Impulse truncation for wireless OFDM systems

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Abstract—**In the present paper we are going to discuss some fundamentals about exceeding the guard interval of an OFDM system. One way to reduce the effects of ISI and ICI is pre-equalizing. With knowledge of the current channel constellation, we will introduce methods to perform a time domain pre-equalizing-filter. Therefore, a channel estimation scheme considering HIPERLAN/2 system parameters will be proposed. The functionality of the introduced time-domain pre-equalization-filter will be proved by simulation results.**

Keywords— **OFDM, muticarrier transmission, pre-equalizer, impulse truncation**

I. Introduction

In the last few years many research has been done on mobile OFDM techniques, because OFDM is intended to be used for the new wireless LAN standards IEEE802.11a and HIPERLAN type 2 [1], [2]. Furthermore OFDM has been fixed for broadcast standards like DAB and DVB-T and for wired xDSL and powerline technology. Here, we mainly concentrate on using OFDM for wireless LAN (WLAN) in 5 GHz band. One basic aspect of OFDM are orthogonal subcarriers. To prevent inter-symbol-interference (ISI) and inter-carrier-interference (ICI) a guard interval, containing a cyclic prefix, is included. The guard length must be equal or greater than the maximum relative channel delay. In section II some properties of the guard interval are shown. It is well known that ISI and ICI increase by exceeding the guard interval length. Some examples, considering situations and scenarios where this effects may affect, are as well presented in section II. In section III some possibilities of impulse truncation are presented. Assuming ideal knowledge of the channel impulse response a time-domain pre-equalization-filter is derived. But the estimation of the channel impulse response is a fundamental problem when computing the equalizers coefficients. Considering the preamble structure of HIPERLAN/2, a channel estimation scheme is proposed in section IV. In section V some performance simulations of a HIPERLAN/2 system are shown.

II. The guard interval of OFDM

In future indoor communication systems, OFDM has established to be the basic modulation scheme of the physical layer (PHY). Assuming complex symbols $d_n(i)$ on each of N subcarriers (n denotes the subcarrier index of the i^{th} OFDM symbol), the complex OFDM baseband signal is

$$s(i,k) = \sum_n d_n(i) \cdot e^{j2\pi \frac{nk}{N_f}}, \tag{1}$$

where N_f denotes the used FFT length. The time index k is defined by

$$k \in \{-N_g, ..., 0, ..., N_f - 1\}. \tag{2}$$

It follows that one OFDM symbol consists of $N_g + N_f$ samples where N_g denotes a cyclic prefix as guard interval. Considering the sample rate f_a, the guard length is $T_g = N_g/f_A$, the FFT window length is $T_s = N_f/f_a$, and the total symbol duration is $T = (N_g + N_f)/f_a$.
In common case every received symbol is influenced by all subcarriers of all OFDM symbols. If we assume the maximum channel delay time is shorter than one OFDM symbol we only have ISI between the actual (i) and the previous ($i-1$) OFDM symbol. The received symbols result to

$$\hat{d}_n(i) = \sum_{\mu=0}^{N-1} C_{n,\mu}^{(0)} \cdot d_\mu(i) + \sum_{\nu=0}^{N-1} C_{n,\nu}^{(-1)} \cdot d_\nu(i-1) \tag{3}$$

where $\hat{d}_n(i)$ denotes the received symbol of the n^{th} subcarrier and C marks the frequency domain channel coefficient. Assuming time invariant conditions, the channel coefficients do not dependent from i, so $C_{\mu,n}^{(0)}$ describes ICI between the μ^{th} and n^{th} subcarrier of the i^{th} OFDM symbol. The second part of (3) is the ISI described by $C_{\nu,n}^{(-1)}$. For complete symbol detection a very complex equalizer has to be used. But, if the overall channel impulse response is not longer than the guard interval ($\tau_{\max} \le T_g$), the channel influence can be described by a simple complex multiplication in frequency domain

$$\hat{d}_n(i) = C_{n,n}^{(0)} \cdot d_n(i). \tag{4}$$

All other channel coefficients of $C_{\mu,n}^{(0)}$ and $C_{\nu,n}^{(-1)}$ are zero in case of ISI and ICI free transmission. This is a very important aspect, because one advantage of OFDM is an easy equalization by multiplying $1/(C_{n,n}^{(0)})$ on each subcarrier separately. Thus, the guard time has to be longer than the maximum relative channel delay. A negative aspect of the guard interval is the bandwidth efficiency decreasing with the guard length. Since the guard interval contains no (new) information, the efficiency decreases by $(1 - T_g/T)$. Thus, developing an OFDM system must be finding a compromise between efficiency and additional ISI/ICI noise. To minimize the ISI/ICI influence and the bandwidth efficiency loss, decreasing the channel impulse response length may be a possible solution.

III. Impulse truncation for OFDM

It is a well known fact, mentioned in section II, that reducing the negative influence of very long channel impulse responses is an important criterion for system development. Before we start presenting the impulse truncation algorithm, we would like to introduce a simplified OFDM block diagram including the pre-equalizer in figure 1.

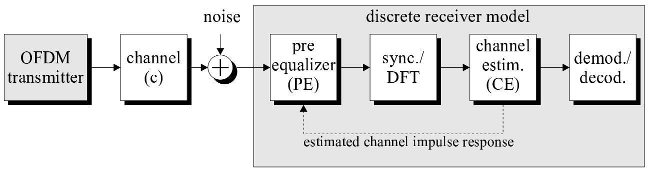

Fig. 1. block diagram of OFDM system

After leaving the transmitter, the OFDM signal passes a very slow fading mobile radio channel, represented by a discrete block ‘channel’. All characteristics of necessary components like digital/analog conversion (DAC), baseband to bandpass conversion, etc. are described by one discrete vector

$$\mathbf{c} = [\mathbf{c}(\mathbf{0}), \mathbf{c}(\mathbf{1}), \mathbf{c}(\mathbf{2}), ..., \mathbf{c}(\mathbf{m})]^*, \tag{5}$$

where * denotes the transjugation operator. We assume that $m > N_g$. If not, impulse truncation is unnecessary because the guard interval can eliminate the influence of multipath propagation completely. All other noise influences are modelled by additive white Gaussian noise (AWGN). The time domain pre-equalizer for impulse truncation is placed before synchronization (positioning the FFT window) and DFT computation. In case of using a FIR filter, the pre-equalizer is described by

$$\mathbf{e} = [\mathbf{e}(\mathbf{0}), \mathbf{e}(\mathbf{1}), \mathbf{e}(\mathbf{2}), ..., \mathbf{e}(\mathbf{p})]^* \,. \tag{6}$$

Further on it is well known, that the convolution $\mathbf{c_e} = \mathbf{c} * \mathbf{e}$ yields to an impulse length $(m + p + 1)$ if all elements of $\mathbf{c}$ and $\mathbf{e}$ are non-zeros

$$\mathbf{c_e} = [\mathbf{c_e}(\mathbf{0}), \mathbf{c_e}(\mathbf{1}), \mathbf{c_e}(\mathbf{2}), ...\mathbf{c_e}(\mathbf{p} + \mathbf{m} - \mathbf{1})]^* \,. \tag{7}$$

In figure 1 a block ‘channel estimation’ has been placed which will be considered in section IV. Here, we assume ideal knowledge of the channel impulse response.

Considering the non-ideal characteristics of FIR equalizers, we developed pre-equalizers based on IIR filters. But in case of noise and non-ideal channel knowledge, the performance of the recursive pre-equalized OFDM system deteriorates, so we focus our further research activities on FIR filter development.

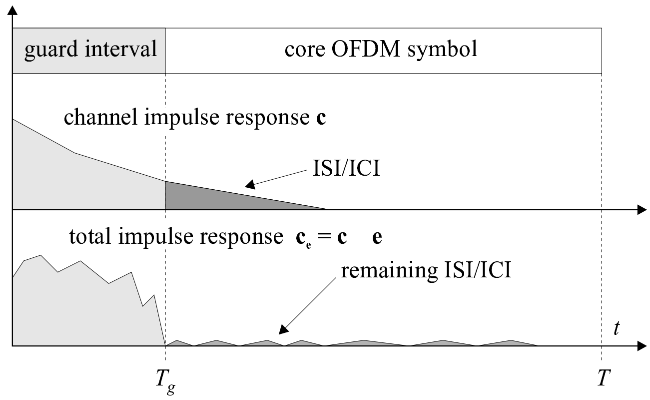

Fig. 2. impulse response before and after pre-equalizing

Figure 2 shows schematically the channel impulse response before and after applying the pre-equalizer. Concerning the new impulse response $\mathbf{c_e}$, we have to minimize the power of all elements $c_e(k)$, where $k > N_g$. Using known MMSE algorithms for developing T-equalizers [4], we have to create a convolution matrix $\mathbf{F}$ of size $((m + p + 1) \times (p + 1))$

$$\mathbf{F} \cdot \mathbf{e} = \mathbf{c_e} \,. \tag{8}$$

For $\mathbf{c_e}$, following assumptions are made:

$$c_e(k) = \begin{cases} 1 & k = 0 \\ \star & 0 < k \leq N_g \\ 0 & k > N_g \end{cases} . \tag{9}$$

The first condition $c_e(0) = 1$ prevents a zero solution; of course any non-zero complex value can be assumed for $c_e(0)$. Since all elements indexed by $k = \{1, ..., N_g\}$ and signed by $\star$ are non-relevant, we may reduce the conditions for $\mathbf{c_e}$ by excluding all arbitrary elements:

$$c_{re}(k) = \begin{cases} 1 & k = 0 \\ 0 & k \geq 1 \end{cases} . \tag{10}$$

Considering reducing (9) to (10), a new reduced convolution matrix $\mathbf{F_r}$ of size $((p + m + 1 - N_g) \times (p + 1))$ can be defined

$$\mathbf{F_r} \cdot \mathbf{e} = \mathbf{c_{re}} \,, \tag{11}$$

where $\mathbf{F_r}$ is equal to $\mathbf{F}$ except for the lines with index $\{1, ..., N_g\}$. With (10), we have to find an equalizer $\mathbf{e}$ fulfilling

$$\mathbf{F_r} \cdot \mathbf{e} = \mathbf{d} + \delta \,. \tag{12}$$

The destination vector

$$\mathbf{d} = [\mathbf{1}, \mathbf{0}, ..., \mathbf{0}]^* \tag{13}$$

results from (10) and the error vector

$$\delta = [\delta(\mathbf{0}), \delta(\mathbf{1}), \delta(\mathbf{2}), ..., \delta(\mathbf{p} + \mathbf{m} - \mathbf{N_g})]^* \qquad (14)$$

describes the remaining error, which has to be minimized. A well known solution for equations like (11) is the minimum mean square error (MMSE) technique [4], [5], so the equalizer can be computed by

$$\mathbf{e} = (\mathbf{F_r}^* \cdot \mathbf{F_r})^{-1} \cdot \mathbf{F_r}^* \cdot \mathbf{d}\,. \qquad (15)$$

Just channel knowledge to create $\mathbf{F_r}$ and the trivial destination vector $\mathbf{d}$ are required to compute the FIR equalizer coefficients. In case of additional noise (AWGN), the pre-equalizer will raise the noise power, so optimal MMSE solutions consider the effective signal to noise ratio (SNR)

$$\mathbf{e} = (\mathbf{F_r}^* \cdot \mathbf{F_r} + \gamma_{\mathbf{pe}}^{\mathbf{2}} \cdot \mathbf{I})^{-1} \cdot \mathbf{F_r}^* \cdot \mathbf{d}\,, \qquad (16)$$

with γ_{pe}^2 = 1/SNR and the identity matrix $\mathbf{I}$. To demonstrate the functionality of (15), a short example is introduced. We assume a channel impulse response $c(k)$ with length $m + 1 = 15$ shown in figure 3a. Thus, $c(k)$ has 15 non-zero coefficients. Further on, we assume a guard interval of length $N_g = 8$.

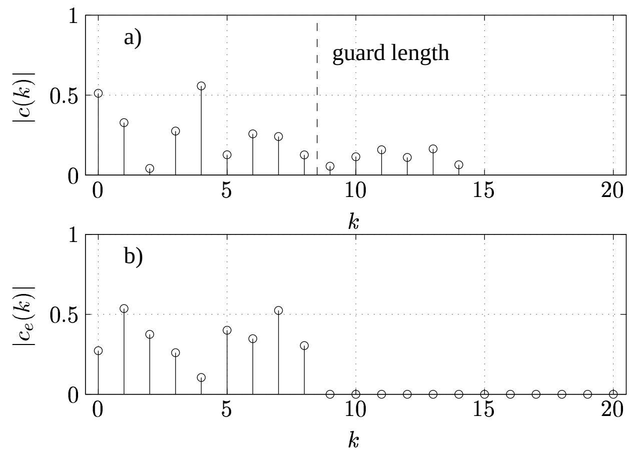

Fig. 3. a) channel impulse response c(k); b) convolution of channel impulse response and pre-equalizer

After pre-equalizing, all unwanted coefficients for $k > 8$ have to be eliminated. In this example, no extra noise is added ($\gamma_{pe}^2 = 0$). The convolution of the channel $c(k)$ and the computed FIR pre-equalizer $e(k)$ with order $p = 31$ has been printed in figure 3b. It can be clearly seen, that the result of pre-equalizing contains quasi-zero coefficients only for $k > 8$.

Figure 4 demonstrates some statistical analyses of the impulse truncation algorithm. The total power delay profiles of truncated channel impulse responses are shown. This figure is based on more than 5000 random channels with an exponential power delay profile and average delay spread $\Delta\tau = 250$ns. The remaining ISI/ICI decreases by raising the pre-equalizers size p. Again, no noise and ideal channel knowledge have been assumed.

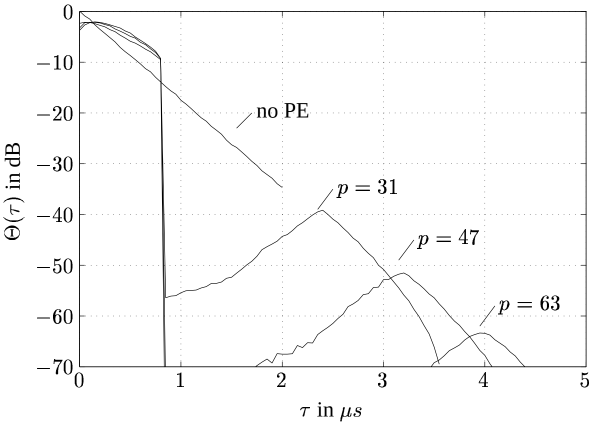

Fig. 4. power delay spectrum of the total channel impulse response ($c_e(k), \Delta\tau = 250$ns) with different pre-equalizer lengths

IV. Impulse truncation with channel estimation for a HIPERLAN/2 system

In section III a MMSE solution for pre-equalizer development has been shown. The central equation (16) is based on channel knowledge. In practical systems, a channel estimation in time or frequency domain is required for coherent demodulation.

The indoor communication standard HIPERLAN/2 contains two training-symbols in order to perform frequency domain channel estimation. To prevent ISI/ICI-disturbance in case of insufficient guard length, the training-symbols are provided with a double length guard interval (GI2). In figure 5a the training structure of a HIPERLAN/2 burst is shown [1], [2].

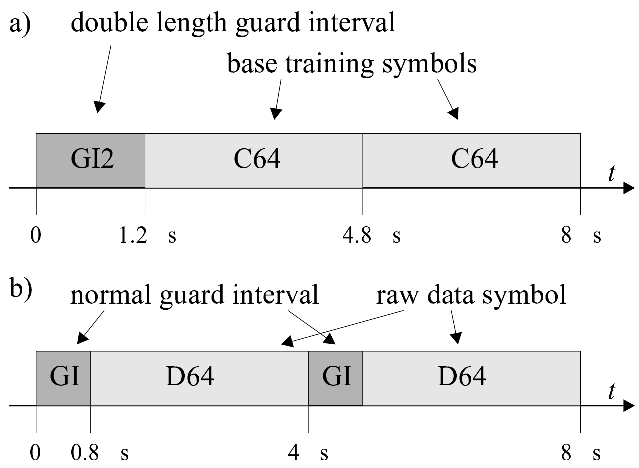

Fig. 5. a) training symbols and double length guard interval; b) "normal" data OFDM symbols

The two equal base training symbols (C64) consist of

each 64 samples (20 MHz sample frequency). With a guard length of 32, it is possible to estimate channels up to length 32 without ISI/ICI-disturbance. If the estimated channel impulse response is longer than the guard interval length of normal data OFDM symbols, the standard provides a guard length of 16 samples (800 ns), we have to compute a pre-equalizer (3.6) to truncate the effective impulse length of channel and pre-equalizer filter.
The training structure of HIPERLAN/2 allows to estimate 52 subcarrier coefficients $\hat{C}_n$. In case of f_a = 20 MHz (FFT length 64), it is not possible to compute the estimated channel impulse response $\tilde{c}(k)$ by

$$\tilde{\mathbf{c}} = \mathrm{IDFT}\left\{\tilde{\mathbf{C}}\right\} \tag{17}$$

$$\tilde{\mathbf{c}} = \mathbf{W_{IDFT}} \cdot \tilde{\mathbf{C}} \tag{18}$$

due to unknown frequencies ($\mathbf{W_{IDFT}}$ is the IDFT matrix). Figure 6 shows the HIPERLAN/2 training structure in frequency domain.

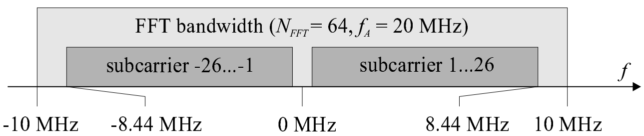

Fig. 6. training symbols in frequency domain

To compute the channel impulse response, we assume a maximum channel impulse response length $m+1$ (=32), and q (=52) is the number of known subcarrier coefficients. We define the vector $\tilde{\mathbf{C}}^{\mathbf{k}}$ containing all known coefficients, and the vector $\tilde{\mathbf{C}}^{\mathbf{u}}$ contains all unknown channel coefficients. In time domain we define $\tilde{\mathbf{c}}^{\mathbf{1}}$ as the first $m+1$ elements of $\tilde{c}(k)$, and $\tilde{\mathbf{c}}^{\mathbf{0}}$ includes all coefficients of the impulse response supposed to be zero. By resorting $\tilde{\mathbf{c}}$ and $\tilde{\mathbf{C}}$ in (17), the IDFT can be written as

$$\begin{bmatrix} \tilde{\mathbf{c}}^{\mathbf{1}} \\ \tilde{\mathbf{c}}^{\mathbf{0}} \end{bmatrix} = \begin{bmatrix} \mathbf{W_{11}} & \mathbf{W_{12}} \\ \mathbf{W_{21}} & \mathbf{W_{22}} \end{bmatrix} \cdot \begin{bmatrix} \tilde{\mathbf{C}}^{\mathbf{k}} \\ \tilde{\mathbf{C}}^{\mathbf{u}} \end{bmatrix} \tag{19}$$

The submatrixes $\mathbf{W_{11}}$,$\mathbf{W_{12}}$,$\mathbf{W_{21}}$, and $\mathbf{W_{22}}$ contain all resorted twiddle factors $e^{j2\pi\frac{nk}{N}}$ of the IDFT matrix. Considering, $\tilde{\mathbf{c}}^{\mathbf{0}}$ should contain only zeros, we may develop the wanted channel impulse response $\tilde{\mathbf{c}}^{\mathbf{1}}$ by

$$\tilde{\mathbf{c}}^{\mathbf{1}} = \left(\mathbf{W_{11}} - \mathbf{W_{12}} \cdot \mathbf{W_{22}}^{-\mathbf{1}} \cdot \mathbf{W_{21}}\right) \cdot \tilde{\mathbf{C}}^{\mathbf{k}} \tag{20}$$

$$= \mathbf{W_{1k}} \cdot \tilde{\mathbf{C}}^{\mathbf{k}} \tag{21}$$

with $\mathbf{W_{22}}^{-\mathbf{1}}$ is the pseudo inverse of $\mathbf{W_{22}}$. By precomputing the $((m+1) \times q)$ matrix $\mathbf{W_{1k}}$, the runtime costs are only a vector/matrix multiplication (21). In practical scenarios the real channel impulse response may be longer than $m+1$ and additive noise falsifies the estimated channel coefficients. Here, the assumption that $\tilde{\mathbf{c}}^{\mathbf{0}}$ contains only zeros will cause numerical overflows due to $\mathbf{W_{22}}^{-\mathbf{1}}$. To prevent these effects, a noise considering factor γ^2 has been introduced:

$$\mathbf{W_{22}}^{-\mathbf{1}} = \begin{cases} \mathbf{W_{22}}^{*} \cdot \left(\mathbf{W_{22}} \cdot \mathbf{W_{22}}^{*} + \gamma^{\mathbf{2}} \cdot \mathbf{I}\right)^{-\mathbf{1}} \\ \qquad \text{if} \quad m+1 > q \\ \left(\mathbf{W_{22}}^{*} \cdot \mathbf{W_{22}} + \gamma^{\mathbf{2}} \cdot \mathbf{I}\right)^{-1} \cdot \mathbf{W_{22}}^{*} \\ \qquad \text{if} \quad m+1 < q \end{cases} \tag{22}$$

In the analyzed HIPERLAN/2 system a fixed $\gamma^2 = -30$dB has been emerged to be sufficient. Thus, the pre-equalizer can be used after initial channel estimation. The transfer function of the complete impulse response $\mathbf{c_e}$ can be computed in frequency domain by multiplying $\tilde{\mathbf{C}}$ and DFT $\{\mathbf{e}\}$.

V. Performance Analyses

The pre-equalizer is not needed, if the channel impulse response doesn't exceed the guard length. In this case, there exists no advantage for applying a FIR pre-equalizer.
To show the influence of the pre-equalizer, we have chosen a OFDM parameter set providing 54 MBit/s. With a distance of 312.5 kHz, 48 subcarriers are used for data transmission. The complex symbols on each subcarrier are taken form the 64-QAM alphabet. The required error protection is handled by a R=3/4 punctured convolutional code. The block interleaver size is 288; this deals with the number of coded bits per OFDM symbol. The needed channel coefficients are taken from a channel estimation, considering the training structure of HIPERLAN/2 by a simple least squares estimation without applying any noise reduction scheme.
The used mobile radio channel is based on a channel model, developed in connection with HIPERLAN/2 standardization. This channel models contain 5 different exponential power delay profiles with delay spreads from 50 ns up to 250 ns. To demonstrate the effect of pre-equalizing, we used the channel type E ($\Delta\tau = 250$ ns).
Since we have chosen our system parameters considering actual WLAN standards, packet oriented data transmission is required. In our simulations, we used data packets of length 432 bits. A packet error occurs, if only one bit is wrong after viterbi decoding. In a real system defective packets have to be transmitted again. Thus, we define a packet error rate (PER) to compare the performance of different components.

A. *Performance with ideal known channel impulse response*

In section III one example proving the functionality of the pre-equalizer has been presented. The effect of pre-equalizing on the error rate is a fundamental criterion for applying. One parameter, we are going to optimize is the pre-equalizer length $(p+1)$. Considering realistic conditions, we assume a noise level of E_b/N_0

= 23 dB.
Figure 7 shows the simulated packet error rate using a pre-equalizer of length $p+1$. If the equalizer has been computed by equation (15), the system performance becomes worse for high equalizer lengths. In case of applying equation (16) with $\gamma_{pe}^2 = -28$ dB, the number of transmission errors decreases for high numbers of equalizer coefficients. As a compromise concerning realization effort and system performance, we have chosen a equalizer length of 64 coefficients.

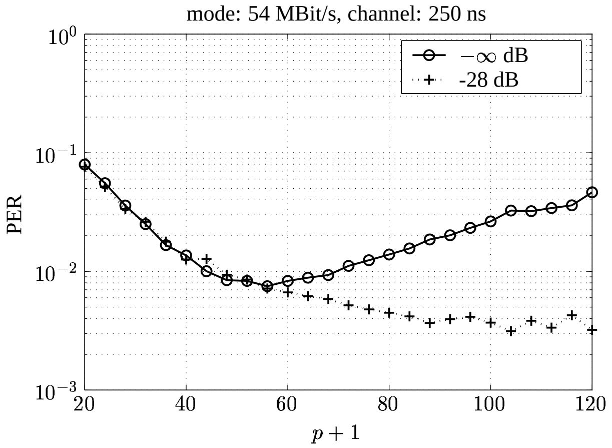

Fig. 7. packet error rate versus equalizer length with $\gamma_{pe}^2 = -\infty$ dB (no noise consideration) and $\gamma_{pe}^2 = -28$ dB

B. Performance analysis using channel estimation

In section III the enormous impact of pre-equalizing, using the ideal known channel impulse response, has been shown. In practical systems, the channel impulse response has to be estimated. One possibility to overcome the problems of channel estimation is the technique described in section III. Especially equation (21) is fundamental for exploiting the training-symbol based OFDM channel estimation of HIPERLAN/2.
Before we may start channel estimation, the channel impulse response length must be fixed for pre-computing $\mathbf{W_{1k}}$. Here, we assumed a channel length $(m+1)$ = 36, so $\mathbf{W_{1k}}$ becomes a matrix with dimensions (36×52). Since indoor channels are very slow fading channels $(f_d < 50Hz)$ and bursts are relatively short (< 1 ms), we have not yet considered any tracking techniques.
In figure 8 some simulation results are shown. In case of no pre-equalizer (no PE), a very high error floor is the consequence of ISI and ICI. For the sake of completeness, the case of pre-equalizing with ideal known channel impulse response has been printed in figure 8, too (PE, no CE). If we consider a realistic channel estimation (PE and CE), the performance loss is < 1 dB considering the ideal known case and a PER of 10^{-2} (packet error rate). Compared to the non pre-equalized case, the gain of pre-equalizing including channel estimation is infinite high due to a very high error floor. The very bad system performance without pre-equalizing can be clearly seen.

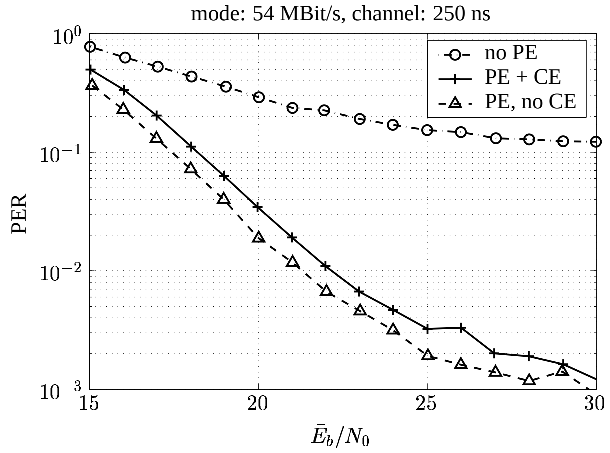

Fig. 8. packet error rate without pre-equalization (PE), with pre-equalization and ideal known channel impulse response (PE, no CE), and pre-equalization with training symbol based channel estimation (PE and CE)

VI. Conclusions

In the present paper, a method to reduce the effect of ISI and ICI in case of OFDM modulation has been illustrated. After introducing the characteristic of the guard interval in section II, we developed an impulse truncation algorithm in section III. It is based on the well known MMSE technique. In practical systems like HIPERLAN/2, the impulse truncation technique must be combined with time-domain channel estimation. In section IV we have shown a channel estimation scheme, exploiting the HIPERLAN/2 training symbols in frequency domain. The performance analyses in section V demonstrate the functionality of the impulse truncation technique considering HIPERLAN/2 system parameters. In section V-B the enormous gain of applying pre-equalization combined with channel estimation can be seen. Impulse truncation is unnecessary, if the guard interval is long enough compared to the channel impulse response. But, if ISI/ICI effects due to very long channel delay times cause a high error floor, the impulse truncation technique is a very attractive method to save the systems functionality.

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