Quantization and its Effects on OFDM Concepts for Wireless Indoor Applications

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ABSTRACT

In this paper, we are going to discuss some fundamental aspects of the implementation of an OFDM modem for wireless indoor applications. One basic component of common OFDM systems is the computation of the DFT. Therefore, some efficient FFT algorithms like the Split-Radix and the Winograd-FFT algorithm have been implemented and their applicability for these modern communication systems has been proved. In the second part, a simple fixed point model of an OFDM system will be presented.

1. INTRODUCTION

In the last few years, the multicarrier technique OFDM (**O**rthogonal **F**requency **D**ivision **M**ultiplexing) has grown to an important alternative for wireless indoor communications. All over the world different standards for broadband wireless LANs (WLAN) have been created. The European Telecommunications Standards Institute (ETSI) and the American equivalent IEEE have decided for OFDM to be the modulation scheme for the new indoor mobile communications standards HIPERLAN/2 and IEEE802.11 in the 5 GHz band. Of course, modern concepts for digital audio broadcasting (DAB) or digital video broadcasting (DVB) are using OFDM, too.

One large advantage of this technology is its robustness in case of multipath propagation. The high liability of OFDM to frequency shifting and Doppler spreading seems to be unimportant because of slowly fading indoor channels. OFDM is the most important representative of the multicarrier technique, since it is easy to implement due to efficient FFT algorithms. But, the implementation costs for these algorithms are, compared to the rest of an OFDM modem, comparatively high, so the most efficient structure has to be chosen.

In this paper we are going to depict some fundamental aspects of OFDM system implementation. First, in section 2, we will introduce the analyzed OFDM structure. The opportunity of FFT structures in OFDM systems is the main aspect of section 3. Here, we present some fundamentals about different FFT algorithms like the Radix-2 [2], the Split-Radix and the Winograd-FFT algorithm [3]. In section 4, we will define a fixed point model of an OFDM receiver. The derivation of the most important parameters will be shown by simulation results. The impact of quantization to the out of band radiation will be discussed in section 5.

Compressed postscript files of our publications are also available from our WWW server `http://www.comm.uni-bremen.de`.

2. OFDM SYSTEM STRUCTURE

In future indoor communication systems OFDM has established to be the basic modulation scheme of the physical layer (PHY). OFDM evolves its true mightiness in case of mobile radio channels with short power delay profiles and very low Doppler frequencies. Looking at a mobile indoor channel we preserve both. Here, we consider the power delay profile of a typical, quasi time invariant indoor mobile radio channel. In the following sections the most important parameters of our analyzed OFDM system are presented.

2.1. Transmitter structure

The parameters of the used OFDM transmitter have been fixed by different WLAN standards. Figure 1a) shows the block diagram of the used transmitter structure. The convolutional channel encoder has a total code rate of $R_{cc} = \frac{3}{4}$. After QPSK modulation the complex symbols are mapped to N=64 subcarriers. The equally spaced subcarriers have a distance of 312.5 kHz. This leads to a raw symbol duration of 3.2 μs. To simplify the necessary analog filter in the digital analog converter (DAC), we use an FFT length of 256. After including a guard interval, the oversampled signal ($f_a = 80$ MHz) has to be converted to a time continuous analog signal. Due to burst by burst data transmission, we have to add a short preamble for channel estimation. This transmitter model leads to a total data rate of 24 MBit/s using a total bandwidth of $\approx$ 20 MHz.

2.2. Receiver structure

Figure 1b) shows the most important parts of the analyzed OFDM receiver. The received analog baseband I/Q signal has to be adapted to the input range of the analog digital converter (ADC). The automatic gain control (AGC) estimates the power of the received preamble and amplifies the signal. Here we assume ideal synchronization. To minimize the costs of the complete OFDM modem, receiver and transmitter are using the same FFT block. The channel transfer function $H(i)$ can be estimated by a simple division, considering the known preamble. In case of QPSK we need no true equalization (multiplication with $1/H(i)$), instead we can combine the OFDM equalization with softbit weighting (multiplication with $|H(i)|^2$) to maximum ratio combining (MRC):

$$e(i) = d'(i) \cdot H^*(i) \tag{1}$$

The following QPSK softbit decision results to

$$b_1(i) \quad = \quad \mathrm{Re}\{d'(i) \cdot H^*(i)\} \tag{2}$$

$$b_2(i) \quad = \quad \mathrm{Im}\{d'(i) \cdot H^*(i)\} \tag{3}$$

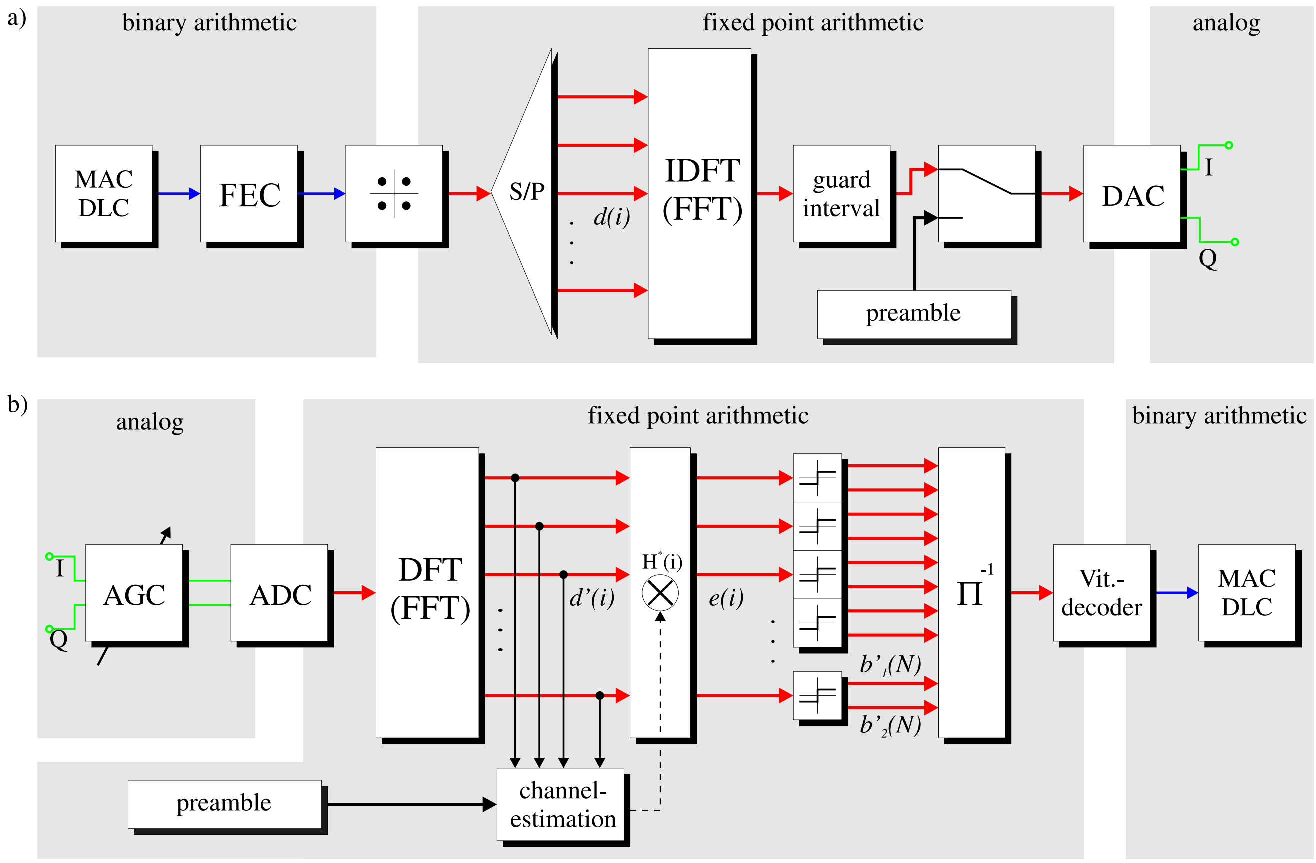

Figure 1: a) OFDM transmitter with channel coding and DAC; b) OFDM receiver with ADC and channel decoding

The softbits $b_1(i)$ and $b_2(i)$ of the i-th subcarrier are decoded by a simple Viterbi decoder. The mean bit duration at its input is about 28 ns.

3. FFT ALGORITHMS

As we have seen in section 2 and in figure 1, the FFT algorithm (Fast Fourier Transform) to compute the DFT (Discrete Fourier Transform) and IDFT is a central part of an OFDM modem. The implementation of a FFT algorithm is very expensive, because it needs many operations. Assuming a total OFDM symbol duration of $4\mu s$, the DFT has to be computed $2.5 \cdot 10^5$ times per second. Several FFT algorithms are disposed to compute the DFT. Here we discuss only 3 very efficient structures.

- The most familiar FFT algorithm is the Radix-2 FFT algorithm (R2FA). In [2] a detailed derivation of the R2FA is given. The easy way to implement this algorithm is a basic characteristic of the R2FA. Due to the recursive structure it is very attractive for DSP implementations. Unfortunately, it is not possible to vary the FFT sequence length N in a flexible way, since N must be a power of 2 ($N = 2^m$, $m \in \mathbb{N}$, $m \geq 1$).

- In a similar but more efficient way the Split-Radix algorithm (SRFA) computes the DFT. Here, the DFT is divided into a Radix 2 and a Radix 4 part, so the efficiency of a Radix 4 algorithm and the flexibility of the R2FA can be exploited by the SRFA. In [3] the structure of this recursively executable algorithms is shown. For applying the SRFA, all FFT sequence lengths $N = 2^m$ ($m \in \mathbb{N}$, $m \geq 2$) are possible.

- A usual unknown FFT representative is the Winograd FFT algorithm (WFTA). Since it is an advancement of the prime factor FFT algorithm (PFTA) [3], the complete DFT can be decomposed into M short DFTs of length N_i, whereas $N = \Pi_{i=0}^{M-1} N_i$. All N_i must be relatively prime factors. The efficiency of the PFTA rises if Winograds short DFT modules are used (WFTA). One disadvantage of this algorithm is the complicated structure, so it is very difficult to implement on a DSP. Due to the prime factor mapping, only a FFT length of 252 is possible. This leads to a total sample rate $f_a = 78.75$ MHz.

3.1. *Costs of Implementation*

The number of needed operations is a very important criterion of decision. The following table shows the number of required real additions, real multiplications, and the maximum path length for different FFT algorithms with input sequence length 256. The WFTA assists only a length 252 FFT.

	length	A	M	max. path
R2FA	256	5634	3076	7 M + 15 A
SRFA	256	5008	1824	3 M + 11 A
WFTA	252	6640	792	1 M + 20 A

A: real additions; M: real multiplications

As a result of comparison, there are 3 aspects to discuss:

1. **total number of operations:**
   The SRFA needs only 6832 real operations for computing a 256 point DFT. So, if the number of operations is the main criterion, the SRFA seems to be most efficient.

2. **number of multiplications:**
   In many cases multiplications are more complex than other operations. To compute a 252 point DFT, the WFTA needs only 792 real multiplications.

3. **maximum data path length:**
   The structure of all FFT algorithms can be divided into parallel branches. For an effective use, all branches should have an equal length. If there is one very long branch, the system has to wait for it. Here, the maximum length data path of a length 256 SRFA contains only 3 real multiplications and 11 additions.

In [5] some relationships between the required operations and the implementation costs are given. For this first approach, we assume an equal word length for all operations.

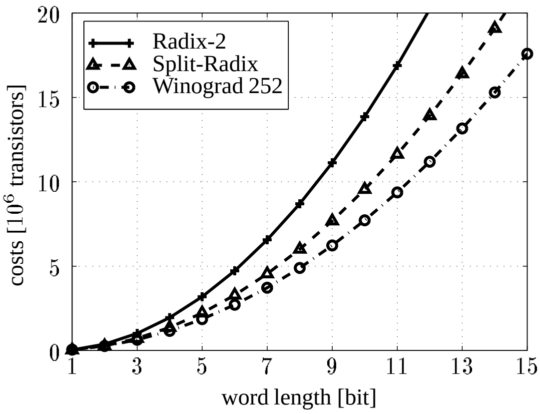

Figure 2: costs of implementation

In Figure 2, the total implementation costs are shown for the mentioned FFT algorithms. Assuming word lengths > 5 bits, the effort of SRFA is about 70 % of R2FA. Due to lower number of multiplications, WFTA will cost about 65 % of R2FA.
If we just consider the required additions and multiplications, the WFTA causes least implementation costs.

3.2. Internal Quantization Effects

The following analyzes show the internal quantization effects of the discussed FFT algorithms. First, we have to define a criterion of comparison.
The mean square error computation (MSE) for different algorithms is shown in figure 3. The equal distributed noise $s(k)$ is quantized with N_Q bit. Since $s(k)$ is a complex signal, we have to quantize the real and imaginary part independently, each with N_Q bits per sample. The MSE is defined as follows:

$$MSE|_{dB} = 10 \cdot \log_{10} \left(\frac{E\{|\Delta S_q(k)|^2\}}{E\{|S_q(k)|^2\}} \right) . \qquad (4)$$

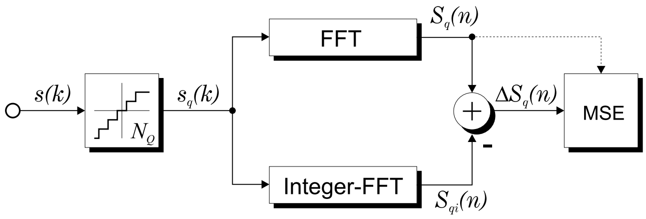

Figure 3: simulation of internal quantization effects

Due to various configurations of multiplications and additions, the algorithms result in different MSE values. Regarding the fixed point implementation, all integer additions lead to non limited integer values, the multiplications with twiddle factors W_N^x (realize that $|W_N^x| = 1$) are rounded to integer values.

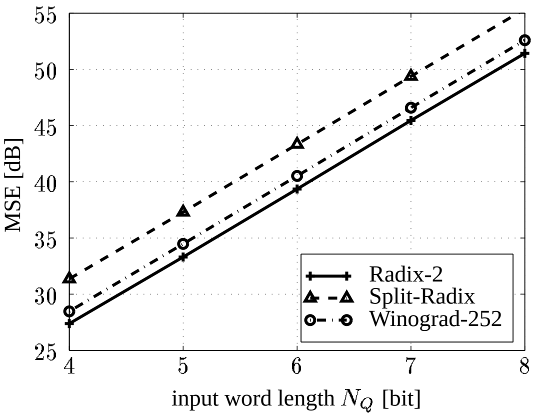

Figure 4: internal quantization effects

The simulated MSE values for input word lengths $N_Q = \{4, 5, 6, 7, 8\}$ are printed in figure 4. It applies to all FFT algorithms that the MSE grows,as expected, with 6 dB/bit. It can be seen that SRFA delivers the to best MSE performance.

4. FIXED POINT OFDM RECEIVER

In section 3, several FFT algorithms have been analyzed. Considering internal quantization effects, the Split-Radix FFT algorithm results in best MSE performance. Thus, we use a length 256 Split-Radix FFT algorithm for the following analyzes. All following parameters have been found by simulations. Therefore, we assume an ideal OFDM transmitter. As a criterion for comparison, we consider a bit error rate (BER) of 10^{-4}.

4.1. Automatic Gain Control

Regarding figure 1b), the first element of our OFDM receiver is an amplifier named automatic gain control (AGC). The major task of the AGC is an optimal saturation of the following analog digital converter (ADC). Thus, we define the factor A_{AGC}, which specifies the distance between ADC limitation and the RMS of the AGC output signal, like shown in figure 5.
The influence of A_{AGC} on the system BER has been found by simulations. The simulation results are shown in the following table:

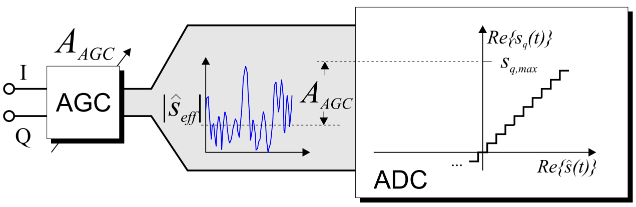

Figure 5: ADC input level A_{AGC}

A_{AGC} [dB]	3	4	5	6	7
SNR-loss [dB]	0.5	0.1	< 0.1	< 0.1	< 0.1

As a good compromise, we choose $A_{AGC} = 6$ dB. A little reserve for estimation errors has been considered.

4.2. *Fixed Point FFT*

A very important parameter of an OFDM fixed point model is the FFT input word length. Utilizing an 8-ary Radix FFT algorithm, the word length usually increases step by step. If we describe the input word length $N_{Q,fft_{in}}$, the word length for the FFT output data is

$$N_{Q,fft_{out}} = N_{Q,fft_{in}} + 8\,. \qquad (5)$$

The FFT input word length has been found by simulations. For different values of $N_{Q,fft_{in}}$, the next table shows the SNR-loss due to the input word length.

$N_{Q,fft_{in}}$ [bit]	3	4	5	6	7
SNR-loss [dB]	1.5	0.3	0.1	< 0.1	< 0.1

We can see that a word length of 6 bit (complex: $2 \cdot 6$ bit) suffices for a negligible SNR-loss of <0.1 dB.

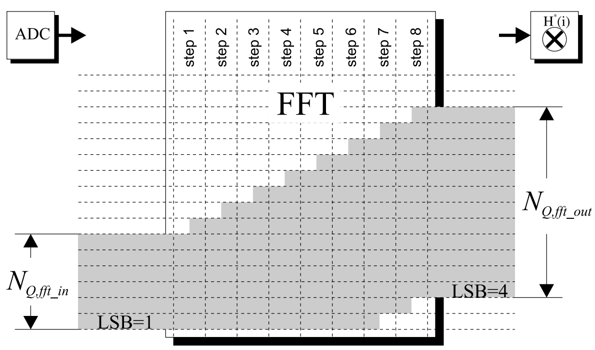

Figure 6: data word length while executing the FFT

Like stated in (5), the output data usually needs a word length of 14 bits. However, simulations have shown that the least significant bit (LSB) is useless, due to internal quantization effects. Here, we can neglect the LSB of the last 2 steps of the FFT algorithm. The following table shows the SNR-loss of LSB cutting:

$N_{Q,fft_{out}}$ [bit]	10	11	12	13	14
SNR-loss [dB]	0.4	0.2	< 0.1	< 0.1	< 0.1

This leads to an acceptable FFT output data worth length of 12 bits. In Figure 6 the needed word lengths during executing the FFT algorithm are shown.

4.3. *Equalization and Decoding*

After executing the FFT, the received data have a large dynamic range. We found out that one can reduce the dynamic range with very low practical consequences. The data word length will be reduced to $N_{Q,eq_{in}}$ bits by clipping. Unfortunately this word length reduction and dynamic limitation can not be done while executing the FFT. In figure 7 the schematic of the various word lengths are printed.

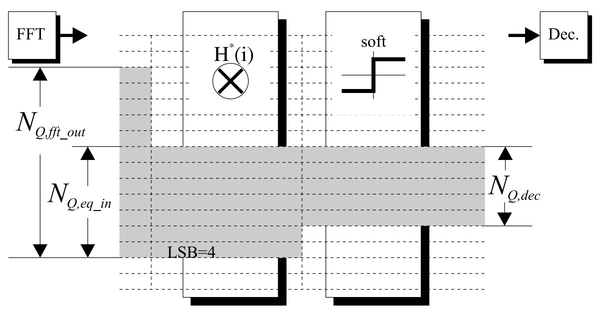

Figure 7: data word length while equalizing

As shown in the table below, a data word length of 7 bits is sufficient.

$N_{Q,eq_{in}}$ [bit]	5	6	7	8
SNR-loss [dB]	> 2	1	< 0.1	< 0.1

The required word length for Viterbi decoding has been found by simulations, too. In this case, a data word length of 5 bits is sufficient.

4.4. *Performance of Fixed Point Receiver*

In the last section, a fixed point model of an OFDM receiver was defined. All parameters have been separately optimized. In this section the simulation results for the whole fixed point model are shown. The following parameters are fixed:

A_{AGC}	$N_{Q,fft_{in}}$	$N_{Q,fft_{out}}$	$N_{Q,eq}$	$N_{Q,dec}$
6 dB	6 bit	12 bit	7 bit	5 bit

Figure 8 shows the simulated bit error rates (BER) of the fixed point model. As a reference the performance of the same OFDM system using 32 bit floating point arithmetic has been printed, too. The resulting SNR-loss is about 0.3 dB regarding a bit error rate of 10^{-3}.

5. FIXED POINT OFDM TRANSMITTER

In section 4, we have defined some quantization parameters to characterize a fixed point receiver. Usually the FFT block of the mobile station can be used for both, uplink (UL) and downlink (DL). So the defined fixed point FFT is available for both transmission directions. If we regard the complete OFDM system for downlink transmission, the transmitter is placed in the base station, and the mobile station contains the OFDM receiver. For the uplink channel, it's just contrariwise. Since it is possible to spent more complexity in the base station, the quantization effects of the transmitter are uncritical in the downlink mode. Using the mobile station as transmitter, we have to assume the same quantization parameters as in the receiver mode. Considering an ideal receiver and a fixed

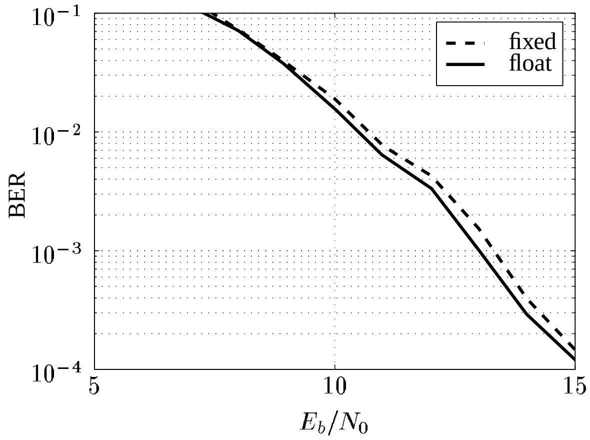

Figure 8: system performance of fixed point model and floating point model

point transmitter, simulations have shown, that the SNR-loss is lower than in the contrariwise case analyzed in section 4. So we don't consider the system performance of a fixed point OFDM transmitter in this section.

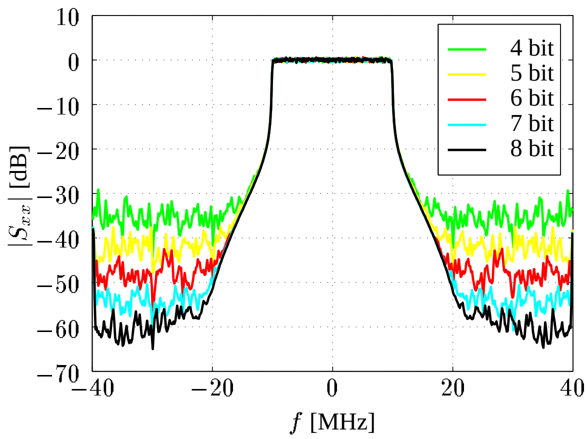

Figure 9: power density spectrum of transmitter output signal

Looking at the OFDM transmitter in figure 1a), we only expect quantization effects while computing the FFT. As we have seen in the last section, we can describe the fixed point model of the FFT by one important parameter: data word length at FFT input.
In figure 9 the power density spectrum of an OFDM transmitter using $256/64$ times oversampling and a cosine-roll-off impulse shaping (in time domain) is shown. The influence of the internal quantization effect of the FFT can be clearly seen. Of course this results will move if a power amplifier and a real analog filter (DAC) will be taken into account.
The negative peak at $f = 0$ shows that the signal contains no DC component. Regarding the shown out of band radiation one can conclude that the number of simulated random symbols doesn't suffice. But increasing the simulation effort will not smooth the spectrum. It is just an effect of the internal quantization effects of the Split-Radix FFT algorithm.

6. CONCLUSIONS

In the presented paper an OFDM system, compatible to modern indoor communication systems, has been analyzed regarding implementation aspects. Since OFDM requires an efficient algorithm to compute the DFT or IDFT, three known FFT algorithms have been analyzed. As a criteria of comparison, two characteristics of length 256 (252) FFT algorithms are shown: implementation costs and internal quantization effects.
Regarding the implementation costs, a length 252 Winograd FFT algorithm is very attractive. Due to a very complicated structure, the Split-Radix FFT algorithm outperforms WFTA considering internal quantization effects. Thus, a Split-Radix FFT algorithm has been chosen for a FFT length of 256.
In section 4, a fixed point model of an OFDM receiver has been defined. The most important parameters have been shown. Regarding the bit error rate, the performance loss of the fixed point receiver has been presented.

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