Reducing the Peak to Average Power Ratio of Multicarrier Signals by Adaptive Subcarrier Selection

Heiko Schmidt and Karl-Dirk Kammeyer

University of Bremen, FB-1, Department of Telecommunications
P.O. Box 33 04 40, D-28334 Bremen, Germany,
Fax: +(49)-421/218-3341, e-mail: schmidt@comm.uni-bremen.de

ABSTRACT

A non-constant envelope is a fundamental problem of the multicarrier technique OFDM. In many cases, non-linear power amplifiers with power saturation lead to high out-of-band radiation. Another characteristic of OFDM is the influence of frequency selective fading of mobile radio channels. To overcome this effect coding schemes are required for low error rates, therefore a certain redundancy has to be introduced. In this paper, there will be presented a combination of an adaptive subcarrier selection as coding scheme with a new algorithm to reduce the out-of-band radiation, exploiting the non-information carrying subcarriers. The efficiency of this technique will be demonstrated by simulation results.

1. INTRODUCTION

In the last few years, the multicarrier technique has grown to an important alternative for wireless indoor communication. One large advantage of this technology is its robustness in case of multipath propagation. OFDM (**O**rthogonal **F**requency **D**ivision **M**ultiplexing) is one representative of the multicarrier technique, since it is easy to implement due to efficient FFT/IFFT algorithms. Here, we mainly discuss two important aspects of the OFDM technique: the loss of the bit error rate (BER) caused by frequency selective fading and the out-of-band radiation due to non-linear amplifiers [3].

The complex envelope of an OFDM signal is an overlay of N complex oscillations with different frequencies, phases and amplitudes. As a consequence, we obtain a time-domain signal with high peak to average power ratio. In order to reduce the costs of mobile communication systems, components with non-ideal characteristics, e.g. low-end amplifiers with power saturation, are used. Overcharging a power-limited amplifier in case of high peak values will lead to high out-of-band radiation.

Starting from a known scheme to lower the BER by adaptive subcarrier selection [4], we can combine this kind of coding scheme with a new algorithm to reduce the peak to average power ratio. Simulation results demonstrate the quality of this new technique.

After a common OFDM indroduction in section 2 the adaptive selection of subcarriers is presented in section 3. In section 4, the significant improvement of the BER by neglecting weak subcarriers can be seen. In section 5, the reduction of the out-of-band radiation by using non-information carrying subcarriers to reduce the crest factor is discussed.

2. THE OFDM CONCEPT

Assuming that N complex information carrying symbols $d_{n,i}$ are taken from a fixed modulation alphabeth, the complex envelope of the total OFDM signal, consisting of N subcarriers, can be expressed as

$$s_{all}(t) = \sum_{i=-\infty}^{\infty} \sum_{n=0}^{N-1} d_{n,i} \cdot g(t - i(T + T_g)) \cdot e^{j2\pi \frac{nt}{T}} \,. \tag{1}$$

The total symbol duration $(T+T_g)$ includes a guard interval with length T_g. This guard interval is a cyclic extention of the OFDM symbol and prevents intersymbol interference (ISI). Therefore, the channel influence leads to a multiplication with a corresponding factor on each subcarrier. The spectral distance between two subcarriers is chosen to be $\Delta f = 1/T$.

For the following derivations, we can concentrate on single OFDM symbols without guard interval by assuming $i = 0$ and, hence, omit the sum over i in (1). The complex envelope of single OFDM symbols is given by

$$s(t) = \sum_{n=0}^{N-1} d_n \cdot g(t) \cdot e^{j2\pi \frac{nt}{T}} \,, \quad 0 \le t \le T \,. \tag{2}$$

The crest factor C of one OFDM symbol is defined by the peak to RMS magnitude ratio of (1)

$$C_{dB} = 10 \cdot \log_{10} \left(\frac{\max\limits_{t \in [0,T]} |s(t)|^2}{\frac{1}{T} \int_{t=0}^{T} |s(t)|^2 \, dt} \right) . \tag{3}$$

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Furthermore, sampling the time-domain OFDM signal $s(t)$ with a period of $T_s = \frac{T}{N}$ and assuming a rectangular function $g(t)$ of length T, yields a transmitted signal

$$x(k) = \sum_{n=0}^{N-1} d_n \cdot e^{j2\pi\frac{nk}{N}}\ , \quad 0 \le k < N-1 \qquad (4)$$

using a IDFT or IFFT.
Assuming M-PSK or M-DPSK modulation, the denominator in (3) equals to N. The numerator of (3) depends on the constellation of the information symbols d_n, so its maximum is equal to N^2 and so $C_{dB} \le 10 \cdot \log_{10}(N)$.
The subcarrier decoding of the received signal $\hat{s}(k)$ can be computed by a FFT. This leads to N complex symbols $\hat{d}_n$ containing the information.

3. SELECTION OF SUBCARRIERS

Since all subcarriers are orthogonal, we can exclude those with lowest SNR. Lower data rates but an enormously reduced BER are the consequences of this technique, as shown in section 4. The implemented redundancy can be used by the crest factor algorithm described in section 5 since it is based on dividing the subcarriers in two groups: information carrying and non-information carrying subcarriers. Therefore, a binary vector of length N called subcarrier allocation scheme is defined, pointing to the information carrying subcarriers.

$$\mathbf{c} = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{N-1} \end{bmatrix} \quad , \quad c_n \in \{0, 1\} \qquad (5)$$

With (5) an inverse subcarrier allocation scheme $\overline{\mathbf{c}}$ can be defined, pointing to non-information carrying subcarriers. For the following derivations, the number of information carrying subcarriers is called N_c and excluded subcarriers are counted by $N_n = N - N_c$. The following methodes to generate $\mathbf{c}$ or $\overline{\mathbf{c}}$ are using an amplitude estimation of the received symbols $|\hat{d}_n|$ on each subcarrier.

1. The main criterion for subcarrier allocation scheme generation is a threshold decision, so N_c depends on the current mobile radio channel constellation. This technique will guarantee a low BER but as a consequence the data rate will vary.

2. To guarantee a constant data rate, the number of information carrying subcarriers has to be locked to a certain N_c. In this case, a ranking algorithm selects subcarriers, but the quality of data transmission will vary depending on the current channel parameters.

A mobile indoor communication system offers the possibility of bi-directional transmission, hence the subcarrier allocation scheme will be known by the transmitter. It is necessary to adapt the allocation scheme always to time variant channel parameters. Assuming indoor transmission, a low adaption rate suffices.

4. LOW BIT ERROR RATES BY LEAVING OUT WEAK SUBCARRIERS

4.1. System Description

The subcarriers of an OFDM system are orthogonal while a guard interval eliminates ISI. Due to multipath channels with frequency selective fading, single subcarriers are disturbed differently by Gaussian noise, hence, most wrong data decisions happen on subcarriers with low amplitude. Applying the selection methods described in section 3, thus using only “good” channels for data transmission, we obtain a lower total BER. Due to leaving out “bad” subcarriers, the total data rate and also the bandwidth efficiency is lessened to

$$u = \log_2(M)\left(1 - \frac{T_g}{T + T_g}\right)\left(\frac{N_c}{N}\right), \qquad (6)$$

using M-ary modulation. To compensate the disadvantage of lower data rate, the modulation scheme can be adapted for a constant bandwidth efficiency. To minimize the $\frac{E_b}{N_0}$-loss due to a lower bandwidth efficiency, the non-information carrying subcarriers have to be filled with zero symbols.

4.2. Simulation results with a subcarrier subset

The effect discribed above can be shown for an OFDM system in 5.2 GHz band with N=64 subcarriers, 8-DPSK modulation and a total bandwidth of 20 MHz. A mobile radio channel with frequency selective Rayleigh fading and a maximum echo delay of 300 ns is assumed, so a guard interval of length $T_g = \frac{1}{9}T$ will be sufficient. The total data rate using all subcarrier for data transmission is about 50 MBit/s. By leaving out weak subcarriers, the data rate is lessened with the ratio $\frac{N_c}{N}$, but fig. 1a) clearly indicates the gain of the BER applying the subcarrier selection technique. If we focus on BER $\le 10^{-3}$, the $\frac{E_b}{N_0}$-gain is about 14 dB, assuming $\frac{N_c}{N} = \frac{1}{2}$.
To show the connection between data rate and modulation scheme we will concentrate on two system constellations with a data rate of 26.66 Mbit/s. Therefore, an 8-DPSK using $N_c = 32$ of $N = 64$ subcarriers is compared with a DQPSK modulation using $N_c = 48$ of $N = 64$ subcarriers.

In fig. 1b) can be seen that the BER of 8-DPSK with $\frac{N_c}{N} = \frac{1}{2}$ and DQPSK with $\frac{N_c}{N} = \frac{3}{4}$ are equal.

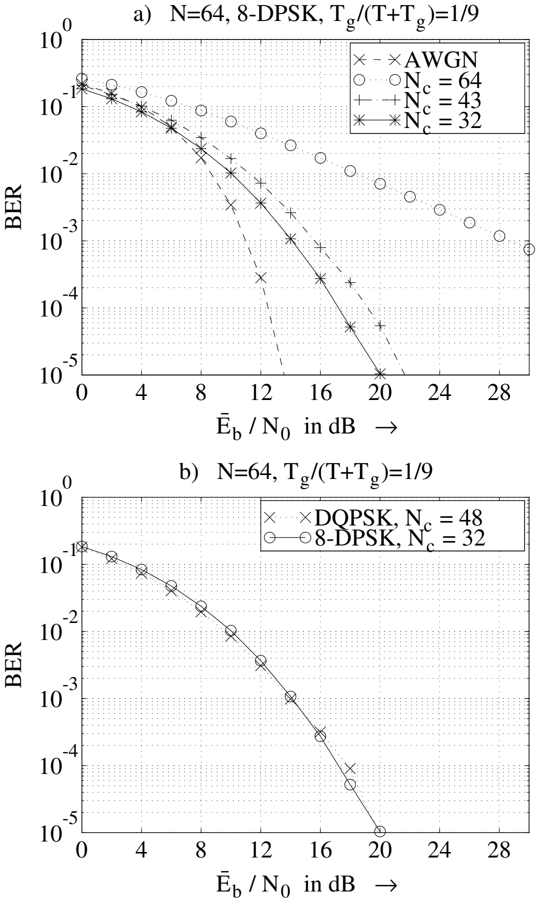

Figure 1: a) Bit error rate of an OFDM system with N=64 subcarriers and 8-DPSK modulation;
b) Bit error rate of two system constellations with an equal total data rate $R = 26.66$ Mbit/s

5. ALGORITHM TO REDUCE THE CREST FACTOR

Assuming an OFDM system with N_c of N subcarriers for data transmission, we can concentrate on the rest of them to reduce the crest factor of the time-domain OFDM signal. We have to find a complex time-domain function, furtheron named “reducing function”, which reduces the peak values by overlaying. As a marginal condition, only excluded subcarriers (frequencies) are allowed for the reducing function. Another possibility is to modulate the excluded subcarriers depending on the current subcarrier constellation to reduce the peak values in time-domain. So, the information carrying subcarriers are ***not*** distorted, only the additional power of the reducing function implicates an $\frac{\bar{E}_b}{N_0}$-loss.
The subcarrier allocation scheme $\mathbf{c}$ has to be applied by the transmitter. We assume that all non-information carrying subcarriers are filled with zero symbols. Now, an iterative technique to reduce the peak values of each OFDM symbol will be presented. The main idea is the overlay of a reduction function $\Delta s(t)$, which only consists of non-information carrying frequencies. The new algorithm starts by generating the raw OFDM symbol $s_0(k_w)$ with all used subcarriers. To make sure that we can detect all peak values, an oversampling rate w is defined. Further, the oversampled signal is marked as a function of k_w. Simulations have shown that an oversampling rate of $w \geq 4$ suffices to detect and remove all peak values. The iteration loop with loop index r is described as follows:

1. At the first step, an ideal reducing function $\Delta s_{r,ideal}$ is definded by a decision with a specified threshold $S_{max,r}$.

$$\begin{aligned}\Delta s_{r,ideal}(k_w) &= 0\,,\ |s_{r-1}(k_w)| \leq S_{max,r} &(7)\\ &= (S_{max,r} - |s_{r-1}(k_w)|) &(8)\\ &\quad \cdot e^{j\ arg(s_{r-1}(k_w))},\\ &\quad |s_{r-1}(k_w)| > S_{max,r}\,,\\ &\quad k_w = 0, ..., w \cdot N - 1\end{aligned}$$

2. The oversampled function $\Delta s_{r,ideal}(k_w)$ includes unwanted higher frequencies, so first we have to sample down $\Delta s_{r,ideal}(k_w)$ by applying an ideal lowpass filter. The next step has to be done in frequency domain, so the spectrum of the resampled function $\Delta s_{r,ideal}(k)$ has to be computed by a DFT. Its frequency spectrum $\Delta d_{r,ideal}(n)$ includes the information carrying subcarriers. They can be excluded by a multiplication with the inverse subcarrier allocation scheme $\bar{\mathbf{c}}$.

$$\begin{aligned}\Delta d_{r,real}(n) &= \sqrt{\frac{N}{N_c}} \cdot \Delta d_{r,ideal}(n) \cdot \bar{\mathbf{c}}(n), &(9)\\ &\quad n = 0, ..., N-1\end{aligned}$$

The necessity of the factor $\sqrt{\frac{N}{N_c}}$ in (9) results from reducing the power of Δs while excluding some frequencies. The real reducing function in frequency-domain $\Delta d_{r,real}(n)$ leads to a time-domain function $\Delta s_{r,real}(k)$. With w times oversampling, we get the real reducing function $\Delta s_{r,real}(k_w)$.

3. The overlay in time-domain of the real reducing function $\Delta s_{r,real}(k_w)$ and $s_{r-1}(k_w)$

$$\begin{aligned}s_r(k_w) &= s_{r-1}(k_w) + \Delta s_{r,real}(k_w), &(10)\\ &\quad k_w = 0, ..., w \cdot N - 1\end{aligned}$$

creates an OFDM symbol with a reduced crest factor.

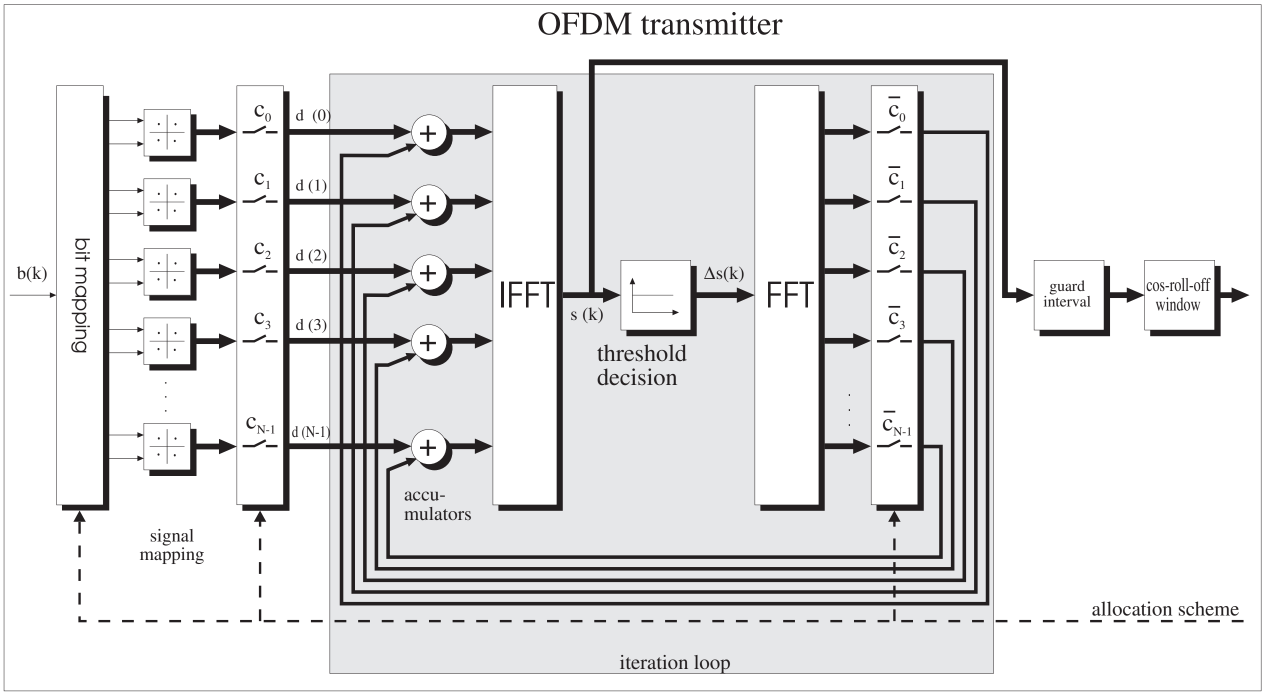

Figure 2: Block diagram of an OFDM transmitter with iterative crest factor reducing algorithm considering a subcarrier allocation scheme.

4. Furthermore, the crest factor reduction can be continued by an iteration loop beginning with step 1.

To end iteration loop, we can either define a maximal crest factor, with the disadvantage of many possible iterations, or a maximum iteration counter is obtained with the result of a suboptimal crest factor in some cases.

Furthermore, a variable threshold $S_{max,r}$ at step 1 optimizes the iteration speed. In case of more than two iterations, the efficiency of the algorithm raises by using a low threshold for the first iterations.

Fig. 2 shows the block diagram of an OFDM transmitter considering the adaptive subcarrier selection. It begins with a bit mapping box, where the data are mapped to "good" subcarriers. The next block generates complex symbols on each subcarrier using $\log_2(M)$ bits per symbol. Now weak subcarriers are filled with zero symbols. After executing the IFFT, a threshold decision generates the first reducing function. After filtering the ideal reducing function, the inverse subcarrier allocation scheme can be applied. To save one IFFT-block, the real reducing function in frequency-domain is accumulated to the complex symbol vector. There are no differences between an overlay in time- or frequency-domain because of the linearity of the IFFT.

5.1. Simulation Results Applying the Crest Factor Reducing Algorithm

Here, the effect of this new crest factor reducing algorithm will be demonstrated. In fig. 3a) a polar plot of one OFDM symbol using $N_c = 32$ of $N = 64$ subcarriers for data transmission is shown. It can be seen that many peaks exceed the marked 3 dB limit enormously. After using the reducing algorithm, the OFDM symbol mutates as shown in fig. 3b). The 3 dB limit is only touched by the complex magnitude. The additional power is about 20%, so the $\frac{\bar{E}_b}{N_0}$-loss is < 1 dB.

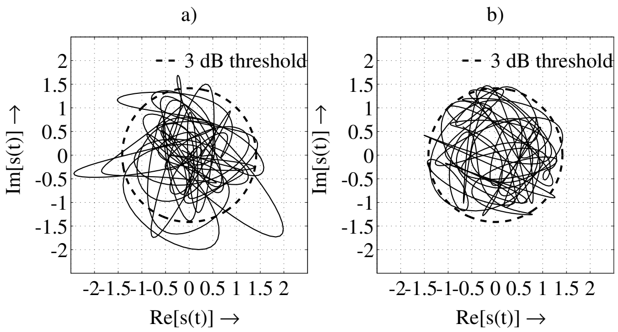

Figure 3: Polar plot of one OFDM symbol s(t); a) using 8-PSK, $N_c = 32$ and $N = 64$; b) after 9 iteration loops the crest factor is about 3 dB (addtitional power $\approx 20\%$).

5.2. *Out-Of-Band Radiation Applying the Crestfactor Reducing Algorithm*

The rect-impulses of OFDM implicate $\frac{\sin(x)}{x}$-formed power density spectra (PDS). So high out-of-band radiation is a consequence of this kind of impulse shaping. To overcome this effect, the rect-impulses of OFDM are smoothed by cos-roll-off functions, so the theoretical out-of-band radiation can be reduced enormously.
But real amplifiers with a saturation point in their non-linear characteristics as shown in fig. 4 cause high out-of-band radiation due to cutting the OFDM signal. To overcome this effect, the peak values have to be reduced. The ratio between normalized maximum output power and average input power is called input backoff (IBO).

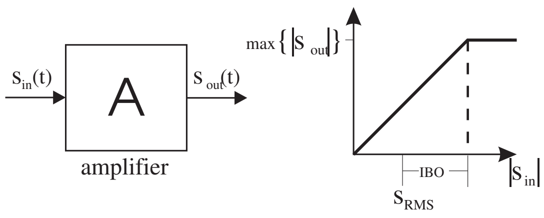

Figure 4: Amplifier characteristic with power saturation.

Assuming an amplifier characteristic as shown in fig. 4, the out-of-band radiation depends on the crest factor and the chosen IBO. The functionality of the crest factor reducing algorithm can be shown with constant IB0 = 3 dB. Therefore, some PDS of a OFDM signals with $\frac{N_c}{N} = \frac{32}{64}$ are computed while changing the iteration number. In fig. 5, four PDS are shown with variated iteration depth. The PDS of an untreated OFDM signal has the iteration depth 0. The reduction of the out-of-band radiation subjects to an increasing number of iterations, but on the other hand, the gain per iteration decreases.

6. CONCLUSION

In this paper, a crest factor reducing algorithm for an OFDM based multicarrier system combined with an adaptive subcarrier selection is presented. It is well known that OFDM requires any type of coding scheme. Instead of using channel coding, one can improve the performance by leaving out weak subcarriers. Simulations have shown the effect of this technique on the bit error rate. For many applications, this reduction of the bit error rate sufficies, so a coding scheme with high numerical effort is not necessary.
Furthermore, the introduced redundancy can be used, see section 5, to reduce the crest factor and hence to lower the out-of-band radiation. Here, the non-information carrying

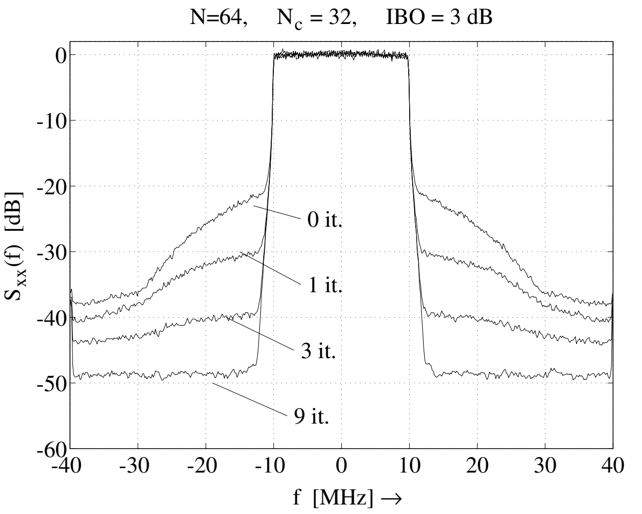

Figure 5: Spectrum and out-of-band radiation of an OFDM signal with an IBO=3 dB amplifier
a) without any crest factor reducing (0 it.),
b) with 1 iteration,
c) with 3 iterations,
d) with 9 iterations.

subcarriers are modulated depending on a threshold decision, so the overlay of the resulting reducing function and the information carrying signal significantly reduces the crest factor. The reduced out-of-band radiation of OFDM signals affirms the functionality of this technique. In this paper, an iterative algorithm with two FFT-blocks per iteration is used to compute the reducing function. Additional research has to be done to decrease the numerical effort.

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