AN ALTERNATIVE IMPLEMENTATION OF THE SUPERDIRECTIVE BEAMFORMER

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ABSTRACT

In this contribution we introduce a new implementation of superdirective beamformers. The new structure has the advantage of reduced computational complexity. This advantage is due to a GSC-like (Generalized Sidelobe Canceller) scheme. Unlike the conventional GSC, the filters in the sidelobe cancelling path are fixed and can be computed in advance by using the Wiener solution. The new structure yields exactly the same noise reduction performance as the superdirective beamformer does.

1. INTRODUCTION

Adaptive arrays have often been used for speech enhancement in the last decade, but the reported results are not very encouraging. On the other hand, fixed superdirective beamformers have become a renewed research field in the speech processing area [1, 2]. The main advantage of fixed beamformers is that no superresolution problems occur, and the problem of sensor noise can be solved by constrained design. Cox et al. [3] have shown that the design of superdirective beamformers and Frost's adaptive algorithm [4] are based on the same optimization criterion. Thus, the Frost-algorithm converges to the superdirective beamformer in an isotropic (diffuse) noise field. Furthermore, many authors [5, 6, 7] have shown that the Frost-algorithm is equivalent to the generalized sidelobe canceller (GSC) [8] under special conditions. Therefore, it is possible to implement the superdirective beamformer in a GSC-like structure having fixed filters in the sidelobe path. These filters can be computed in advance by using the Wiener optimization criterion.

In the next section, the Wiener solution for the GSC will be derived in terms of the complex coherence function of the input sensors. Therefore, the optimal solution for theoretically well-defined noise fields can be computed. In the third section, examples for broadside and endfire applications will be given and we can show that the computational complexity can be greatly reduced by our new structure. The last section shows the equivalence between the standard superdirective beamformer and our new GSC-like implementation.

2. ALGORITHMS

In a first step, we will introduce some slight changes in the design procedure for superdirective beamformers (SDB). This is necessary in order to compare the SDB with the new structure and to design the broad-band filter in terms of the complex coherence function.

Superdirective designs always include a propagation vector of the desired signal d. In line-arrays it is set to the delay or the phase shift between adjacent sensors. The second design variable is the cross-power-spectral-density (CPSD) matrix between all sensors. If we are using the spatial coherence function instead, a unified view on the design of SDBs can be given. Figure 1 shows a typical SDB-structure. We want to design the filter $A_i(\omega)$ in such a way, that only the additional superdirectivity is in the filter coefficients. Therefore, the coherence function after the phase shift has to be taken for the design and

$$\mathbf{d} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$$

Thus, the filter is given by

$$\mathbf{A}(\omega) = \frac{\Gamma^{-1}\mathbf{d}}{\mathbf{d}^{\mathbf{T}}\Gamma^{-1}\mathbf{d}} \tag{1}$$

and

$$\Gamma = \begin{pmatrix} 1 & \Gamma_{X_0X_1} & \dots & \Gamma_{X_0X_{N-1}} \\ \Gamma_{X_1X_0} & 1 & \dots & \Gamma_{X_1X_{N-2}} \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{X_{N-1}X_0} & \Gamma_{X_{N-1}X_1} & \dots & 1 \end{pmatrix}$$

is the coherence matrix. The coherences for a diffuse noise field behind the delay elements are given by

$$\Gamma_{X_iX_j}(\omega) = \frac{\sin(\frac{\omega d_{ij}}{c})}{\frac{\omega d_{ij}}{c}} e^{-j\frac{\omega d_{ij}\cos\Theta}{c}} \tag{2}$$

where d_{ij} and Θ denote the distances between the sensors and the direction of the desired signal, respectively. c represents the speed of sound.

The solution of equation 1 cannot be used directly for array design, because the result requires infinite precision of the sensors. In order to avoid this, Gilbert and Morgan [9] recommended to add a small scalar at the main diagonal of the cross-correlation matrix. We suggest a slightly different solution. The noise variance of the sensors can be included in the coherence function. For example, in a diffuse noise field, plus an uncorrelated noise with variance σ_n^2, the coherence is

$$\Gamma_{X_iX_j}(\omega) = \frac{\sin(\frac{\omega d_{ij}}{c})}{\frac{\omega d_{ij}}{c}(1+\frac{\sigma_n^2}{P_{nn}(\omega)})} e^{-j\frac{\omega d_{ij}\cos\Theta}{c}} \tag{3}$$

where $P_{nn}(\omega)$ is the assumed noise power spectral density of the diffuse noise field. The constrained design in eq. 3 gives suitable coefficients for an SDB. Typical values for the ratio between the room noise and the sensor noise are -20dB to -40dB.

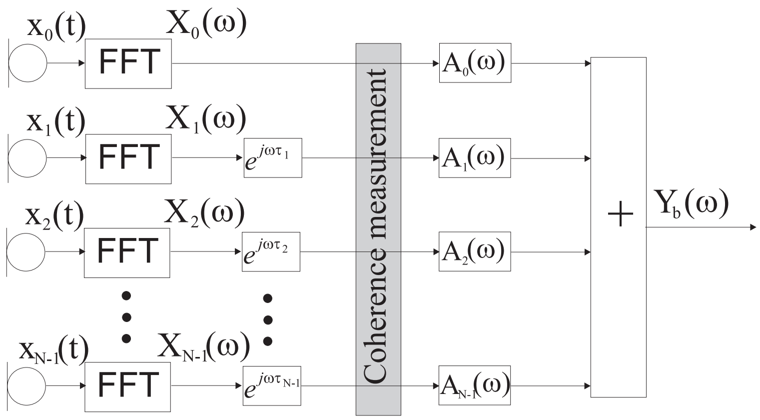

Figure 1: Block diagram of a superdirective beamformer in a frequency domain implementation, including measurepoints of coherence

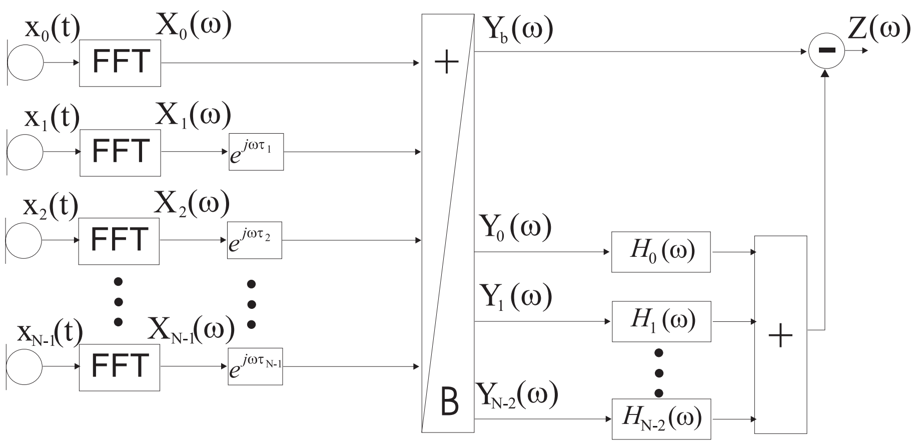

Figure 2: Block diagram of a GSC-like superdirective beamformer in a frequency domain implementation

2.1. Wiener Solution for GSC

The second step in the design of a GSC-like structure for superdirective beamformers needs a frequency domain structure of the GSC (see figure 2). Nordholm et al. [10] have derived a broadband Wiener solution for the GSC in the frequency domain in terms of the beamformer output $Y_b(\omega)$ and the blocking signals $Y_i(\omega)$. The fixed filters H_i in the sidelobe cancelling path are given by the cross-power spectral density (CPSD) matrix of the blocking signals

$$\mathbf{P_{Y_iY_i}} = \begin{pmatrix} P_{Y_0Y_0} & P_{Y_0Y_1} & \cdots & P_{Y_0Y_{N-2}} \\ P_{Y_1Y_0} & P_{Y_1Y_1} & \cdots & P_{Y_1Y_{N-3}} \\ \vdots & \vdots & \ddots & \vdots \\ P_{Y_{N-2}Y_0} & P_{Y_{N-2}Y_1} & \cdots & P_{Y_{N-2}Y_{N-2}} \end{pmatrix} \qquad (4)$$

and the CPSD-vector between the beamformer output and the blocking signals (for clarity, the frequency variable ω is omitted)

$$\begin{pmatrix} H_0 \\ H_1 \\ \vdots \\ H_{N-2} \end{pmatrix} = \mathbf{P_{Y_iY_i}}^{-1} \begin{pmatrix} P_{Y_bY_0} \\ P_{Y_bY_1} \\ \vdots \\ P_{Y_bY_{N-2}} \end{pmatrix} \qquad (5)$$

In order to express the CPSDs of the sidelobe path signals Y_i in terms of the CPSDs of the input signals X_i, the shading coefficients of the beamformer $A_i(\omega)$ and the $(N-1) \times N$ blocking-matrix

$$B = \begin{bmatrix} b_{00} & b_{01} & \cdots & b_{0N-1} \\ b_{10} & b_{11} & \cdots & b_{1N-1} \\ \vdots & \vdots & \ddots & \vdots \\ b_{(N-2)0} & b_{(N-2)1} & \cdots & b_{N-2N-1} \end{bmatrix} \qquad (6)$$

have to be taken into account. This results in

$$P_{Y_iY_j}(\omega) = \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} b_{ik}^* b_{jl} X_k^*(\omega) X_l(\omega) \qquad (7)$$

and

$$P_{Y_bY_i}(\omega) = \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} a_k^* b_{il} X_k^*(\omega) X_l(\omega) \qquad (8)$$

for all $0 \leq i, j \leq N-2$. If we now assume that the PSD of the noise field P_{nn} is the same at each sensor, we can rewrite equation (7) and equation (8) in terms of the coherence function $\Gamma(\omega)$ of the input noise field:

$$P_{Y_iY_j}(\omega) = P_{nn} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} b_{ik}^* b_{jl} \Gamma_{X_kX_l}(\omega) \tag{9}$$

$$P_{Y_bY_i}(\omega) = P_{nn} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} a_k^* b_{il} \Gamma_{X_kX_l}(\omega) \tag{10}$$

Therefore, the filters H_i do not depend on P_{nn} and only the spatial characteristics of the noise determine the filters.

To compute the solution for a superdirective beamformer the coherence function in equation (9) and (10) is set in order to describe a diffuse noise field (see eq. (3)).

3. EXAMPLES

3.1. Two Sensors

As a first example we assume that only two channels are available. We set the coefficients to

$$A(\omega) = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & -1 \end{bmatrix} \quad .$$

In this case, there is only one filter H_0. It can be computed directly and is given by [11]

$$H_0(\omega) = j \frac{\Im\{\Gamma_{X_0X_1}(\omega)\}}{1 - \Re\{\Gamma_{X_0X_1}(\omega)\}} \tag{11}$$

The filter consists only of an imaginary part. The real part is zero at all frequencies. Equation (11) shows that no noise reduction can be achieved in the broadside case, since the coherence is real valued only. Figure 3 shows the complex coherence function for the endfire case (d = 5cm). The advantages of our new structure

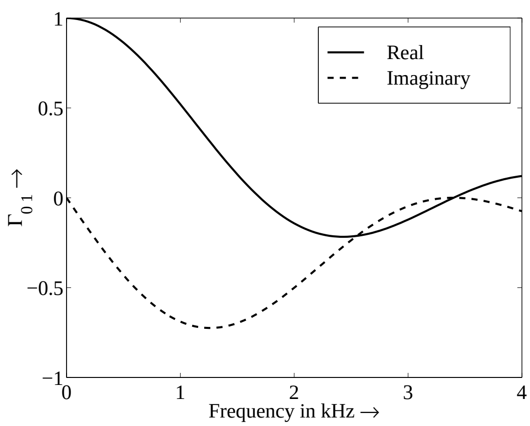

Figure 3: Complex coherence function in a diffuse noise field after endfire steering ($d = 5cm$)

compared to the SDB are:

- Instead of two filters only one is used.
- A delay-and-sum-beamformer output is available at the upper path
- The lower path can be used to estimate the noise level, since the blocking signals contain no desired signal.

3.2. Four Sensors

Our second example consists of four omnidirectional microphones. The conventional beamformer is set to the standard delay-and-sum-beamformer. In this example the influence of the blocking matrix is very important. We are using the standard Griffith-Jim blocking matrix

$$B_1 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \tag{12}$$

and the Walsh-blocking matrix

$$B_2 = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \tag{13}$$

In the broadside case the filters are real-valued only. Interestingly, however, if we examine B_1, the filter H_1 will be zero at all frequencies, and by using B_2 the filters H_0 and H_2 will be zero. Therefore, the computational complexity can be reduced from four filters in the SDB to one filter and a few additions in our new structure. The results for endfire steering are similar. B_2 reduces the complexity from four complex filters in SDB to three filters, whereas H_0 and H_2 are imaginary-valued only and H_1 is real-valued only.

4. ANALYSIS

In order to show the equivalence between SDBs and our new structure, we examine the noise reduction performances (NR). NR for beamformers can be computed by [12] (ω is omitted)

$$NR_b = \frac{\left| \sum_{i=0}^{N-1} a_i \right|^2}{\sum_{i=0}^{N-1} |a_i|^2 + 2 \sum_{i=0}^{N-2} \sum_{j=i+1}^{N-1} \Re\left\{ a_i a_j^* \Gamma_{X_iX_j} \right\}} \tag{14}$$

We are only interested in the additional NR of the superdirective part. Therefore, we have to compute the NR according to equation (14) for the delay-and-sum-beamformer and for the superdirective beamformer. The additive NR is given by the ratio between the two results. Figure 4 shows some results for broadside, and figure 5 for endfire arrays. The ratio for the white noise gain constraint is set to 0.01, and a perfectly diffuse noise field is assumed (directivity index).

The NR of the GSC-part is given by

$$\begin{aligned} NR_{gsc}(\omega) &= \frac{P_{Y_bY_b}(\omega)}{P_{ZZ}(\omega)} \\ &= \frac{P_{Y_bY_b}(\omega)}{P_{Y_bY_b}(\omega) - \sum_{i=0}^{N-2} \sum_{j=0}^{N-2} H_i(\omega)^* H_j(\omega) P_{Y_iY_j}} \\ &= \frac{1}{1 - \sum_{i=0}^{N-2} \sum_{j=0}^{N-2} H_i(\omega)^* H_j(\omega) \frac{P_{Y_iY_j}}{P_{Y_bY_b}}} \end{aligned} \tag{15}$$

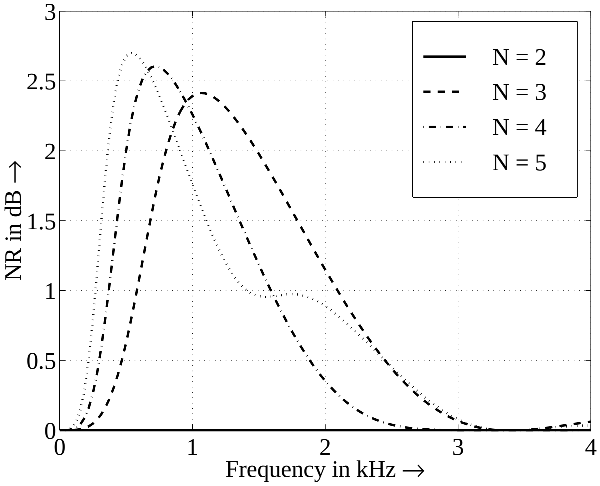

Figure 4: Additional noise reduction performance of SDBs ($d = 5cm$) for different numbers of sensors N (broadside) (note that there is no additional noise reduction for $N = 2$)

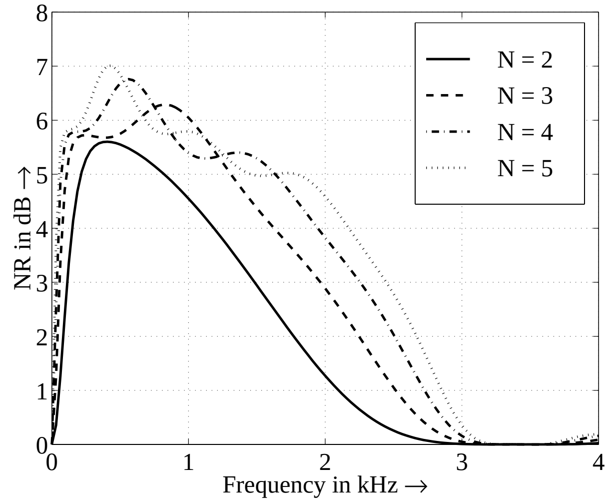

Figure 5: Additional noise reduction performance of SDBs ($d = 5cm$) for different numbers of sensors N (endfire)

where

$$P_{Y_bY_b}(\omega) = P_{nn} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} a_k^* a_l \Gamma_{X_k X_l}(\omega)$$

is the PSD of the beamformer output.

Using the same constrained design parameter (ratio = 0.01) and computing the directivity index shows that the superdirective array and the fixed GSC beamfomer have exactly the same noise reduction performance (figure 4 and 5).

5. CONCLUSION

In this paper we have derived the Wiener-solution of the generalized sidelobe canceller in terms of the input coherence function. We have shown that the results lead to a superdirective beamformer. The reduction of computational complexity and the possibility to extend the structure are main advantages of our new scheme. Furthermore, we have described a unified view of the design of superdirective beamformers in terms of the complex coherence function.

6. REFERENCES

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