BLIND IDENTIFICATION OF MIXED-PHASE FIR SYSTEMS WITH APPLICATION TO MOBILE COMMUNICATION CHANNELS

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ABSTRACT

We investigate the applicability of two algorithms for the blind identification of mixed-phase linear time-invariant FIR systems to the estimation of mobile radio channels on GSM conditions. One approach is based on *Second Order Cyclostationary Statistics (SOCS)*, whereas the other exploits *Higher Order Stationary Statistics (HOSS)* of the received signal. While the former class of algorithms suffers from "singular" systems which can not be identified, the latter class is said to require an excessive number of samples of the received signal to achieve comparable performance levels.
The purpose of this paper is two-fold: first, we demonstrate that "singular" systems represent a severe limitation to SOCS-based methods when it comes to the estimation of time-*variant* mobile radio channels from a small number of received samples. Secondly, we reveal that the approach relying on 4th order statistics yields a superior estimation performance: At a signal-to-noise-ratio of 10 dB, all channel examples can be identified from 142 samples of a GSM burst within a normalized mean square error bound of 7 per cent.

1. INTRODUCTION

LET us regard the problem of system identification from the viewpoint of channel estimation in a digital communication system. *Maximum Likelihood Sequence Estimation* (MLSE) represents the optimum procedure to remove intersymbol interference from a received digital communication signal corrupted by linear channel distortions and additive white noise. It requires the knowledge, i.e. the estimation, of the possibly mixed-phase equivalent symbol-rate impulse response of the multipath radio channel which, in a *mobile* environment, is *time-variant*. In many applications, time-variance is relatively slow so that the channel can be estimated repeatedly in periods of time where it can be assumed time-*in*variant (*piecewise* time-invariant).

Application: Consider the channel estimation scheme used in state-of-the-art mobile communication systems according to the GSM standard *(Global System for Mobile commun.)*. Information symbols are transmitted in *bursts* where each "normal" burst contains two packets of 58 data symbols (bits) surrounding a training sequence of 26 bits (see Fig. 1).

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Symbol-rate channel estimates can be derived from the cross-correlation between the received (corrupted) and the stored (ideal) training sequences, i.e. from a 2nd order statistical property of (piecewise) stationary sequences. However, the repeated transmission of training sequences leaves a GSM system with an overhead capacity of $26/116 = 22.4\%$ which could be used for other purposes such as channel coding, if the channel estimation problem was solved *blindly*.

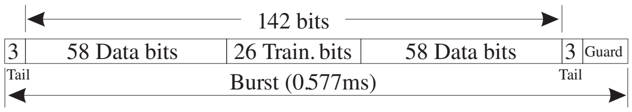

Figure 1: GSM "normal" burst

The fundamental idea of *blind* system identification is to derive the channel characteristics from the received signal only, i.e. *without* training sequences. Depending on the different ways to extract information from the received signal, two classes of algorithms can be distinguished:

Class HOSS: When the received signal is sampled at symbol rate $1/T$, the resulting sequence is (piecewise) stationary. Since second order statistics of a stationary signal are inadequate for the identification of the complete channel characteristics (including phase information), class HOSS methods are based on *Higher Order Stationary Statistics*. Higher order *cumulants* contain the complete information on the channel's magnitude and phase provided that the distribution of the channel input signal is non-Gaussian.

Class SOCS: When the sampling period is a fraction of T *(time diversity)*, or alternatively, the symbol-rate sampled signals received by several sensors are interleaved *(antennae diversity)*, the resulting received sequence is (piecewise) *cyclostationary*. Generally, *Second Order Cyclostationary Statistics* (SOCS) are sufficient to retrieve the complete channel characteristics, but there are "singular" channel classes which can *not* be identified this way.

Remark: Although the non-blind GSM channel estimation scheme assumes time *in*variance of the channel during the transmission of the 26 training bits only, the resulting estimate is used by MLSE (Viterbi) on the adjacent data fields. As the channel coefficients might already have changed in the data fields, there is an implicit assumption of *piecewise time-invariance over one burst* in this concept. Therefore, blind channel estimation approaches may also suppose this.

In summary, a blind channel estimation algorithm for such an application should satisfy the following requirements:

(1) Exploit SOCS or 4-th order statistics. Methods based on 3rd order stationary statistics were discarded due to the zero skewness of digital communication signals.
(2) **Reliable (complex) channel estimates must be obtained from 142 symbol-rate samples, only.**
(3) As the time-variant effective channel order is unknown, an overestimation must not represent a problem.
(4) Robustness with respect to additive Gaussian noise at S/N ratios down to (at least) 10 dB, if possible 7 dB.

Selected algorithms: Among the SOCS approaches we have considered for application, the TXK method suggested by Tong et al. [1] gave the best results. The SUBCHANNEL RESPONSE MATCHING algorithm by Schell et al. [2] does not meet requirement (3) while the SUBSPACE ALGORITHM by Moulines et al. [3] suffers from the problem of differentiating between signal and noise subspace eigenvalues, which is quite sensitive to additive noise. Within the HOSS class, the EIGENVECTOR APPROACH TO BLIND IDENTIFICATION (EVI) by Boss et al. [4] was found to outperform the W-SLICE method[1] by Fonollosa et al. [5].

After a detailed problem statement in section 2, we will demonstrate in section 3, how TXK and EVI perform on realistic mobile radio channels on GSM conditions.

2. PROBLEM STATEMENT

2.1. Blind estimation of time-invariant FIR systems

Assumptions: Consider a digital communication system where, each symbol period T, the i.i.d. random sequence $d(k)$ takes a value from a finite set[2]. In a stationary propagation scenario, the equivalent baseband representation of the composite channel (physical multipath radio channel as well as pulse shaping and receive filters) is given by a continuous-time time-invariant impulse response $h_c(\tau)$, where the subscript 'c' stands for 'continuous-time'.

Sampling the channel output $x_c(t) = \sum_k d(k)\, h_c(t-kT)$ at M times the symbol rate to obtain $x(i) = x_c(t)|_{t=iT/M}$ can be described by convolving the upsampled transmitted sequence with the discrete-time channel impulse response

$$h(i) \ \stackrel{\Delta}{=}\ h_c(\tau)|_{\tau=iT/M} \ , \qquad (1)$$

which we assume to have finite length (see Fig. 2). Note that the time indices k and i refer to samples spaced T and T/M seconds apart, resp.. The channel output sequence

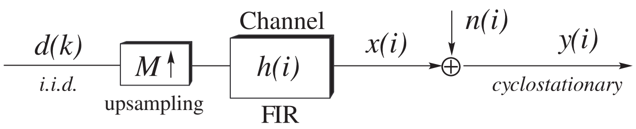

Figure 2: Equivalent discrete-time comm. system model

$x(i)$ is corrupted by independent stationary additive Gaussian noise $n(i)$, which has been colored by the upsampled receive filter impulse response. Just as $x(i)$ and $x_c(t)$, the upsampled received sequence $y(i)$ is *cyclostationary*.

[1] A performance comparison will be published in the near future.
[2] For notational simplicity, we assume zero mean processes.

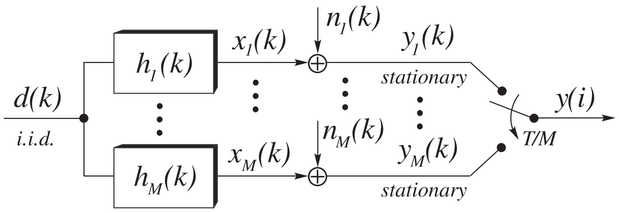

Figure 3: Stationary polyphase representation of $y(i)$

According to Gardner's *Time Series Representation*, $y(i)$ can be decomposed into M *stationary* sequences (Fig. 3)

$$y_\mu(k) = x_\mu(k) + n_\mu(k) = d(k) * h_\mu(k) + n_\mu(k) \qquad (2)$$

with $\mu = 1, \cdots, M$. The sequences $x_\mu(k)$, $n_\mu(k)$, and $y_\mu(k)$ with T-spaced samples denote the μ-th *polyphase component* of the respective signal sampled at rate M/T, e.g. $y_\mu(k) = y(i)|_{i=kM+\mu-1}$. Equally, $h_\mu(k) = h(i)|_{i=kM+\mu-1}$ represents the μ-th *polyphase subchannel* of $h(i)$.

Objective: Given solely $y(i)$ (or equivalently, $y_\mu(k)$), estimate $h(i)$ or $h_\mu(k)$, $\mu = 1, \cdots, M$, respectively. Note that *all* polyphase subchannels are to be estimated in this paper, although MLSE just requires *one* symbol-rate estimate.

Remark: Tong et al. and Tugnait have proven ([6], e.g.) that $h(i)$ is *not* identifiable by class SOCS algorithms from the cyclostationary correlation sequence of $y(i)$, if, e.g., the z transform $H(z) = \mathcal{Z}\{h(i)\}$ has a set of M zeros spaced equidistantly on a circle with center in $z=0$. In this case, the M subchannels $H_\mu(z) = \mathcal{Z}\{h_\mu(k)\}$ have *at least one common zero*. Such channels will be called "singular".

2.2. Mobile radio communication channel model

On the assumption of a stationary propagation scenario, we have derived two equivalent discrete-time (time-invariant) models of a digital communication system (see Fig. 2, 3). In a *mobile* setting, however, the channel is *time-variant*. Thus, its impulse response $h_c(\cdot)$ not only depends on the time difference τ between the observation and excitation instants, but also on the (absolute) observation time t.

We adopt a stochastic *Gaussian Stationary Uncorrelated Scattering (GSUS)* model for the physical multipath channel. For *slow* time-variance, the composite channel's equivalent baseband impulse response can be approximated by [7]

$$h_c(\tau, t) = \frac{1}{\sqrt{N_e}} \sum_{\nu=1}^{N_e} \exp[j(2\pi f_{d,\nu} t + \Theta_\nu)] \cdot g_{TR}(\tau - \tau_\nu)\,, \ (3)$$

where N_e indicates the number of elementary echo paths and $g_{TR}(\tau)$ denotes the combined transmit/receive filter impulse response. Sample channel impulse responses can be calculated from (3) by independently drawing *(i)* N_e Doppler frequencies $f_{d,\nu}$ from a random variable with Jakes probability density function (pdf), *(ii)* N_e initial phases Θ_ν from a uniformly distributed random variable in $[0, 2\pi)$, and *(iii)* N_e echo delay times τ_ν from a random variable with a pdf proportional to the mean power delay spectrum of a propagation environment defined by COST-207[3], e.g.: *Typical Urban (TU), Bad Urban (BU), Hilly Terrain (HT) ...*

[3] Cooperation in the field of Scientific and Technical research.

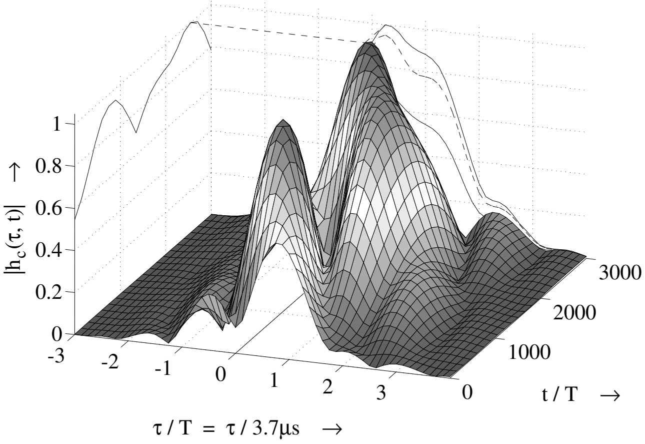

Figure 4: Impulse response of a *Bad Urban* channel

Fig. 4 shows a (non-causal) sample magnitude impulse response $|h_c(\tau,t)|$, obtained from (3) with $N_e = 100$, of a *Bad Urban* channel with raised cosine transmit and receive filters ($r = 0.5$). Both time axes are normalized to the GSM symbol (bit) period $T \approx 3.7\ \mu$s. The velocity of the mobile unit is $v = 100$ km/h. Assuming a carrier frequency of 950 MHz, this leads to a maximum Doppler shift of $f_{d,max} = 88$ Hz. Equation (3) was evaluated over a t range covering one minimum Doppler period $T_{d,min} = 1/f_{d,max} = 3080T$.

Assuming *piecewise time-invariance over one burst*, $h_c(\tau,t)$ is sampled on the t axis each 150 symbols (c.f. Fig. 1). This produces 21 slices within the t range of $3080T$, which can be seen in Fig. 4 as surface lines parallel to the τ axis. Each slice is sampled at $\tau = iT/M$ and then constrained to the range $\mathcal{I}$ of indices i where the associated sample power delay spectrum exceeds the threshold of 1% of its peak value.

$$h(i,\xi) \triangleq h_c(\tau,t) \text{ for } \begin{cases} \tau = iT/M \text{ with } i \in \mathcal{I} \\ t = \xi\, 150T \text{ with } \xi = 0,\cdots,20. \end{cases} \quad (4)$$

The simulation results we present in the following section are based on *linear* modulation, although GSM prescribes *non-linear GMSK (Gaussian Minimum Shift Keying)*. However, as linearity is also assumed in typical GSM receivers, the results for GMSK are not expected to change much.

3. SIMULATION RESULTS

On the assumptions stated in sec. 2.2, nine different sample GSUS composite channels were obtained from (3) by combining three COST-207 propagation environments *(TU, BU, HT)* with three raised cosine transmit/receive filters: roll-off factors $r \in \{0.9, 0.5, 0.1\}$. Let *"BU–(0.5)"* denote the *Bad Urban* channel with roll-off factor $r = 0.5$, e.g.. Using double symbol-rate sampling ($M = 2$) in eq. (4), each channel $h_c(\tau,t)$ was decomposed into 21 slices $h(i,\xi)$. Referring to Fig. 2 with $h(i,\xi)$ substituted for $h(i)$, a burst of 150 i.i.d. *BASK (Binary Ampl. Shift Keying)* symbols $d(k)$ was propagated through each channel slice. The resulting cyclostationary sequence $x(i)$ was limited to a block of $ML = 2 \cdot 142$ steady state samples. Finally, independent additive Gaussian noise $n(i)$, colored by the receive filter sampled at rate $2/T$, was added according to a given mean signal-to-noise ratio $\bar{S}/N$ to obtain ML samples of $y(i)$.

TXK was applied to $y(i)$, while EVI operates at symbol rate and was therefore executed on the two polyphases $y_1(k)$ and $y_2(k)$ according to Fig. 3. In either case, $L = 142$ symbol periods of the received signal were taken into account for the estimation of the required correlation and cumulant sequences by unbiased sample averaging. Both approaches were given the effective length of the sample power delay spectrum, which is equivalent to the mean length of the channel impulse response. Note that the actual effective length of a channel slice may well be shorter due to time selective fading. Since the noise sequence $n(i)$ is colored, no algorithm attempted to compensate for the noise influence. In any case, enabling the TXK white noise cancellation scheme did not improve the results quoted below.

Estimation quality measure: Let $\hat{h}^{(1)}(i,\xi)$ denote the estimate of $h(i,\xi)$ based on a given input burst. In the frame of Monte-Carlo simulations, a total of MC = 100 different BASK bursts was propagated through each channel slice to obtain MC estimates $\hat{h}^{(\gamma)}(i,\xi)$, $\gamma = 1,\cdots,\text{MC}$. For each slice index ξ, estimation quality was assessed on the basis of the averaged *Normalized Mean Square Error*[4]

$$\text{NMSE}(\xi) \triangleq \frac{1}{\text{MC}} \sum_{\gamma=1}^{\text{MC}} \frac{\sum_i |\hat{h}^{(\gamma)}(i,\xi) - h(i,\xi)|^2}{\sum_i |h(i,\xi)|^2}\,. \quad (5)$$

Figure 5: From the set of 9 sample channels, we have selected four "critical" examples with relatively long impulse responses: *HT–(0.5)*, *BU–(0.1):* $7T$; *HT–(0.1):* $8T$; *BU–(0.5):* $5T$. For each channel, Fig. 5 shows the NMSE(ξ)-values (in per cent) of TXK's and EVI's estimates for different values of $\bar{S}/N$, where the noiseless case is marked by "∘" symbols, while "×" and "+" stand for $\bar{S}/N = 10$ dB and 7 dB, respectively. The NMSE(ξ)-values for TXK [EVI] are connected by solid [dotted] lines, respectively, where the TXK results at 7 dB are suppressed to enhance clarity.

We realize from Fig. 5a and b that in the noiseless case ("∘") both approaches can principally estimate the *HT* channels very well (NMSE(ξ) from 3 to 5%). However, TXK can *not* identify slices $\xi = 1$ to 4 of *HT–(0.5)* (Fig. a) and slice 9 of *HT–(0.1)* (Fig. b). This is due to subchannel zeros of $H_1(z,\xi)$ and $H_2(z,\xi)$[5], which accidentally are very close to each other in these slices (distances of 0.002 and 0.008, resp.). Comparing the results at $\bar{S}/N = 10$ dB ("×"), it is obvious that EVI outperforms TXK: if we average NMSE(ξ) over all slices ξ to obtain $\overline{\text{NMSE}}$, TXK delivers values (19.4 and 23.3% for Fig. a and b) which are more than 3 times higher than those of EVI (5.5 and 7.3%, resp.). Even at 7 dB ("+"), EVI performs better than TXK at 10 dB.

With the *BU* channels in Fig. 5c,d (note that the channel used for Fig. 5c was shown in Fig. 4), EVI's NMSE-values at 10 dB can barely be distinguished from those in the noiseless case, because $\overline{\text{NMSE}}$ is inferior to 3% (Fig. c) and 5%

[4] As all blind system identification algorithms can *not* identify one complex factor, each estimate was multiplied with the optimum complex constant (minimizing the Euclidean distance from the true channel slice) before NMSE was calculated.

[5] where $H_\mu(z,\xi)$ is the z transform of subchannel μ of $h(i,\xi)$.

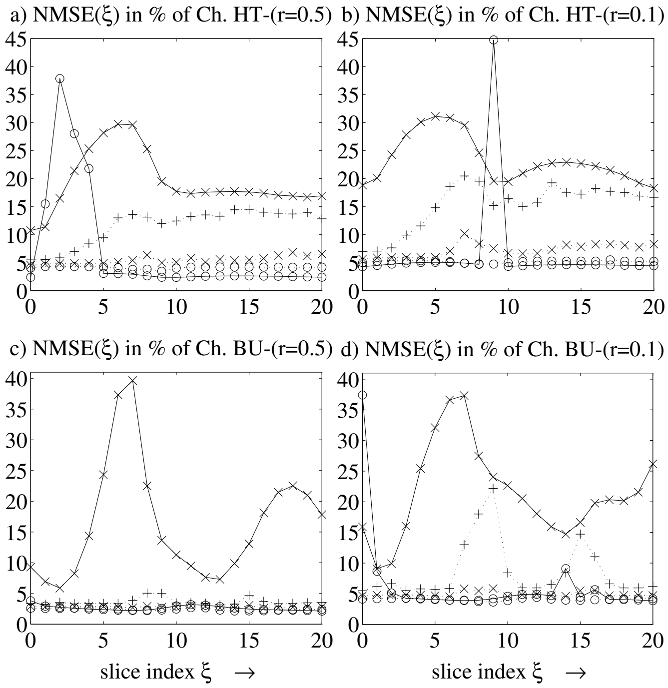

Figure 5: NMSE(ξ) in % of TXK (solid) and EVI (dotted): "o": $\bar{S}/N = \infty$, "×": 10 dB, "+": 7 dB

(Fig. d). On the other hand, TXK's performance is heavily affected at 10 dB, since $\overline{\text{NMSE}}$ rises to 16.2% and 21.4%, resp.. Thus, EVI's estimation performance is superior by an average factor of 5. Moreover, EVI's NMSE(ξ) levels are smaller at 7 dB ("+") than those of TXK at 10 dB, again.

$\bar{\text{S}}$/N	*HT–(0.9)*	*HT–(0.5)*	*HT–(0.1)*	Fac.
∞ dB	(2.3) / 3.4	(7.0) / 4.1	(4.7) / 5.2	1.1
10 dB	15.1 / 4.7	19.4 / 5.5	23.3 / 7.3	3.3
7 dB	16.0 /10.6	20.4 /11.6	22.6 /15.2	1.6
$\bar{\text{S}}$/N	***BU–(0.9)***	***BU–(0.5)***	***BU–(0.1)***	**Fac.**
∞ dB	1.3 / 1.7	2.6 / 2.4	(6.4) / 4.1	1.1
10 dB	13.3 / 2.0	16.2 / 2.8	21.4 / 4.9	5.6
7 dB	15.0 / 2.6	17.2 / 3.6	21.7 / 8.6	4.4
$\bar{\text{S}}$/N	***TU–(0.9)***	***TU–(0.5)***	***TU–(0.1)***	**Fac.**
∞ dB	0.3 / 0.5	1.9 / 1.5	4.4 / 3.3	1.1
10 dB	4.0 / 0.6	12.0 / 1.8	18.2 / 3.9	6.0
7 dB	7.9 / 0.8	12.6 / 2.3	18.6 / 5.2	6.3

Table 1: $\overline{\text{NMSE}}$ in % for TXK's / EVI's estimates of *HT* (above), *BU* and *TU* (below) channels

Table 1 provides the $\overline{\text{NMSE}}$-values for the entire channel set. In columns 2-4, the first (2nd) entry refers to TXK (EVI), where those heavily affected by outliers are given in parenthesis. Compared with Fig. 5, we realize that channels with a short impulse response (resulting from a propagation environment with a small delay spread or transmit/receive filters with a high roll-off factor r) are less critical for both algorithms. However, the statements concerning the performance comparison remain unaffected, as can be seen from the last column, where the mean factor by which EVI outperforms TXK is quoted for each $\bar{S}/N$ and environment.

It should be noted that TXK suffers from a threshold problem, because it needs to distinguish between signal and noise subspace singular values of a correlation matrix. To improve performance in the noisy case, TXK was given the mean noise power (averaged over all channel slices), which is, in the strict sense, illegal a-priori knowledge.

4. CONCLUSIONS AND FURTHER WORK

We have demonstrated that "singular" channels represent a significant limitation to SOCS-based methods because in mobile environments, subchannel zeros can *not* be prevented from colliding. We have also shown that it *is* possible with an algorithm exploiting HOSS to blindly estimate (within an $\overline{\text{NMSE}}$-bound of about 7%) realistic mobile radio channels from 142 received samples at $\bar{S}/N$ levels of 10 dB. Even at 7 dB, many sample channels could be identified.

While this paper concentrated on the quality of blind channel estimation, further work will be directed towards a comparison of the post-MLSE *bit error rates (BER)* attainable from blind and non-blind channel estimates. The decisive question is whether BER is lower *(i)* if non-blind estimates of the mean channel coefficients (averaged over 26 bit periods) are applied to MLSE of 116 data bits, or *(ii)* if blind estimates of the mean channel coefficients (averaged over $142T$) are utilized for the whole sequence of 142 data bits. First simulation results were obtained recently and will be published in the near future (also see our WWW server).

5. REFERENCES

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