MULTICHANNEL NOISE REDUCTION –ALGORITHMS AND THEORETICAL LIMITS–

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ABSTRACT

In this contribution we present a theoretical study on the performance of several multichannel noise reduction algorithms. It is known that the magnitude squared coherence (MSC) determines the performance of a class of adaptive algorithms i.e. active noise control or noise reduction with a reference microphone. However, the MSC is not sufficient for performance evaluation of other noise reduction methods like the Generalized Sidelobe Canceller (GSC) or adaptive post–filter techniques. We propose to measure the spatial complex coherence (CC) or normalized cross power spectrum of the sound field and show that it can be used as a much more general tool for performance analysis. First of all, we summarize the results of previous studies and present new results for the performance of the Generalized Sidelobe Canceller (GSC) as a function of the complex coherence. In the second part we examine different noise fields to show theoretical limits of multichannel noise reduction schemes.

1 INTRODUCTION

Noise reduction for hands–free telephones is still a problem for a low signal–to–noise–ratio (SNR). Several noise reduction systems were introduced over the last few years. Multichannel approaches have advantages, especially if the noise has a non–stationary characteristic. In this contribution we will give some formulas to compute theoretical limits for the noise reduction performance of multichannel techniques. Furthermore, we show that the performance only depends on the spatial complex coherence (CC) function of the noise field.

2 PERFORMANCE ANALYSIS

Noise reduction performance is an important feature for speech enhancement systems. In the following we will show that the theoretical limits of the performance of many multichannel solutions can be evaluated by measuring the complex coherence of the noise field.

2.1 Complex Coherence

The complex coherence function is defined as

$$\Gamma_{X_1X_2}(\omega) = \frac{P_{X_1X_2}(\omega)}{\sqrt{P_{X_1X_1}(\omega)P_{X_2X_2}(\omega)}} \tag{1}$$

where P_X denotes cross– or auto–power–spectral–densities. Furthermore, we also use the average complex coherence function of the noise field for all sensor pairs $i \neq j$.

$$\bar{\Gamma}(\omega) = \frac{2}{N^2-N} \sum_{i=0}^{N-2} \sum_{j=i+1}^{N-1} Re\left\{\Gamma_{ij}(\omega)\right\} \tag{2}$$

where N denotes the number of microphones.

2.2 Delay and Sum Beamformer

The improvement of the SNR or noise reduction factor $NR(\omega)$ of the Delay and Sum beamformer is given by [1]

$$NR_b(\omega) = \frac{1}{\frac{1}{N} + \left(1 - \frac{1}{N}\right)\bar{\Gamma}(\omega)} \tag{3}$$

The noise reduction depends on the number of microphones and the average real part of the complex coherence. The Delay and Sum Beamformer performs best for small or negative real parts of the coherence.

2.3 Adaptive Post–Filter

Assuming that the transfer function of an optimal post–filter can be estimated as

$$\begin{aligned} \widehat{W}(\omega) &= \frac{\frac{2}{N^2-N} \sum_{i=0}^{N-2} \sum_{j=i+1}^{N-1} Re\left\{X_i(\omega)X_j^*(\omega)\right\}}{\left|Y_b(\omega)\right|^2} \\ &= \frac{\frac{2}{N^2-N} \sum_{i=0}^{N-2} \sum_{j=i+1}^{N-1} Re\left\{P_{X_{ij}}\right\}}{P_{Y_bY_b}} \end{aligned} \tag{4}$$

it can be shown that [2]

$$\widehat{W}(\omega) = \frac{SNR_b(\omega)}{1+SNR_b(\omega)} + \frac{NR_b(\omega)}{1+SNR_b(\omega)}\bar{\Gamma}(\omega) \tag{5}$$

where $SNR_b(\omega)$ is the signal to noise ratio at the output of the beamformer. The noise reduction performance is

$$NR_w(\omega) = \frac{1}{\widehat{W}^2(\omega)} \quad (6)$$

The relation to the result for the Delay and Sum Beamformer is obvious. If the Delay and Sum Beamformer works well the adaptive filter is effective for low SNRs.

2.4 Noise Canceller [3]

The noise reduction performance of Widrow's noise cancelling approach is [4]

$$NR_{nc} = \frac{1}{1 - |\Gamma_{X_i X_r}(\omega)|^2} \quad (7)$$

where $|\Gamma_{X_i X_r}(\omega)|^2$ denotes the magnitude squared coherence function (MSC) between the main input X_i and the reference input channel X_r.

2.5 Generalized Sidelobe Canceller [5]

For the special case of a two–channel GSC-structure (see figure 1) we present a new solution for the noise reduction performance in terms of the complex coherence of the noise field. The output of the GSC is given by

$$Y_o(\omega) = Y_b(\omega) - H(\omega) Y_i(\omega) \quad (8)$$

Minimizing the output power results in

$$H_{opt}(\omega) = \frac{P_{Y_i Y_b}(\omega)}{P_{Y_i Y_i}(\omega)} \quad (9)$$

where $P_{Y_i Y_i} = 2P_{NN}(1 - Re\{\Gamma_{X_1 X_2}\})$ [6]. The cross–power–spectral–density between the sum Y_b and the difference path Y_i is given by

$$P_{Y_i Y_b} = P_{X_1 X_1} - P_{X_2 X_2} - P_{X_1 X_2} + P_{X_2 X_1} \quad (10)$$

If we assume that no desired signal is present and that the noise is $P_{NN} = P_{X_1 X_1} = P_{X_2 X_2}$, it follows

$$\begin{aligned} P_{Y_i Y_b} &= -P_{X_1 X_2} + P_{X_2 X_1} = 2j Im\{P_{X_1 X_2}\} \\ &= 2j P_{NN} Im\{\Gamma_{X_1 X_2}\} \end{aligned} \quad (11)$$

The noise reduction factor of the sidelobe-cancelling path is now given by

$$NR_{GSC_2} = \frac{P_{Y_b Y_b}(\omega)}{P_{Y_b Y_b}(\omega) - |H_{opt}|^2 P_{Y_i Y_i}(\omega)} \quad (12)$$

$$\begin{aligned} &= \frac{1}{1 - \left| \frac{2j P_{NN} Im\{\Gamma_{X_1 X_2}(\omega)\}}{2P_{NN}(1 - Re\{\Gamma_{X_1 X_2}(\omega)\})} \right|^2 \frac{P_{NN}(1 - Re\{\Gamma_{X_1 X_2}(\omega)\})}{P_{NN}(1 + Re\{\Gamma_{X_1 X_2}(\omega)\})}} \\ &= \frac{1}{1 - \frac{Im\{\Gamma_{X_1 X_2}(\omega)\}^2}{1 - Re\{\Gamma_{X_1 X_2}(\omega)\}^2}} \end{aligned} \quad (13)$$

Equation 13 shows, that the performance depends on the real and imaginary part of the coherence. The MSC does not provide enough information to predict the noise reduction factor.

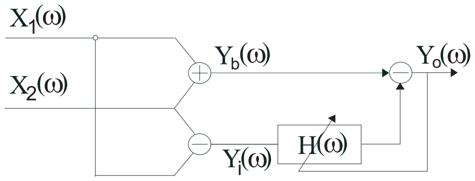

Figure 1: Two–channel GSC structure

3 PERFORMANCE LIMITS

In this section we show some analytical limits of multichannel noise reduction systems. We assume that only two channels are available. A completely coherent, an incoherent and a diffuse noise field will be examined.

3.1 Coherent Noise Field

Assuming a single noise source in the far field of the sensor array the complex coherence function is given by

$$Re\{\Gamma_{X_1 X_2}(\omega)\} = \cos\left(\frac{\omega \cos(\theta) d}{c}\right) \quad (14)$$

$$Im\{\Gamma_{X_1 X_2}(\omega)\} = -\sin\left(\frac{\omega \cos(\theta) d}{c}\right) \quad (15)$$

where d denotes the microphone distance, θ the angle of arrival and c the speed of sound.

- **Delay and Sum Beamformer:** According to equations 3 the noise reduction performance is

$$NR_b(\omega) = \frac{2}{1 + \cos\left(\frac{\omega \cos(\theta) d}{c}\right)} \quad (16)$$

- **Adaptive Post–Filter:** The adaptive post–filter depends on the SNR at the output of the Delay and Sum Beamformer. But due to these dependencies there is no noise reduction for frequencies with an average coherence $\bar{\Gamma}(\omega) = 1$.
- **Noise Canceller:** The magnitude–squared coherence is one and the noise reduction performance reaches ∞.
- **GSC:** Equation 13 shows that the noise reduction performance is ∞ in the theoretical analysis, but the performance is reduced by practical problems such as finite length filters and sensor noise.

3.2 Incoherent Noise Field

The coherence in the incoherent noise field is zero for all frequencies. This may be caused by sensor noise, for example.

- **Delay and Sum Beamformer:** The noise reduction performance depends only on the number of sensors. This is a well–known property of Delay and Sum Beamformers.

- **Adaptive Post–Filter:** In this case equation 5 is a Wiener optimal solution, which can have a high noise reduction factor for frequencies with low SNR:

$$W(\omega) = \frac{SNR_b(\omega)}{1 + SNR_b(\omega)} \quad (17)$$

- **Noise Canceller:** For the incoherent noise field there is no capability to suppress noise. The noise reduction is always zero dB.

- **GSC:** The results for the adaptive part of the GSC are similar, but there is still the noise reduction of the fixed beamformer.

3.3 Diffuse Noise Field

In a diffuse noise field the coherence is real valued only and given by

$$Re\{\Gamma_{X_1X_2}(\omega)\} = si\left(\frac{\omega d}{c}\right) \quad Im\{\Gamma_{X_1X_2}(\omega)\} = 0 \quad (18)$$

- **Delay and Sum Beamformer:** For the diffuse noise field the noise reduction performance is the well–known array gain. For low frequencies there is only a very small NR. For higher frequencies the NR in dB is approximately $10\log_{10} N$.

- **Adaptive Post–Filter:** Its performance is quite similar to the Delay and Sum Beamformer; no or only a little NR for low frequencies, but a very good NR performance where $\bar{\Gamma}(\omega)$ is close to zero.

- **Noise Canceller:** There is a very high potential of noise reduction, but only for low frequencies because of

$$NR_{nc} = \frac{1}{1 - si^2\left(\frac{\omega d}{c}\right)} \quad . \quad (19)$$

  Furthermore, there is a problem with signal cancellation effects due to small amounts of desired signal in the reference sensor.

- **GSC:** Equation 13 shows that the noise reduction performance is zero dB for all frequencies, since the imaginary part of the complex coherence in the diffuse noise field is zero.

4 SIMULATION

To give some realistic performance measurements we simulated a room with a reverberation time $\tau_{60} = 400ms$ according to the image method by Allen and Berkley[7]. The microphone distance is 20cm and the noise source is stationary white Gaussian noise with a direction of arrival of 34 degree. The input SNR is -6dB. Figure 2 shows the real and imaginary part of the complex coherence $\Gamma_{X_1X_2}$. For this room and this type of noise source the noise reduction performance is depicted in figures 3 to 6. The results show that none of the examined multichannel noise reduction schemes is able to suppress low frequency noise ($f < \frac{c}{2d}$), except for the noise canceller approach. However, this structure fails if the desired signal is present, due to signal cancellation.

5 CONCLUSION

In this paper we have presented a theoretical study of several multichannel noise reduction techniques. New formulas for the noise reduction limits of the GSC structure were introduced. The performance analysis shows that no multichannel approach performs well at low frequencies. Furthermore, we found that the complex coherence function is a valuable tool to examine the theoretical limits of this class of noise reduction schemes.

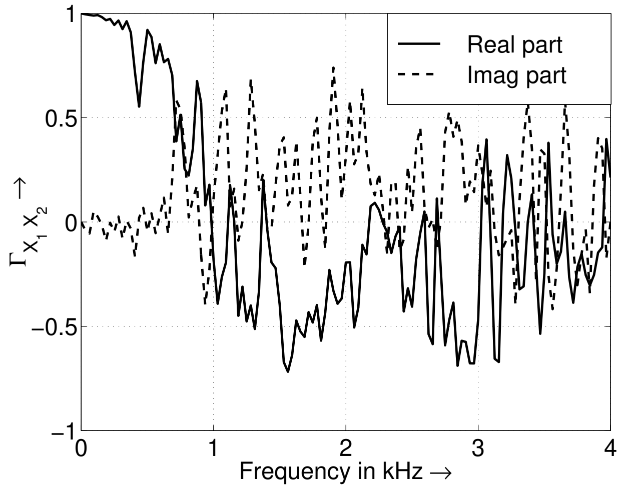

Figure 2: Real and imaginary part of the complex coherence

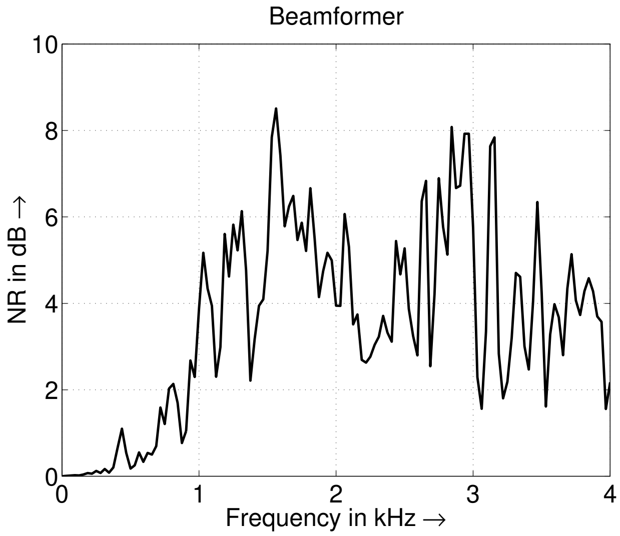

Figure 3: Noise reduction performance of the Delay and Sum Beamformer

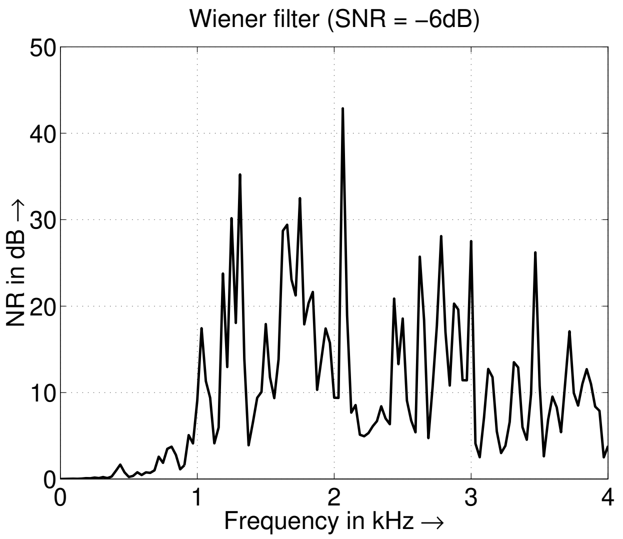

Figure 4: Noise reduction performance of the optimal post–filter

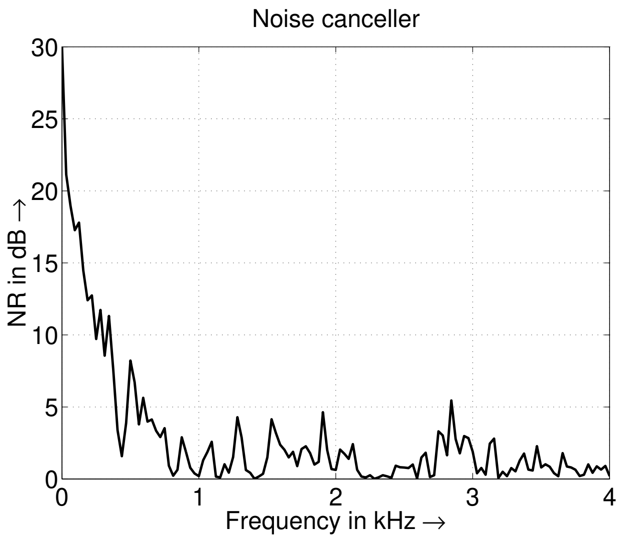

Figure 5: Noise reduction performance of the noise canceller with a reference microphone

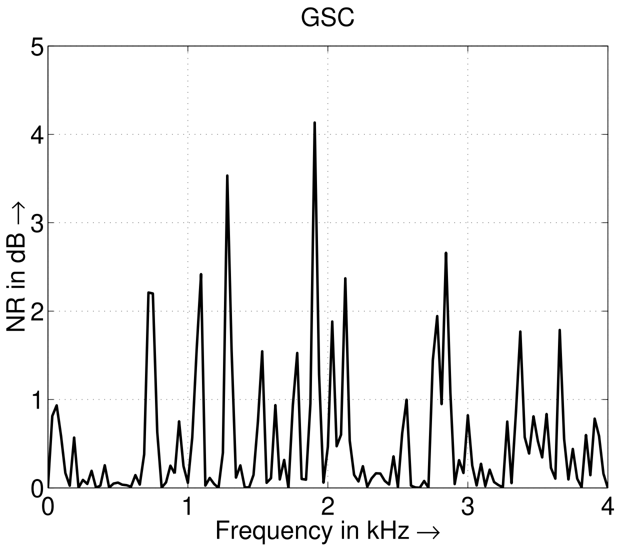

Figure 6: Noise reduction performance of a two-microphone Generalized Sidelobe Canceller

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