BLIND GSM CHANNEL ESTIMATION BASED ON HIGHER ORDER STATISTICS

Dieter Boss and Karl-Dirk Kammeyer

University of Bremen, FB-1, Department of Telecommunications

P.O. Box 33 04 40, D-28334 Bremen, Germany, Fax: +(49)-421/218-3341, e-mail: boss@comm.uni-bremen.de

Abstract: The performance of many comm. systems could be improved if the transmission channel was estimated *blindly*, i.e. without training sequences. As an example, we investigate in this paper whether, on GSM conditions, the blind channel estimation method EVI (EIGENVECTOR APPROACH TO BLIND IDENTIFICATION) can compete with the non-blind least squares scheme based on the cross-correlation. For Gaussian stationary uncorrelated scattering channels, we give simulated bit error rates (BER) after Viterbi detection in terms of the mean signal-to-noise ratio ($\overline{\mathrm{SNR}}$) for blind, non-blind, and ideal channel estimation. Averaged over three COST-207 propagation environments, EVI leads to an $\overline{\mathrm{SNR}}$ loss of 1.1 dB only, which is quite remarkable for an approach based on higher order statistics, as just 142 samples can be used for blind channel estimation.

1. INTRODUCTION: LINEAR GSM CHANNEL MODEL

CONSIDER the equivalent baseband representation of a GSM communication system in Fig. 1, where source and channel coding are omitted to enhance clarity. In the transmitter, coded information bits $d(k)$ together with reference bits are assembled into *bursts* of 142 bits, where a value from $\{-1, 1\}$ is taken each bit period $T = 48/13\ \mu\mathrm{s} \approx 3.7\ \mu\mathrm{s}$. Each burst is encoded differentially to facilitate demodulation. Then, it is modulated by *Gaussian Minimum Shift Keying (GMSK)* with $f_{3dB} = 0.3/T$ and transmitted over the radio channel.

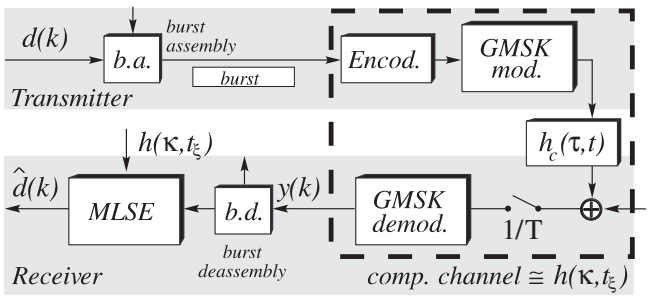

Figure 1: GSM communication system

In a mobile setting, the physical multipath radio channel is *time-variant* with a baseband impulse response depending on the time difference τ between the observation and excitation instants as well as the (absolute) observation time t. We adopt the stochastic zero mean *Gaussian Stationary Uncorrelated Scattering (GSUS)* model leading to the following impulse response of the channel including the receive filter [1]

$$h_c(\tau, t) = \frac{1}{\sqrt{N_e}} \sum_{\nu=1}^{N_e} e^{j(2\pi f_{d,\nu} t + \Theta_\nu)} \cdot g_{Rc}(\tau - \tau_\nu)\,, \qquad (1)$$

where N_e is the number of elementary echo paths, $g_{Rc}(\tau)$ denotes the receive filter impulse response, and the subscript in $h_c(\cdot)$ suggests its continuous-time property. 3D sample impulse responses can easily be determined from (1) by independently drawing N_e Doppler frequencies $f_{d,\nu}$, N_e initial phases Θ_ν, and N_e echo delay times τ_ν from random variables with Jakes, uniform, and piecewise exponential probability density functions, respectively. As for the echo delay times τ_ν, we use standard COST-207[1] *Typical Urban (TU)*, *Bad Urban (BU)* and *Hilly Terrain (HT)* profiles.
With time-variance being relatively slow in both GSM-900 and DCS-1800 applications ($f_{d,max}^{-1} = (226\ \mathrm{Hz})^{-1} \approx 4.4\ \mathrm{ms} \gg T$), the channel can be assumed *quasi (piecewise) time-invariant*, i.e. time-invariant over a certain number Δk of bit periods T

$$h_c(\tau, t) \approx h_c(\tau, t_\xi) \quad \text{for} \quad |t - t_\xi| \le \Delta k\, T/2\ . \qquad (2)$$

According to Figure 1, each received burst is corrupted by additive Gaussian noise $r(t)$ colored by the 5th order Butterworth receive filter $g_{Rc}(\tau)$ with cut-off frequency at 75 kHz which we use throughout the paper for reasons of adjacent channel suppression (GSM carrier spacing is 200 kHz). Upon symbol-rate sampling, a simple derotation demodulator can be used to obtain the quasi stationary sequence $y(k)$. If at most Δk consecutive samples of $y(k)$ are observed at once, $h_c(\tau, t)$ in Fig. 1 can be replaced with the current channel slice $h_c(\tau, t_\xi)$ according to (2).

Maximum Likelihood Sequence Estimation (MLSE) represents the optimum procedure to remove intersymbol interference from a received digital communication signal such as $y(k)$. However, it assumes the "composite channel", i.e. the equivalent symbol-rate system between $d(k)$ and $y(k)$ (see the dashed frame in Fig. 1), to be *(R1)* linear with finite impulse response which *(R2)* must be known. *(R3)* Moreover, the noise component of $y(k)$ is supposed to be white. Strictly speaking, none of these requirements is met in typical GSM systems.

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[1] European Cooperation in the field of Scientific and Technical research.

However, as for requirement *(R1)*, it can be shown that the composite channel can be approximated by the *linear* model

$$h(\kappa, t_\xi) = \begin{cases} j^{-\kappa} \cdot g_0(\kappa T, t_\xi) & \text{for } \kappa \in \mathcal{K} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

with

$$g_0(\tau, t) \stackrel{\Delta}{=} c_0(\tau) * h_c(\tau, t)\,, \quad (4)$$

where $c_0(\tau)$ represents the Laurent approximation [2] of the (nonlinear) GMSK modulator, "$*$" denotes convolution and the factor $j^{-\kappa}$ takes the combined effects of differential encoding and derotation demodulation into account. To obtain an FIR model, let $h(\kappa, t_\xi)$ be constrained to the range $\mathcal{K}$ of "relevant" indices κ.

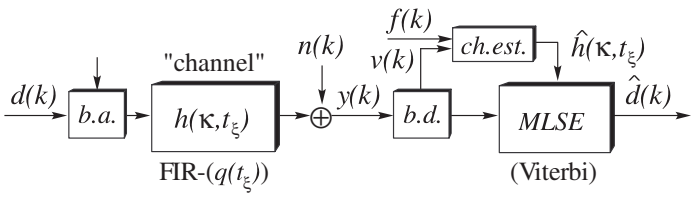

Figure 2: GSM system with FIR model of the composite channel

Using the linear model $h(\kappa, t_\xi)$ according to (3) for the composite channel, Fig. 1 can be redrawn according to Fig. 2. In the sequel, $h(\kappa, t_\xi)$ will simply be termed "channel". Note that for any fixed value of t_ξ, the channel slice $h(\kappa, t_\xi)$ may be mixed-phase. Let $q(t_\xi)$ denote its effective order, which may vary from slice to slice due to time selective fading. The noise sequence $n(k)$ emerges from $r(t)$ in Fig. 1 by symbol-rate sampling and demodulation. Although $n(k)$ is colored due to the Butterworth receive filter, this is tolerated in typical GSM receivers (c.f. requirement *(R3)*).

According to *(R2)*, MLSE requires the knowledge of $h(\kappa, t_\xi)$ in order to equalize a block of Δk samples of $y(k)$. As this knowledge is not available, the problem of *channel estimation* arises. This is indicated in Fig. 2 by a box termed channel estimation ("ch.est."), which is to deliver an estimate $\hat{h}(\kappa, t_\xi)$ of $h(\kappa, t_\xi)$.

In the following section, we describe the principal possibilities to estimate $h(\kappa, t_\xi)$. We focus on two algorithms which we will compare in section 3 in terms of their channel estimation quality as well as the resulting bit error performance after MLSE.

2. BLIND AND NON-BLIND CHANNEL ESTIMATION

Channel estimation is a particular form of *system identification*. In our case, the system is assumed mixed-phase linear (quasi) time-invariant with finite impulse response $h(\kappa, t_\xi)$. Such a system can be identified with or without reference data (training sequences).

As an example for an approach based on training sequences, consider the least squares channel estimation scheme which could be used in GSM receivers. According to Figure 3, each "normal" burst contains a training sequence $f(k)$ of 26 bits surrounded by two packets of 58 data bits emerging from the information sequence $d(k)$. On the assumption of time-invariance over 26 bit periods, channel estimates $\hat{h}(\kappa, t_\xi)$ can be derived from the received sequence sampled at symbol-rate by using the sample cross-correlation between the demodulated (corrupted) and the stored (ideal) training sequences $v(k)$ and $f(k)$, respectively (see Fig. 2). For any given order $\hat{q}$ of the FIR system to be estimated, the optimum estimate in the least squares (LS) sense is given by

$$\begin{bmatrix} \hat{h}(0, t_\xi) \\ \hat{h}(1, t_\xi) \\ \vdots \\ \hat{h}(\hat{q}, t_\xi) \end{bmatrix} = \begin{bmatrix} f(\hat{q}) & \cdots & f(0) \\ \vdots & \ddots & \vdots \\ f(25-\hat{q}) & \ddots & f(\hat{q}) \\ \vdots & \ddots & \vdots \\ f(25) & \cdots & f(25-\hat{q}) \end{bmatrix}^{\dagger} \begin{bmatrix} v(\hat{q}) \\ v(\hat{q}+1) \\ \vdots \\ v(25) \end{bmatrix}, \quad (5)$$

where $\dagger$ denotes the matrix pseudoinverse. Eq. (5) will be referred to as the LS channel estimation scheme. Note that state-of-the-art GSM receivers may apply simpler (suboptimum) approaches. N.B.: Although the LS channel estimation scheme assumes time-*in*variance of the channel over a period of $\Delta k = 26$ training bits only, the resulting estimate is used for MLSE on the adjacent data fields. As the channel coefficients might already have changed in the data fields, there is an implicit assumption of *piecewise time-invariance over one burst* ($\Delta k = 142$) in this concept.

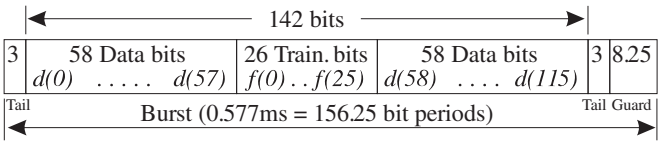

Figure 3: GSM "normal" burst

The repeated transmission of training sequences leaves a GSM system with an overhead capacity of $26/116 = 22.4\%$ which could be used for other purposes such as channel coding if the channel estimation problem was solved *blindly*.

The fundamental idea of *blind* system identification is to derive the complete channel characteristics (including the phase) from the received signal only, i.e. *without* training sequences. As just one symbol-rate sampled received sequence is available in common GSM mobile units, in the downlink this can only be accomplished by exploiting *Higher Order Statistics (HOS)* of the (piecewise) stationary demodulated sequence $y(k)$. Higher order *cumulants* contain the complete information on the channel's magnitude and phase provided that the distribution of $d(k)$ is non-Gaussian (which is true for digital communication applications).

Remark: If the sampling period was a fraction of T, or alternatively, the symbol-rate sampled signals received by several antennae were interleaved (in the uplink, e.g.), the resulting demodulated sequence is (quasi) *cyclostationary*. Generally, *Second Order Cyclostationary Statistics* (SOCS) are sufficient to retrieve the complete channel characteristics. However, there are "singular" channel classes such as those with common zeros in all polyphase subchannels which can *not* be identified [3]. We have demonstrated in [4] that singular channels represent a severe limitation to SOCS-based methods because the channel estimation performance from few samples is heavily affected if subchannel zeros are just "close" to each other. Because in a mobile environment, subchannel zeros can *not* be prevented from approaching each other, SOCS-based algorithms are neglected in this paper.

In summary, we have used the following criteria for the selection of a blind channel estimation algorithm:

(1) As the eff. channel order is unknown (and just *quasi* time-invariant), an order misfit must not represent a problem.

(2) **Reliable complex channel estimates must be obtained from 142 samples of the demodulated sequence $y(k)$.**

(3) This should apply to arbitrary channels (even if there are zeros 'on' or 'close to' the complex plane's unit circle).

(4) The estimates should be as robust as possible with respect to additive Gaussian noise.

We have selected the EIGENVECTOR APPROACH TO BLIND IDENTIFICATION (EVI) by Jelonnek, Boss and Kammeyer [5, 6], which is based on 4th (and 2nd) order statistics[2], because recent simulation results have shown that it meets the above criteria. For linear modulation with raised cosine transmit and receive filtering, we have demonstrated in [7] that EVI can blindly estimate COST-207 channels from 142 samples within a normalized mean square error bound of about 5 per cent (at a constant SNR of 7 dB). Notice that EVI's estimation performance was also compared with methods based on SOCS [4] and HOS [7]. On the above conditions, EVI's estimation performance was found to be superior in both cases.

3. SIMULATION RESULTS

Fig. 4 shows the magnitude impulse response $|g_0(\tau, t)|$ according to (4), where $h_c(\tau, t)$ is a sample COST-207 *Hilly Terrain (HT)* channel obtained from (1) with $N_e = 100$. Both time axes are normalized to the GSM bit period $T \approx 3.7\,\mu$s. The maximum Doppler frequency is set to $f_{d,max} = 200$ Hz. For GSM-900 and DCS-1800, this corresponds to velocities of the mobile unit in the ranges $225 \ldots 243$ km/h and $115 \ldots 126$ km/h, respectively. For Figure 4, equation (4) is evaluated over a t range covering three minimum Doppler periods $T_{d,min} = 1/f_{d,max} = 5$ ms, i.e. 15 ms, or equivalently, 4062 bit periods or 26 burst periods.

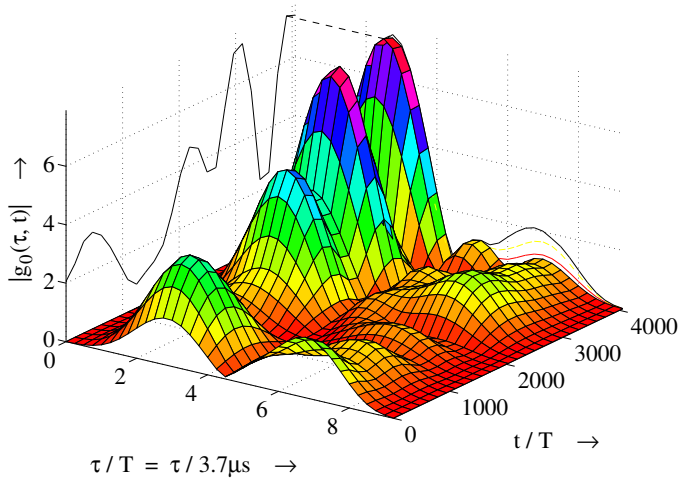

Figure 4: Linear model of a sample *Hilly Terrain* composite channel

Assuming *quasi time-invariance over one burst*, we can sample $g_0(\tau, t)$ at $t_\xi = \xi \cdot 156.25\,T$ (c.f. Fig. 3). This produces 26 slices $g_0(\tau, t_0), \cdots, g_0(\tau, t_{25})$ within the t range of $4062T$, which can be seen in Fig. 4 as surface lines parallel to the τ axis. Note that the magnitude impulse response $|h(\kappa, t_\xi)|$ of the linear model (3), which is the FIR system to be estimated for MLSE, can also be seen in Figure 4, if $|g_0(\tau, t_\xi)|$ is sampled at $\tau = \kappa T$.

For the simulation of a GSM data transmission link according to Fig. 1, the impulse response $h_c(\tau, t_\xi)$ is required rather than $g_0(\tau, t_\xi)$. However, $|h_c(\tau, t)|$ looks pretty much like Fig. 4 because of the bell-shaped form of the Laurent impulse $c_0(\tau)$. Just the "hills" are slightly narrower (in direction of τ).

Since at most each 4th burst is sent to/from the same mobile station, just each 4th slice $h_c(\tau, t_{4\xi})$ should be taken into account. As this would leave us with just six remaining slices within the period of time considered above, we use a much wider range of t for the simulations to ensure a satisfactory approximation of a GSUS channel by a sample impulse response $h_c(\tau, t)$. For the simulation results given below, a total of 1000 channel slices $h_c(\tau, t_{4\xi})$ with $\xi = 0, \cdots, 999$ is used. Note that this covers a t range of 2.3s, i.e. 4000 burst periods ($625000T$) or 462 minimum Doppler periods.

For the following description of the simulation procedure, let D_ξ denote the ξ-th burst containing 156 bits with values from $\{-1, 1\}$. Also assume that we have selected a sample impulse response $h_c(\tau, t)$ and a value of the mean signal-to-noise ratio $\overline{\mathrm{SNR}}$.
Referring to Fig. 1 with $h_c(\tau, t_{4\xi})$ substituted for $h_c(\tau, t)$, the burst D_ξ is encoded, modulated, and then propagated through the channel slice $h_c(\tau, t_{4\xi})$. Additive Gaussian noise $r(t)$, colored by the receive filter, is added according to $\overline{\mathrm{SNR}}$. Upon symbol-rate sampling and demodulation, 148 samples of $y(k)$ are obtained.
Then, both the non-blind least squares (LS) and the blind EVI channel estimation schemes are applied to $y(k)$, where the former utilizes the training midamble of 26 bits (see eq. (5)) while the latter uses 142 samples[3] of $y(k)$. Both approaches are given the number $L = \hat{q}+1$ of channel coefficients to be estimated, where L is calculated from the effective length of the sample power delay spectrum of the respective GSUS channel. Note that the actual effective length $q(t_{4\xi}) + 1$ of the channel slice $h(\kappa, t_{4\xi})$ may well be shorter (or longer) due to time selective fading (see Fig. 4 at $t \approx 1200T$, e.g.). The resulting channel estimates $\hat{h}(\kappa, t_{4\xi})$ are then passed to the Viterbi detector to obtain the est. bursts $\hat{D}_\xi$.
In the frame of Monte-Carlo runs, this procedure is executed for 1000 bursts $D_0, \cdots, D_{999}$ and $\overline{\mathrm{SNR}}$ values in the range from 0 to 16 dB. Finally, the bit error rate (BER) is calculated from all bursts $\hat{D}_\xi$ and D_ξ transmitted at a given $\overline{\mathrm{SNR}}$.

Each channel estimate $\hat{h}(\kappa, t_{4\xi})$, $\xi = 0, \cdots, 999$ is assessed on the basis of the *Normalized Mean Square Error (*NMSE*)*

$$\mathrm{NMSE}(\xi) \triangleq \frac{\sum_\kappa |\hat{h}(\kappa, t_{4\xi}) - h(\kappa, t_{4\xi})|^2}{\sum_\kappa |h(\kappa, t_{4\xi})|^2} . \qquad (6)$$

$\mathrm{NMSE}(\xi)$ is then averaged over all values of ξ in order to obtain an overall measure (denoted NMSE) for each value of $\overline{\mathrm{SNR}}$.

Figure 5: For three sample COST-207 GSUS channels, Fig. 5 displays the performance of both the channel estimation algorithms and the Viterbi detector in terms of $\overline{\mathrm{SNR}}$. The upper row of subplots displays the NMSE values on a logarithmic scale, while the bottom row shows the bit error probabilities. For the

[2] Note that third order statistics can not be used due to the zero skewness of $d(k)$.

[3] For EVI, the training sequence was replaced with 26 data bits.

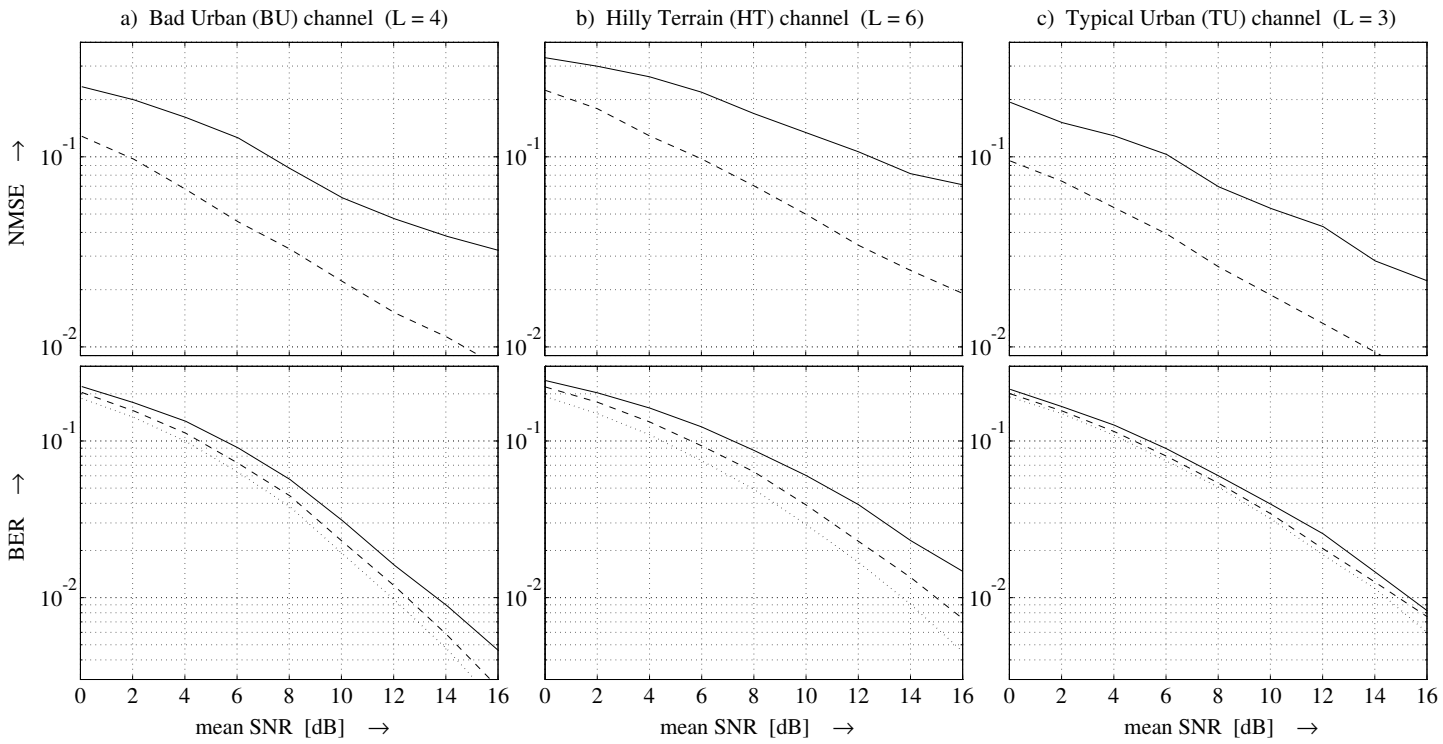

Figure 5: NMSE (above) and BER (below) in terms of $\overline{\mathrm{SNR}}$ for the estimates of three sample COST-207 GSUS channels
solid: blind EVI channel estimation, dashed: non-blind LS channel estimation, dotted: ideal channel estimation

Figures 5a, b, and c, a *Bad Urban (BU)*, a *Hilly Terrain (HT)* and a *Typical Urban (TU)* sample channel is used, where the number of coefficients of each estimated slice is set to $L = 4$, 6, and 3, respectively. The solid lines refers to EVI, while the dashed lines indicate the performance of the LS approach.

From the upper subplots, we realize that, for EVI, NMSE is two to three times as high as for LS, where the factor 2 holds for low values of $\overline{\mathrm{SNR}}$, while 3 applies above 12 dB. In general, the NMSE values degrade as the number L of coefficients increases.

With the bottom subplots of Fig. 5, a dotted line is added for comparison. It is obtained when using ideal channel estimates for MLSE, i.e. $\hat{h}(\kappa, t_{4\xi}) \equiv h(\kappa, t_{4\xi})$ for $\kappa = 0, \cdots, L-1$. Note that all curves suffer from the linearity assumption for the GMSK modulator as well as from the order misfit between L and the actual effective length of the current channel slice. For the *BU* channel, we realize that at BER $\approx 1\%$, blind EVI channel estimation requires about 1 dB more in $\overline{\mathrm{SNR}}$ than the non-blind LS scheme. While this loss increases to about 2 dB for the sample *HT* channel, it falls to 0.2 dB for the *TU* channel. If we average over the three sample channels, this leads to a loss of 1.1 dB.

Figure 6: For the *Bad Urban* simulation results at $\overline{\mathrm{SNR}} = 14\,\mathrm{dB}$, Figures 6 and 7 provide a detailed view for the 81 bursts with indices $\xi = 185$ to 265. The first subplot of Fig. 6 displays the current SNR for each burst, i.e. $\mathrm{SNR}(\xi)$. Although $\overline{\mathrm{SNR}}$ was set to 14 dB, "instantaneous" $\mathrm{SNR}(\xi)$ varies between 2.6 and 19 dB from burst to burst due to time selective fading[4]. The 2nd subplot shows $\mathrm{NMSE}(\xi)$ according to (6) of the LS estimates, where the values exceeding a certain threshold level (dotted line) are marked with vertical dashed lines which are continued across the subplots above and below to facilitate orientation. Clearly, low values of $\mathrm{SNR}(\xi)$ lead to relatively bad LS estimates (see $\xi = 207$ and 237), although there are exceptions to this rule ($\xi = 260$). From the 3rd subplot we realize that the Viterbi algorithm is quite robust with respect to channel estimation errors: considering the critical estimates (vertical dashed lines), it is evident that modest channel estimates lead to a "burst" of bit errors only if $\mathrm{SNR}(\xi)$ is low (see $\xi = 195$). On the other hand, if $\mathrm{SNR}(\xi)$ collapses ($\xi = 260$), even a good channel estimate may lead to errors. From the last subplot of Fig. 6, it can be seen that even ideal channel estimates would just slightly improve on the BER performance of LS.

[4] All 1000 bursts considered, $\mathrm{SNR}(\xi)$ varies even from -2.4 to 21 dB.

Figure 7 depicts the behavior with blind EVI channel estimation. From the second subplot, we realize that NMSE is about three times as high as with LS. Once more, low values of $\mathrm{SNR}(\xi)$ in general lead to bad estimates, although there are exceptions again ($\xi = 207$). The final subplot gives the distance d_{min} from the unit circle of the closest zero of $H(z, t_{4\xi}) = \mathcal{Z}\{h(\kappa, t_{4\xi}\}$ for all values of ξ. As d_{min} falls below 10^{-2} for many slices, our criterion (3) for the selection of a blind channel estimation algoritm is underlined. Note that at $\xi = 219$, EVI's performance does not collapse although d_{min} falls below 10^{-6}, here.

4. CONCLUSIONS AND FURTHER WORK

For GSM data transmission over GSUS mobile communication channels, we have demonstrated that the HOS-based blind channel estimation method EVI entails an $\overline{\mathrm{SNR}}$ loss of 1.1 dB only (averaged over three COST-207 propagation environments) compared with the non-blind least squares (LS) channel estimation

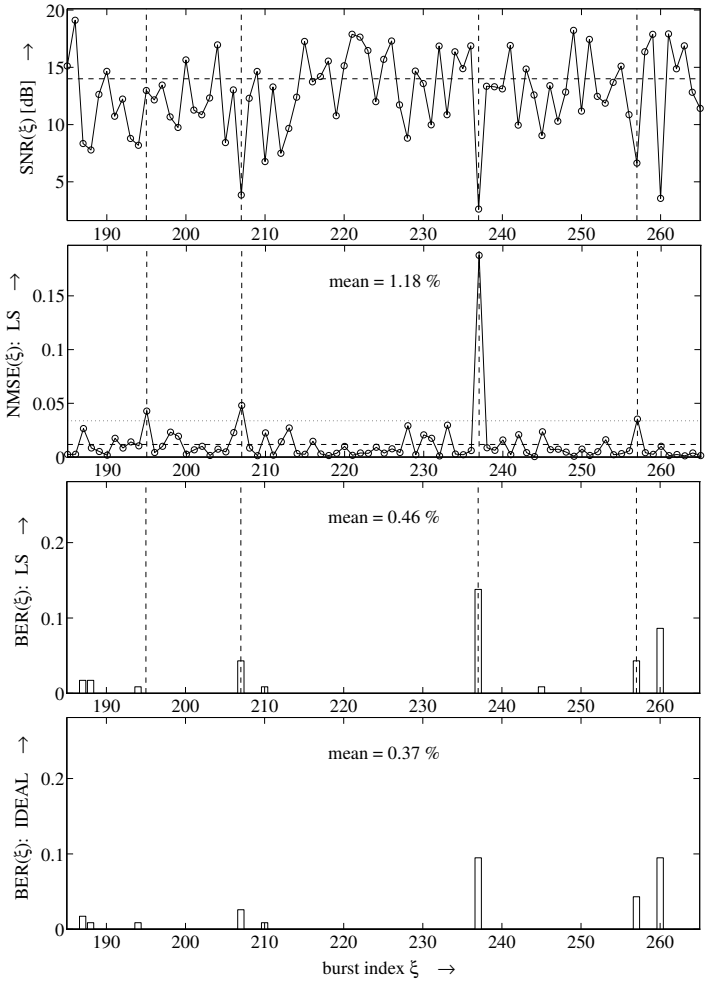

Fig. 6: SNR, NMSE and BER in terms of the burst index ξ for the LS and ideal estimates of the *BU* channel at $\overline{\mathrm{SNR}} = 14$ dB

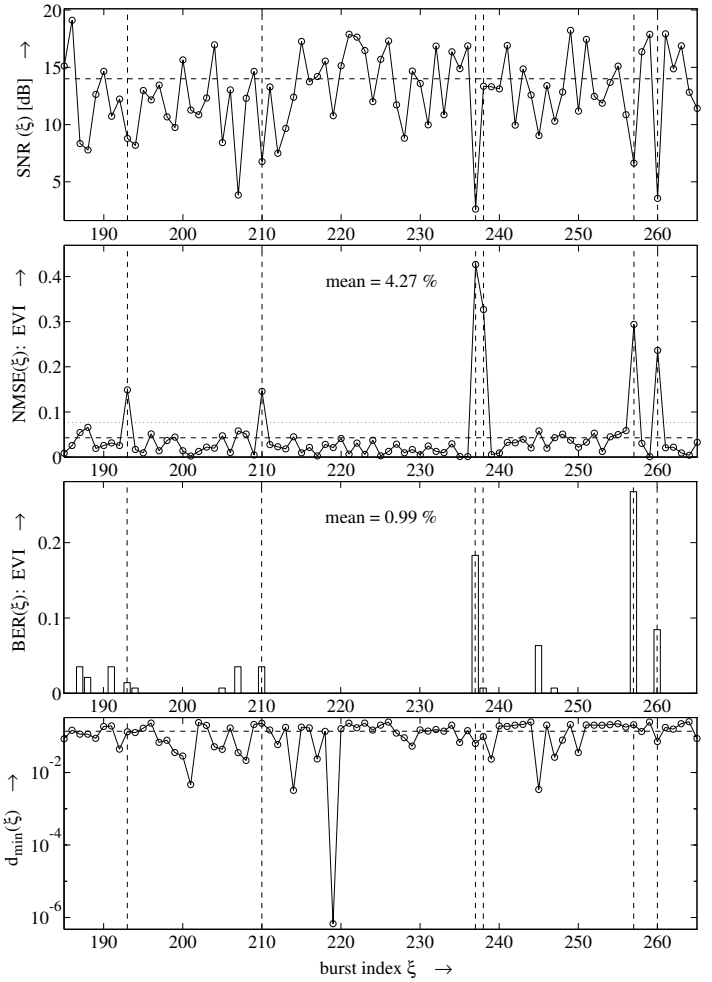

Fig. 7: SNR, NMSE and BER in terms of the burst index ξ for EVI's estimates of the *BU* channel at $\overline{\mathrm{SNR}} = 14$ dB

scheme. Note that these results are quite remarkable for a HOS-based algorithm, as just 142 samples of the demodulated sequence can be used for channel estimation.

While the results presented in this paper are based on a quasi time-invariance assumption of the channel over one GSM burst, further work will take the time-variant behavior within each burst period into account. In this case, BER performance for the LS channel estimates is expected to degrade. The decisive question is whether BER is lower *(i)* if non-blind estimates of the mean channel coefficients (averaged over 26 bit periods) are applied to MLSE of 116 data bits, or *(ii)* if blind estimates of the mean channel coefficients (averaged over $142T$) are utilized for the whole sequence of 142 data bits. First simulation results were obtained recently [8].

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