Modified Subspace Smoothing Technique for Multipath Direction Finding

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ABSTRACT

Recently a new subspace smoothing algorithm for estimation of directions of arrival (DOA) in coherent signal environment by using antenna arrays is presented. Unlike the existing forward and backward spatial smoothing algorithm, the new subspace smoothing algorithm is not restricted to uniformly linear arrays, and $M-1$ DOA's of direct path and multipath components can be estimated, where M is the number of the sensors. Unfortunately the steering vectors have to be estimated several times in that algorithm, which is computationally very intensiv. In order to reduce the computational cost but keep the advantages of the new idea, a modified version of the subspace smoothing algorithm will be presented in this paper. We will show that there is equivalence between the modified subspace smoothing algorithm and the well-known forward spatial smoothing algorithm. We will also show that the array aperture is extended virtually by using the modified subspace smoothing algorithm. Finally, simulation results will demonstrate the features of our modified algorithm.

1 INTRODUCTION

In mobile communication systems we often work with coherent signal environment. When a signal wavefront reflects off of a surface, the original wavefront and the reflected wavefront impinge both on the receiving array of sensors, although from different directions. If the delay difference between the two paths is sufficiently small, we can say, that the two wavefronts are coherent. Normally we have to deal with more than two coherent wave fronts. The existing high-resolution direction finding algorithms based on subspace techniques such as MUSIC [1] and ESPRIT [2] fail in coherent environment. Since the multipath components are coherent with the direct path, the signal eigenvectors fail to span the signal subspace and DOA estimation is not possible. However, the loss of rank in the signal subspace can be restored by using the forward and backward spatial smoothing technique [3], which is the mostly used DOA estimation technique for problems with coherent signals. However, only up to $2M/3$ DOA's of direct path and multipath components can be estimated, where M is the number of sensors, and only uniformly linear array can be used. Another technique for solving the coherent problem is the maximum likelihood approach [4], which is unfortunately computationally very intensive.

Recently a new subspace smoothing algorithm for DOA estimation in coherent signal environment has presented in [5], which is not restricted to uniformly linear array and up to $M-1$ DOA's can be estimated. In that algorithm, received signals at different time instances are used. Due to the large change of the wavefronts in phase for a slight move of the mobile user, the steering vectors at different time instances can be used to restore the rank of the signal subspace, so that the existing high-resolution subspace methods such like MUSIC [1] and ESPERIT [2] can be used for DOA estimation. Unfortunately, the steering vectors have to be estimated first for those time instances in that paper, which is computationally very intensiv.

In order to reduce the computational cost a modified version of the subspace smoothing algorithm will be presented in this paper. In the modified version, a similar procedure as in the forward spatial smoothing algorithm [3] is used to restore the rank of the signal subspace. We will show in this paper that there is equivalence between the two algorithms and by using the modified subspace smoothing algorithm the array aperture is extended virtually.

This paper is organized as follows. In section 2 the data model will be described. In section 3 the modified subspace smoothing algorithm will be presented. In section 4 the features of the new smoothing algorithm will be studied and in section 5 simulation results will be presented. The conclusion follows in section 6.

2 DATA MODEL

Assume that we have P wavefronts from q independent narrow band sources impinging on an M-element array. The received signals of the M array elements can be described as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{A}$ is an $M \times P$ unknown steering matrix, $\mathbf{s}(t)$ is a $P \times 1$ wavefront vector and $\mathbf{n}(t)$ is the independent noise vector, which can be Gaussian and non-Gaussian. Since there are only q independent sources among P wavefronts, we can express the received signal $\mathbf{x}(t)$ in terms of independent sources:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{Q}\mathbf{u}(t) + \mathbf{n}(t) = \mathbf{B}\mathbf{u}(t) + \mathbf{n}(t) \tag{2}$$

where $\mathbf{u}(t)$ is a $q \times 1$ vector with the q independent narrow band sources at time t and $\mathbf{Q}$ is a $P \times q$ matrix with the complex attenuation vectors of the sources.

$$\mathbf{Q} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{c}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{c}_q \end{bmatrix}, \tag{3}$$

here, $\mathbf{c}_i$ is the complex attenuation vector of the ith source $(1 \leq i \leq q)$. As defined in equation (2), $\mathbf{B}$ is equal $\mathbf{AQ}$, which is called the effective steering matrix.

Now if the mobile users move slightly, due to the relatively large distance between the mobile users and base station, the DOA's of all wavefronts do not vary rapidly, but the complex attenuation vector $\mathbf{c}_i$ changes more rapidly, especially the phase [5]. From (3) we can see that $\mathbf{Q}$ would change rapidly, which implies that $\mathbf{B}$ also changes rapidly.

In this paper received signals of a moving mobile user at m time instances $(t = t_0, t = t_0 + T_i, \cdots, t = t_0 + (m-1) * T_i))$ will be considered, in which time interval $(t_0 \leq t \leq t_0 + (m-1) * T_i)$ the DOA's of all wavefronts can be assumed to be unchanged, where t is the continuous time, t_0 is an arbitrary start point of time and T_i is the distance between two time instances. At each time instance the received signals can be written as:

$$\begin{aligned} \mathbf{x}^j(t) &= \mathbf{A} \begin{bmatrix} \mathbf{c}_1^j & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{c}_2^j & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{c}_q^j \end{bmatrix} \mathbf{u}(t) + \mathbf{n}^j(t) && (4) \\ &= \mathbf{A}\mathbf{Q}^j\mathbf{u}(t) + \mathbf{n}^j(t) && (5) \\ &= \mathbf{B}^j\mathbf{u}(t) + \mathbf{n}^j(t), \qquad 1 \leq j \leq m && (6) \end{aligned}$$

3 MODIFIED SUBSPACE SMOOTHING ALGORITHM

Using received signals of M sensors at m time instances with a data length of T_i/T_s each time, where T_s is the symbol duration, we will try here to estimate DOA's of the P wavefronts, where $P \leq M, m$. To do that, the covariance matrix of the received signals at each time instance is computed $(1 \leq j \leq m)$:

$$\begin{aligned} \mathbf{R}_{\mathbf{X}^j} &= \mathcal{E}\{\mathbf{x}^j(\mathbf{x}^j)^H\} && (7) \\ &= \mathbf{A}\mathbf{Q}^j\mathbf{R}_{\mathbf{u}}(\mathbf{Q}^j)^H(\mathbf{A})^H + \sigma_n^2\mathbf{I} && (8) \end{aligned}$$

where $\mathbf{R}_{\mathbf{u}}$ is the $q \times q$ covariance matrix of the q independent sources, σ_n^2 is the noise power and $\mathbf{I}$ is the $M \times M$ identity matrix.

The rank of $\mathbf{R}_{\mathbf{u}}$ is q, and $\mathbf{Q}^j$ is a $P \times q$ matrix with $P \geq q$ (see section 2). That implies that the rank of the $P \times P$ matrix $\mathbf{Q}^j\mathbf{R}_{\mathbf{u}}(\mathbf{Q}^j)^H$ is q. Since A has the size $M \times P$, so the covariance matrix $\mathbf{R}_{\mathbf{X}^j}$ has also rank q, if $M > P, q$. In our data model we have P wavefronts, that means that the rank of the signal subspace is P, so the covariance matrix $\mathbf{R}_{\mathbf{X}^j}$ do not span the signal subspace and can not be used in any subspace based algorithm for DOA estimation.

In order to restore the rank of the signal subspace, a similar procedure as in the forward spatial smoothing algorithm [3] will be used. In the forward spatial smoothing algorithm an uniformly linear array with M sensors is divided into L subarrays with M_0 elements in each subarray, where the L subarrays are shifted versions of each other. Using the average of the covariance matrices of all subarrays, the rank of the signal subspace can be restored, if $L, M_0 > P$.

In our case, we note that covariance matrices of the received signals at m time instances have been computed in (8). Taking the average of them, we can get:

$$\begin{aligned} \mathbf{R}_{\mathbf{X}} &= \frac{1}{m}\sum_{j=1}^{m}\mathbf{R}_{\mathbf{X}^j} && (9) \\ &= \mathbf{A}(\frac{1}{m}\sum_{j=1}^{m}\mathbf{Q}^j\mathbf{R}_{\mathbf{u}}(\mathbf{Q}^j)^H)\mathbf{A}^H + \sigma_n^2\mathbf{I} && (10) \\ &= \mathbf{A}\tilde{\mathbf{R}}_{\mathbf{u}}\mathbf{A}^H + \sigma_n^2\mathbf{I} && (11) \end{aligned}$$

where $\tilde{\mathbf{R}}_{\mathbf{u}} = \frac{1}{m}\sum_{j=1}^{m}\mathbf{Q}^j\mathbf{R}_{\mathbf{u}}(\mathbf{Q}^j)^H$. Since $\mathbf{Q}^j$ changes rapidly for each time instance, the average of the PxP matrices $\mathbf{Q}^j\mathbf{R}_{\mathbf{u}}(\mathbf{Q}^j)^H$ for $1 \leq j \leq m$ would have rank P, if $m \geq P$. That means that $\tilde{\mathbf{R}}_{\mathbf{u}}$ would have the full rank of the signal subspace. Since $rank(\mathbf{R}_{\mathbf{X}}) = \min(rank(\tilde{\mathbf{R}}_{\mathbf{u}}), rank(\mathbf{A}))$ and $rank(\mathbf{A}) = P$, if $M > P$, so the averaged matrix $\mathbf{R}_{\mathbf{X}}$ would have the full rank, too. Using $\mathbf{R}_{\mathbf{X}}$ in a subspace based DOA estimation algorithm such as MUSIC [1], maximum $M-1$ DOA's can be estimated. Note that no restriction to uniformly linear array is made here.

4 VIRTUAL EXTENSION OF ARRAY APERTURE

As shown in section 3, similar procedure as in the forward spatial smoothing algorithm is used in the modified subspace smoothing algorithm to restore the rank of the signal subspace. In this section we will show that in certain cases the two algorithms are equivalent, except that the array aperture is extended virtually by using the modified subspace smoothing algorithm.

With a shifting of one element for each subarray the received signals of the subarrays in the forward spatial smoothing algorithm can be expressed as:

$$\mathbf{x}^l(t) = \mathbf{A}_{M_0}\mathbf{W}^{l-1}\mathbf{s}(t) + \mathbf{n}^l(t), \quad 1 \le l \le L \qquad (12)$$

where $\mathbf{A}_{M_0}$ is the steering matrix of an M_0-element uniformly linear array,

$$\mathbf{W}^{l-1} = \begin{bmatrix} e^{-j\pi\sin\theta_1} & 0 & \cdots & 0 \\ 0 & e^{-j\pi\sin\theta_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-j\pi\sin\theta_P} \end{bmatrix}^{l-1},$$

and $L + M_0 - 1 = M$.

Let's compare the equations (12) and (5). In cases where only direct paths of the q narrow band sources exist, i.e. $P = q$ and $\mathbf{s}(t) = \mathbf{u}(t)$, $\mathbf{Q}^j$ in (4) is a $P \times P$ identity matrix at $j = 1$. At $j = 2$, the mobile user has moved a small distance y. Due to the large distance between the mobile user and the base station we can assume that all path lengths of the q direct paths have changed y. So we have

$$\mathbf{Q}^2 = \begin{bmatrix} e^{-j2\pi\frac{y}{\lambda}\sin\theta_1} & 0 & \cdots & 0 \\ 0 & e^{-j2\pi\frac{y}{\lambda}\sin\theta_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-j2\pi\frac{y}{\lambda}\sin\theta_P} \end{bmatrix},$$

where λ is the wave length. With growing j, the phase of $\mathbf{c}_j^j$ can be assumed as changing linear with the movement of the mobile user,

$$\mathbf{Q}^j = \begin{bmatrix} e^{-j2\pi\frac{y}{\lambda}\sin\theta_1} & 0 & \cdots & 0 \\ 0 & e^{-j2\pi\frac{y}{\lambda}\sin\theta_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-j2\pi\frac{y}{\lambda}\sin\theta_P} \end{bmatrix}^{j-1}.$$

If $y = 0.5\lambda$, $\mathbf{Q}^j$ is identical with $\mathbf{W}^{l-1}$ in (12). So each movement of the mobile user can be interpreted as a shift of the array of one element. In such cases using the average of the covariance matrices of all subarrays is equivalent to using the average of the covariance matrices of all time instances; hence, the forward spatial smoothing algorithm is equivalent to the modified subspace smoothing algorithm. However, there are only M_0 sensors in each subarray in the forward spatial smoothing algorithm, but M sensors are used at each time instance in the modified subspace smoothing algorithm, where $M > M_0$. By collecting received signals at m time instances, the M-element array can be considered as a virtually extended $M + m - 1$-element array.

5 SIMULATION RESULTS

In the experiments an 8-element array is used, which can be uniformly linear and circular. Two QPSK narrow band sources with different DOA's are impinging on the array. White Gaussian noise with SNR $= 10dB$ is assumed for each wavefront. For comparison the forward–backward spatial smoothing algorithm is chosen, which works better than the forward spatial smoothing algorithm [3].

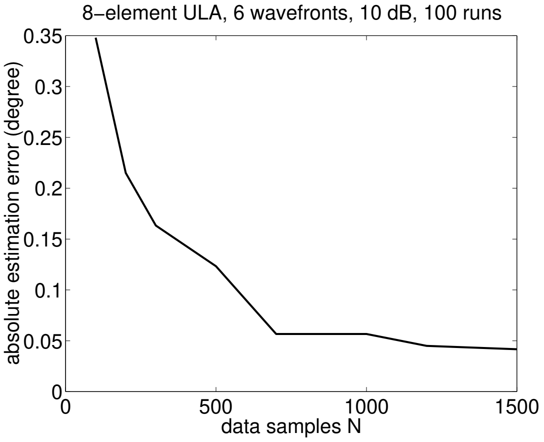

Figure 1: Absolute estimation error of estimated DOA's, plotted as a function of the data samples N.

Experiment 1: In this experiment uniformly linear array is used. The first QPSK source approaches the array at 40^0, 68^0 and -10^0. The corresponding attenuation factors are $\{0.2e^{-j1}, 1, 0.8e^{j0.5}\}$. The second QPSK source approaches the array at 50^0, 10^0, -30^0 and the corresponding attenuation factors are $\{0.4e^{j0.7}, 1, 0.2e^{-j0.3}\}$. So we have totally 6 wavefronts and 6 DOA's have to be estimated. Since $6 > 2/3M$, where $M = 8$, the forward and backward spatial smoothing algorithm doesn't work. Using our modified subspace smoothing algorithm the 6 DOA's can be estimated very well.

In figure 1 100 monte-carlo runs are used for computation of each point. As absolute estimation error the averaged absolute estimation errors of all DOA's over all monte-carlo runs is taken.

$$\Delta = \frac{1}{P}\frac{1}{100}\sum_{i=1}^{100}\sum_{j=1}^{P} |\mathrm{DOA}(j) - \hat{\mathrm{DOA}}^i(j)|, \qquad (13)$$

here Δ is the absolute estimation error, $P = 6$, $\mathrm{DOA}(j)$ is the exact DOA of the jth wavefront and $\hat{\mathrm{DOA}}^i(j)$ is the estimated DOA of the jth wavefront in the ith monte-carlo run. If the absolute estimation error of a monte-carlo run is bigger than 10^0, we say that the using algorithm fails to estimate the DOA's in this run and do not compute it in equation (13). In this experiment only 9 runs (out of 100) failed with 100 data samples.

Experiment 2: In order to enable comparison between our algorithm and the forward–backward spatial smoothing algorithm, only 4 wavefronts of the two sources from experiment 1 are considered here. The DOA's of the 4 wavefronts are 40^0, 68^0, 50^0 and 10^0. The corresponding attenuation factors are the same as in experiment 1.

From figure 2 we can see that under same conditions

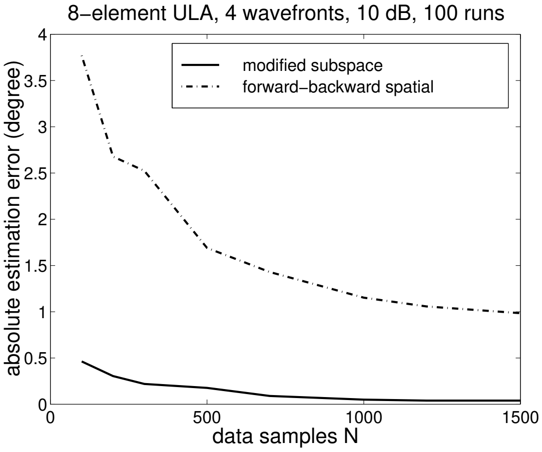

Figure 2: Absolute estimation error of estimated DOA's, plotted as a function of the data samples N.

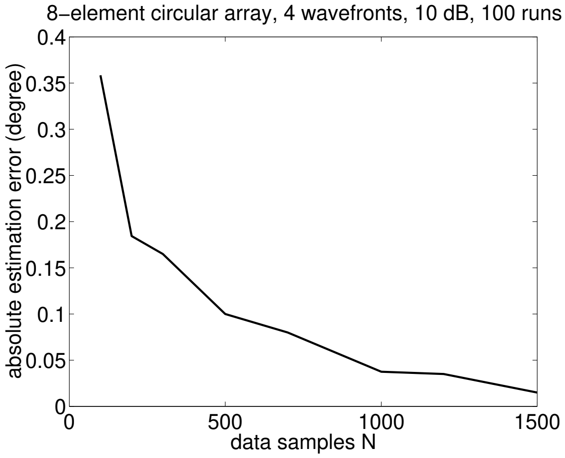

Figure 3: Absolute estimation error of estimated DOA's, plotted as a function of the data samples N.

our algorithm works much better than the forward–backward spatial smoothing algorithm. Comparing the failure rates of the two algorithms (TABLE 1), we can see that the stability of our algorithm is also better.

TABLE I: Failure Rates of the two smoothing algorithms in Experiment 2

data samples N	100	200	300	> 300
modified subspace	5	0	0	0
forward-backward spatial	10	6	4	0

Experiment 3: In this experiment we will demonstrate that our algorithm is not restricted to uniformly linear array. For this experiment the same scenario as in experiment 2 is used, except an 8-element circular array is taken.

Figure 3 shows that using our algorithm DOA's of coherent wavefronts can be estimated very well even with a non uniformly linear array. The failure rate in this experiment is 17% with 100 data samples and only 1% with 200 data samples.

6 CONCLUSION

In this paper a new DOA's estimation algorithm for coherent signal wavefronts is presented,which is a modified version of the existing subspace smoothing algorithm. In the modified subspace smoothing algorithm a similar procedure as in the forward spatial smoothing algorithm is used to restore the rank of the signal subspace. Unlike the forward spatial smoothing algorihm, the modified subspace smoothing algorithm is not restricted to uniformly linear array and maximum $M-1$ DOA's of the coherent wavefronts can be estimated. Under same conditions we have shown in our simulations in section 5 that better results can be obtained with the modified subspace smoothing algorithm than with the forward–backward spatial smoothing algorithm. Since in the modified subspace smoothing algorithm the steering vectors do not need to be estimated first, the computational cost is reduced strongly in comparison to the original subspace smoothing algorithm.

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