoding algorithm [5]. For the second decoding stage, i.e. the outer (convolutional) decoding, the application of a soft input Viterbi algorithm (VA) is optimum in the ML sense and requires reliability information. Therefore, the reliability estimator exploits the M decision variables to calculate the assigned (bit specific) log-likelihood ratio $\mathcal{L}$ which is fed to the Viterbi channel decoder after deinterleaving (Π_t^{-1}). Afterwards, we obtain the estimated data stream with bits $\hat{d}$.

[1] Being aware of a changed symbol duration $T = \log_2(M) R_c T_d$.

3. ML DECISION DECODING

In the following, we will derive the (bit specific) log-likelihood criterion for ML decoding of M-ary orthogonally demodulated signals. We consider coherent OFDM-CDMA demodulation with MRC equalization. The MAI is taken into account as AWGN and independent adjacent fading subcarriers ensured by perfect frequency interleaving (Π_f) are assumed.

3.1. Antipodal Transmission

Let us first assume binary antipodal transmission on a single carrier system with $b_\nu \in \{-1, +1\}$ denoting the ν-th transmitted encoded bit. Hard decision at the receiver leads to the binary encoded bit $z_\nu \in \{-1, +1\}$. For a memoryless discrete binary symmetric channel the likelihood criterion for ML channel decoding is given by [9]

$$\Phi_{ML}^{BSC} = \sum_\nu b_\nu \underbrace{z_\nu \log_2 \frac{P_\nu}{1-P_\nu}}_{\triangleq\ \mathcal{L}_\nu} \stackrel{!}{=} \max\,, \tag{5}$$

where $P_\nu \triangleq P(z_\nu = b_\nu)$ denotes the probability for correct decision. The likelihood criterion can efficiently be solved by the Viterbi algorithm (VA) where the log-likelihood ratio $\mathcal{L}_\nu = z_\nu\, L_\nu$ serves as a soft input signal, and, in this context, the absolute value $L_\nu = \log_2 \frac{P_\nu}{1-P_\nu}$ of the log-likelihood ratio can be interpreted as a reliability indicator for the hard decided code bit z_ν. In case of OFDM-CDMA with antipodal transmission, several approaches has been given for the downlink [4]. Derivations for the uplink result in approximately the same approaches except of individual channel transfer functions for each specific user.

3.2. M-ary orthogonal Walsh modulation

Let us now return to the problem of M-ary orthogonal modulation with MRC equalization. Here, it is also possible to describe the reliability estimator based on hard symbol decision. In the following, we summarize the derivation of the bit specific reliability estimator; detailed description has been presented in [10] as well as for noncoherent reception in a Direct-Sequence-CDMA system in [11, 12].

We start with the calculation of the conditional probability that $\boldsymbol{\omega}^n$ is the transmitted symbol under the observation of $\boldsymbol{q}$. Furthermore, one time instant is taken into account (subscripts are omitted). Applying Bayes law results in

$$P(\boldsymbol{\omega}^n|\boldsymbol{q}) \;=\; \frac{P(\boldsymbol{\omega}^n)}{p(\boldsymbol{q})} \cdot p(\boldsymbol{q}|\boldsymbol{\omega}^n)\,, \tag{6}$$

and for equiprobable transmitted symbols $\boldsymbol{\omega}^n$, we obtain $P(\boldsymbol{\omega}^n) = 1/M$. Assuming statistical independence of the components of $\boldsymbol{q}$, we can express (6) in the form [12]

$$P(\boldsymbol{\omega}^n|\boldsymbol{q}) = \frac{1/M}{p(\boldsymbol{q})} \prod_{m=0}^{M-1} r(q_m) \cdot \frac{R(q_n)}{r(q_n)}, \tag{7}$$

with $n \in \{0, \ldots, M-1\}$, $r(q_m) = p(q_m|\boldsymbol{\omega}^n \neq \boldsymbol{\omega}^m)$ and $R(q_n) = p(q_n|\boldsymbol{\omega}^n)$. In case of the considered coherent MRC equalization, $r(q_m)$ is the *probability density function* (pdf) of a zero mean Gaussian distribution with variance σ_q^2. Consequently, $R(q_n)$ denotes the pdf of a noncentral Gaussian distribution with same variance, where the mean is given by the expected value of the average channel energy over all subcarriers E_h, [10]. The calculation of

$$\frac{R(q_n)}{r(q_n)} = \exp\left(\frac{2\,q_n\,E_h - E_h^2}{2\sigma_q^2}\right) \tag{8}$$

requires the knowledge of the noise variance σ_q^2 and E_h.

These parameters can be estimated by exploiting the multicarrier technique as follows:

(*i*) *noise variance estimation*

Assuming that the decision variable with maximum magnitude, denoted by q_l, represents the true hypothesis, we obviously obtain an estimate of the noise power $\hat{\sigma}_q^2$ by the mean over $M-1$ components $q_m, m \neq l$, containing the false hypotheses:

$$\hat{\sigma}_q^2 = \frac{1}{M-1} \cdot \sum_{\substack{m=0\\ m\neq l}}^{M-1} q_m^2\,. \tag{9}$$

(*ii*) *average channel energy estimation*

If we assume statistically independent fading subcarriers and the same average energy on all subcarriers, one can estimate E_h by the mean over all subcarriers:

$$\hat{E}_h = \frac{1}{N_M} \sum_{i=0}^{N_M-1} |H_i|^2. \tag{10}$$

In order to determine the bit-specific reliability information, the symbols $\boldsymbol{\omega}^n$ are replaced by their corresponding bit patterns $\{b_0, \ldots, b_{\log_2(M)-1}\}$ in antipodal representation. Hence, the soft information of one bit can be calculated by adding all probabilities $P(\boldsymbol{\omega}^n|\boldsymbol{q})$, see (7), belonging to the considered bit. E.g., adding all probabilities belonging to '$b_0 = +1$' results in

$$P(b_0 = +1|\boldsymbol{q}) = \sum_{n=M/2}^{M-1} P(\boldsymbol{\omega}^n|\boldsymbol{q}).$$

This leads to the reliability information for the first bit of the Walsh symbol being '$b_0 = +1$',

$$L_{b_0} = \log_2 \frac{P(b_0 = +1|\boldsymbol{q})}{1 - P(b_0 = +1|\boldsymbol{q})}$$

which serves as a soft input for the Viterbi channel decoder. The other bits $b_1 \ldots b_{\log_2(M)-1}$ are treated equivalently.

4. SYSTEM COMPARISON

4.1. System description

Related to the European HIPERLAN/2 standardization, the following results are given for an indoor Rayleigh fading channel. The bandwidth is $B = 25\,\text{MHz}$ in the $5.2\,\text{GHz}$ range. A velocity of $v_0 = 0.5\,\text{m/s}$ results in a very low maximum Doppler frequency $f_{d,max}$ of about 9 Hz. Hence, long symbol durations T are possible providing the application of high level M-ary orthogonal modulation. Furthermore, due to the very low maximum Doppler frequency, the fading on each subcarrier is highly statistically dependent. To avoid bursty errors, sufficient frequency interleaving (Π_f) is required.

First, the indoor channel is modeled assuming sufficient frequency interleaving. Subsequently, we take into account an exponential delay profile with a maximum delay spread of $\tau_{max} \approx 350\,\text{ns}$ and a coherent bandwidth of approximately $2.8\,\text{MHz}$. I.e. the number of independent subcarriers is reduced dramatically.

The trade-off between coding and spreading is carried out for an unchanged product $B\,T_d$ (3) and is based on binary modulation without FEC with $N_2 = 64$ subcarriers. The guard time is chosen to be $T_g = 640\,\text{ns}$. This system design results in a data rate of $R = 312.5\,\text{kbit/s}$ for all cases of spreading and coding. A block interleaver (Π_t) with 32 rows and 18 columns is used to separate the channel code and the M-ary Walsh modulation. We employed two convolutional codes of rate $R_c = 1/2$ and $R_c = 1/3$, both with constraint length 7 [8]. For all cases, random frequency interleaving (Π_f) is applied, $J = 8$ users are taken into account, and perfect channel estimation as well as perfect synchronization is assumed for the considered user.

4.2. Simulation Results

Fig. 3 shows the BER of the M-ary Walsh modulation for $M = 16, 32$ by sharing P_G between simple spreading N_p and FEC of rates $R_c = 1,\ 1/2$ and $R_c = 1/3$; see (3). Independent of the M-ary modulation, the results indicate that choosing a higher amount of simple spreading N_p combined with less powerful FEC ($R_c = 1/2$) outperforms the scheme with lower amount N_p combined with more powerful FEC ($R_c = 1/3$). Note the gain of approximately 1 dB in $\bar{E}_b/N_0$ at a BER of 10^{-3} for the 32-ary modulation. This fact can be explained for $R_c = 1/3$ by the higher loss due to an unchanged guard

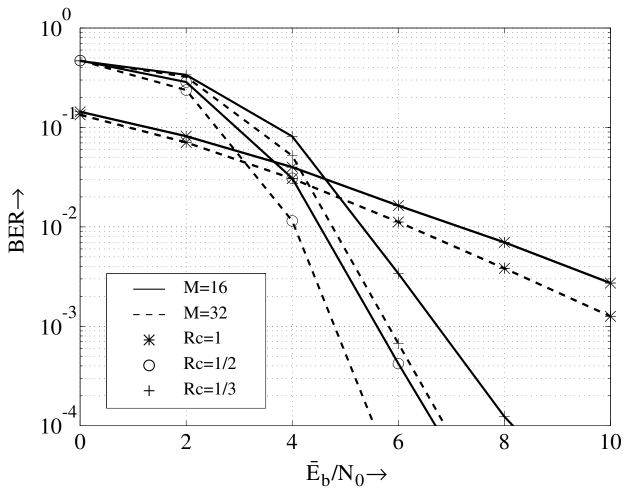

Figure 3: Sufficient frequency interleaving: Trade-off between channel coding, M-ary Walsh modulation and simple spreading for $M = 16,\ 32$ and $R_c = 1,\ 1/2,\ 1/3$.

time T_g, but a shorter symbol duration T, and an additional lower energy of the encoded bit in front of the Viterbi decoder[2]. Moreover, less number of subcarriers ($N_M = M \cdot N_p$) results in higher MAI in case of the used PN-codes. Hence, the more powerful FEC scheme is not able to compensate for the summarized losses.

We now focus on the trade-off between M-ary Walsh modulation and FEC (N_p remains unchanged due to an unchanged $B\,T_d$, see (3)). For example, 32-ary modulated signals with rate of code $R_c = 1/2$ yield a gain of $\bar{E}_b/N_0 \approx 2\,\mathrm{dB}$ in comparison with 16-ary Walsh modulation with $R_c = 1/3$ at BER=10^{-3}. This results from an increasing Euclidian distance (1) for higher values M and less MAI (higher number of subcarriers). To be summarized, the quality of the FHT output is decisive for the overall system performance.

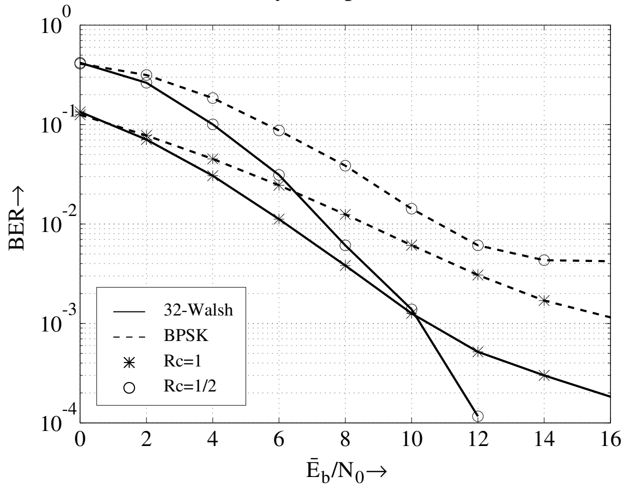

Figure 4: Insufficient frequency interleaving: Comparison of 32-ary Walsh modulation and BPSK, $R_c = 1,\ 1/2$.

The simulation results in Fig. 4 report the BER for the transmission over an indoor channel with coherence bandwidth of approximately 2.8 MHz. BPSK and 32-ary Walsh modulation are considered. In contrast to Fig. 3, there exists a significant loss for the Walsh modulation due to the statistical dependence of the adjacent subcarriers. This degradation is mainly caused by the insufficient frequency interleaving. There exists only a slight degradation due to the assumption of independent subcarriers for the derivations of the log-likelihood ratios.

[2] $\bar{E}_b$ denotes the bit energy at the output of the VA decoder.

Most important, the results shown by Fig. 4 express that BPSK even combined with FEC leads to an unacceptable BER for an OFDM-CDMA uplink transmission over an indoor Rayleigh fading channel. Moderate performance can only be achieved if M-ary orthogonal Walsh modulation is applied.

5. CONCLUSION

In this paper, we have investigated the concatenation of ML detected M-ary orthogonal Walsh modulated signals with ML Viterbi decoding for an OFDM-CDMA uplink transmission over a frequency-selective slowly Rayleigh fading channel. In particular, we have first evaluated the bit specific log-likelihood ratio for the ML Viterbi decoding exploiting the M hypotheses of the ML detected Walsh symbols. Assuming an unchanged bandwidth and data rate, we further analyzed the trade-off between coding and spreading to optimize the BER. Simulation results indicate to better strengthen the inner spreading consisting of M-ary orthogonal modulation and simple spreading with an inherently less powerful channel code. The results are compared with BPSK and emphasize that for an indoor scenario a moderate BER can only be achieved if the inner spreading is optimized, e.g. by applying M-ary orthogonal Walsh modulation. BPSK even combined with channel coding leads to an unacceptable BER.

6. REFERENCES

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