EXPLOITING SECOND ORDER CYCLOSTATIONARITY OR HIGHER ORDER STATISTICS FOR THE BLIND IDENTIFICATION OF MIXED PHASE FIR SYSTEMS ?

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RESUME[2]

Cette communication est consacrée au problème d'identification aveugle de systèmes RIF à phases mixtes. Deux algorithmes basés sur les statistiques d'ordre 2 du signal reçu (qui est suréchantilloné et donc cyclostationnaire) sont comparés à une approche utilisant les statistiques d'ordre 4 du signal stationnaire reçu. Alors que les premiers *ne* peuvent identifier certains systèmes à partir de leurs statistiques cyclostationnaires d'ordre 2, les derniers ont la réputation de nécessiter un nombre plus élevé d'échantillons de signal pour avoir une variance d'estimation comparable. Pourtant, il sera démontré que l'algorithme d'ordre 4 (EVI) ména à une qualité d'estimation supérieure à partir du même nombre d'échantillons lorsqu'il s'agit de canaux de transmission réels dans des systèmes de communication numérique.

ABSTRACT[2]

This paper addresses the problem of blind identification of mixed phase time-invariant FIR systems. Two algorithms based on 2nd order statistics of the oversampled cyclostationary received signal are compared with an efficient approach exploiting 4th order statistics of the stationary received signal sampled at symbol rate. While the former suffer from systems which can *not* be identified from 2nd order cyclostationary statistics, the latter are said to require more samples of the received signal to obtain comparable levels of estimation variance. However, we show in this paper that the approach relying on 4th order statistics (EVI) delivers a superior estimation performance based on the same number of samples when it comes to the identification of realistic transmission channels in digital communication systems.

1 INTRODUCTION

The fundamental idea of *blind* system identification (channel estimation) is to derive the channel characteristics from the received signal only, i.e. *without* access to the source signal. Depending on the different ways to extract information from the received signal, two classes of approaches can be defined:

Class A: When the received signal is sampled at the symbol rate $1/T$, the resulting sequence is stationary. Since 2nd order statistics of stationary signals are inadequate for the identification of the complete channel characteristics (including phase information), class A approaches are based either explicitely or implicitely on *higher order statistics* (HOS). Higher order cumulants contain the complete information on the channel's magnitude and phase provided that the source signal's distribution is non-Gaussian.

Class B: When the sampling period is a fraction of T, the resulting oversampled received sequence is *cyclostationary.* Generally, *second order cyclostationary statistics* (SOCS) are sufficient to retrieve the complete channel characteristics, but there are certain channels that can *not* be identified.

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Many approaches of either class have been proposed in recent years. The purpose of this paper is to investigate the performance of two fast class B algorithms and to compare it with an efficient class A approach:

- SRM: Class B SUBCHANNEL RESPONSE MATCHING algorithm by Schell, Smith and Gardner [1]; a similar appr. was proposed by Baccala et al [2],
- TXK: Class B TXK-METHOD suggested by Tong, Xu and Kailath [3],
- EVI: Class A EIGENVECTOR APPROACH TO BLIND IDENTIFICATION by Jelonnek, Boss and Kammeyer [4, 5] based on 4th order statistics.

Application: In this paper, the problem of blind system identification is regarded from the viewpoint of digital communications. Due to the equivalent baseband representation of the bandpass communication system, all signals and systems take complex values.

MAXIMUM LIKELIHOOD SEQUENCE ESTIMATION (MLSE, Viterbi detection) represents the optimum procedure to remove intersymbol interference from a received digital communication signal. It requires the estimation of the equivalent symbol rate impulse response of the (mixed phase multipath) transmission channel. New horizons are opened, if this system identification problem is solved blindly, i.e. without training sequences used in state-of-the-art communication systems. Note that a major criterion for the selection of the above algorithms is their ability to deliver reliable channel estimates on the basis of few samples, because communication channels can only be considered short-term time-invariant. Also note that class A algorithms based on 3rd order statistics can not be applied due to the zero skewness of most digital communication signals.

After a detailed problem statement in sec. 2, we demonstrate in section 3 the asymptotic "estimation" performance, where true covariance and cumulant sequences are used by the respective approach. The algorithms' behaviour is explained (i) in the case of "singular" channels, (ii) when the channel order is overestimated and (iii) when additive noise is present. Particular attention is devoted to the sensitivity of the channel estimates with respect to these influences when the covariance and cumulant sequences are estimated from a finite number of samples (section 4). Finally, the approaches are applied to a realistic communication channel.

2 PROBLEM STATEMENT

Assumptions: Consider the digital communication model where, each symbol period T, the stationary[3] i.i.d. source sequence $d(k)$ takes a (complex) value from a finite set[4]. The composite transmission channel is described by a continuous-time causal time-invariant finite impulse response $h_c(t)$. Now, sampling the continuous-time channel output signal $x_c(t) = \sum_k d(k)\, h_c(t-kT)$ at M times the symbol rate to obtain $x(i) = x_c(t)|_{t=iT/M}$ can be described by convolving the oversampled source sequence with the discrete-time channel impulse response $h(i) = h_c(t)|_{t=iT/M}$, as depicted in Fig. 1. Let the order of $h(i)$ be denoted q. Note that

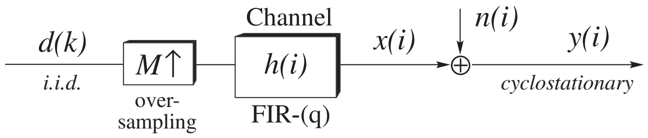

Figure 1: Oversampled digital comm. sequence $y(i)$

the time indices k and i refer to samples spaced T and T/M seconds apart, respectively. The channel output sequence $x(i)$ is corrupted by independent additive stationary noise $n(i)$. The oversampled digital communication sequence $y(i)$ is *cyclostationary* (as is $x_c(t)$ and $x(i)$).

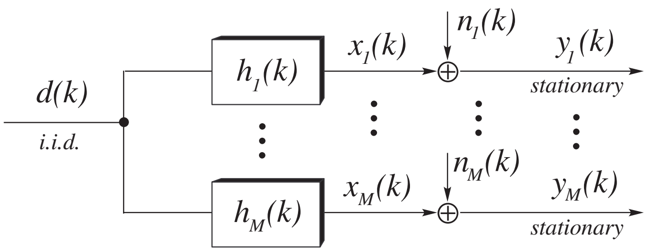

Figure 2: "Time Series Representation" of $y(i)$

[3] Generally, references to stationarity or cyclostationarity imply the respective *wide sense* property (i.e. first and second order). However, class A approaches require higher orders of (cyclo)stationarity.

[4] To simplify the description, all processes are assumed to be zero mean.

According to Gardner's TIME SERIES REPRESENTATION [6, Sec. 12.6], $y(i)$ can be decomposed into M *stationary* sequences $y_1(k), \cdots, y_M(k)$ (see Fig. 2), where

$$y_\mu(k) = x_\mu(k) + n_\mu(k) = d(k) * h_\mu(k) + n_\mu(k). \qquad (1)$$

The sequences $x_\mu(k)$, $n_\mu(k)$, and $y_\mu(k)$ with T-spaced samples denote the μ-th *polyphase component* of the respective signal sampled at M times the symbol rate, e.g.

$$y_\mu(k) = y(i)|_{i=kM+\mu-1} = y_c(t)|_{t=kT+(\mu-1)T/M}\,. \qquad (2)$$

Equivalently, $h_\mu(k)$ represents the μ-th *polyphase subchannel* of $h(i)$. Let $H_\mu(z)$ and $H(z)$ denote their z transforms.

Objective: Given solely the data $y(i)$ (or equivalently, $y_1(k), \cdots, y_M(k)$), estimate $h(i)$ (or $h_1(k), \cdots, h_M(k)$).

3 ASYMPTOTIC ESTIMATION PERFORMANCE

"Singular" channels: Tong et al. and Tugnait have proven [7, 8] that the finite impulse response $h(i)$ with uncorrelated input is *not* identifiable by class B algorithms from the cyclostationary correlation sequence of the M times oversampled seq. $y(i)$, if one of the following condition holds.

(1) $H(z)$ has a set of M zeros located symmetrically on a circle with the complex plane's origin in its center. This is equivalent to stating that the M polyphase subchannels $H_1(z), \cdots, H_M(z)$ have at least one common zero.

(2) The channel consists of time delays that are integer multiples of the symbol period T.

(3) If M is even and the channel consists of time delays equaling integer multiples of $T/2$.

Note that channels according to (1) or (3) can still be identified from its SOCS by altering the oversampling factor M. However, channels (2) are not identifiable irrespective of how the sampling rate is chosen [8].

For a QPSK transmission and two channel examples ($q = 8$), Fig. 3 shows the "estimation" results obtained by SRM, TXK[5] and EVI when true correlation and cumulant values are used and the oversampling factor is chosen to be $M = 3$. True and estimated zeros of $H(z)$ are indicated by circles ("$\circ$") and crosses ("$\times$"), resp., in the complex plane.

The minimum distance between subchannel zeros being 0.17, channel C1 has no common zeros in its polyphase subchannels $H_1^{C1}(z), \cdots, H_3^{C1}(z)$. From the superimposed symbols in Fig. 3a to c, we realize that all channel zeros are perfectly identified. Although no order overestimation is assumed here, TXK and EVI introduce negligible zeros in $z = 0$ and $|z| = \infty$ (some estimated zeros outside the displayed section of the complex plane are given in brackets in Fig. 3b and c).

For channel C3 in Figures 3d to f, one zero in each subchannel $H_\mu^{C1}(z)$ is shifted to coincide in $z = 0.9\, e^{-j\pi/4}$. Thus, $H^{C3}(z)$ does have three zeros located symmetrically on a

[5] where the param. m is the observation period in multiples of T/M.

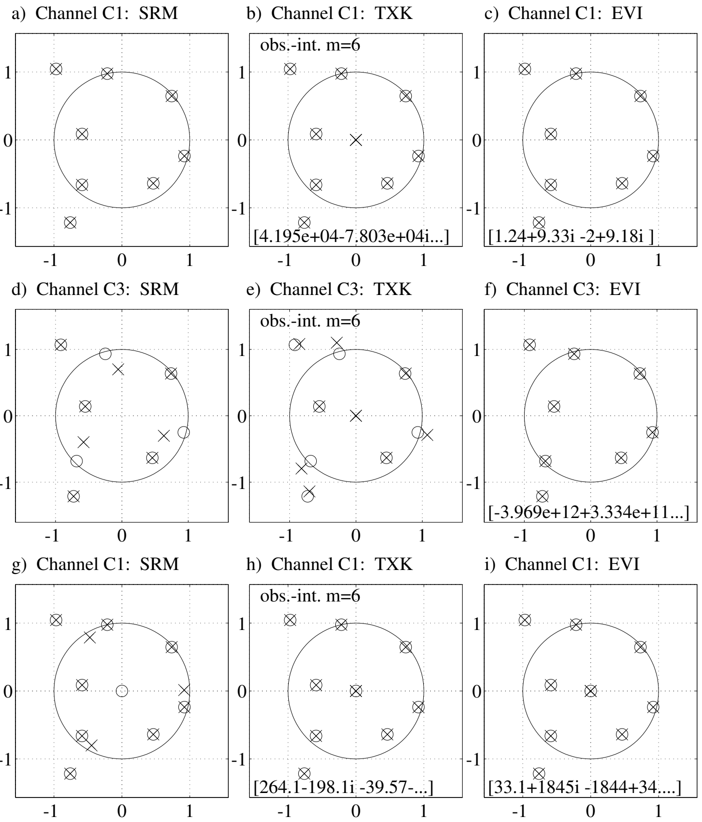

Figure 3: Asymptotic "estimation" results

circle around $z = 0$. From Fig. 3d, we realize that SRM can not identify these zeros. Its cost function assumes the minimum value with two channel estimates, both of which fail to identify the ring of zeros. Instead, an erroneous ring of zeros is introduced. Due to a rank deficiency of a correlation matrix, TXK estimates *all* zeros with a bias, which is non-negligible for the max. phase zeros and those corresponding to common subchannel zeros (Fig. 3e). It is obvious from Fig. 3f that EVI does not suffer from singular channels.

Channel order overestimation: For the Fig. 3g to i, the order q of $h^{C1}(i)$ is overestimated by $\Delta q = \hat{q} - q = 3$. From Fig. 3h and i, we see that TXK and EVI are robust. This must be emphasized, because knowledge of the order is of utmost importance to many identification approaches. As an overestimation of the order of $h(i)$ by $\Delta q \geq M$ introduces $\lfloor \Delta q / M \rfloor$ common zeros in $z = 0$ into the polyphase subchannels, this is a major problem with many class B approaches (such as SRM, see Fig. 3g). Just as in Fig. 3d, SRM fails to identify the common subchannel zeros.

Additive noise: The asymptotic solution of the SRM and TXK methods is not influenced by independent stationary additive *white* noise. The SRM's solution (i.e. the least significant eigenvector of a correlation matrix) does not change with white noise, whereas TXK incorporates an attempt to cancel the degradation of the correlation function due to white noise. Although the subtraction of the estimated noise power from the appropriate correlation values could easily be incorporated into EVI too, the degradation of EVI's solution due to white noise is minor anyway [4]. Finally, *coloured* noise degrades the solution of all algorithms.

4 ESTIMATION PERFORMANCE BASED ON FINITE DATA BLOCKS

As mentioned above, singular channels can *not* be identified by class B approaches, even if the true correlation sequence was available. Now, if it is estimated from a finite number of samples, the variance of the estimated zeros will be increased considerably even if subchannel zeros are "close" to each other rather than being identical. The closer they are allowed to be spaced for a given algorithm to successfully identify them, the more samples are required for the correlation estimates. On the other hand, EVI's performance does not depend on the closeness of subchannel zeros.

For a QPSK transmission and two channel examples, Fig. 4 displays the estimation results based on finite data blocks and $M = 3$. While channel C3 is critical due to its common subchannel zeros in $z = 0.9\, e^{-j\pi/4}$, channel C2 has a distance of 0.073 between the closest subchannel zeros. For each of the 100 Monte-Carlo runs, the steady state channel output sequence $x(i)$ is corrupted by AWGN according to a given SNR. Then, L samples of the resulting sequence $y(i)$ are used to estimate the correlation and cumulant values required by the respective approach. Estimated zeros of $H(z)$ are marked with dots ("$\cdot$") in Figure 4, while their mean values are indicated by asterisks ("$*$").

Without noise and $L = 1200$, the superimposed symbols in Fig. 4a to c reveal that all approaches identify the zeros of $H^{C2}(z)$ quite satisfactorily[6]. However, due to the finite value of L, SRM estimates the three zeros corresponding to closely spaced subchannel zeros with non-zero bias and an increased std deviation ($\sigma \approx 0.2$). Although 1200 samples seem to be sufficient for TXK and EVI, the maximum phase zeros are estimated with increased standard deviation.
For Fig. 4d to f, SNR is set to 10dB. Although 9000 samples of $y(i)$ are used, SRM tends to scatter the estimates of the close subchannel zeros along the unit circle of the complex plane. TXK delivers biased estimates of those and the max. phase zeros. Here, EVI yields the best result (Fig. 4f).

Fig. 4g to l depict the results for the critical channel C3, where Fig. g to i show the noiseless case with $L = 5400$ while Fig. j to l display the noisy case (SNR=10dB, $L = 9000$). As stated above, SRM and TXK are incapable of identifying the ring of 3 equidistant zeros. Instead, SRM estimates with negligible variance 3 wrong zeros on the unit circle (Fig. 4g), which are scattered all along the unit circle when noise is present (Fig. 4j). With or without noise, TXK's estimation of both the ring of zeros and the max. phase zeros is biased (Fig. 4h, k). Again, EVI delivers the most favourable result.

In Fig. 4m to 4o, the order of C2 is overestimated by $\Delta q = 3$. As 1200 samples are used, they can be compared with Fig. 4a to c. From Fig. 4m, we note that SRM estimates a wrong ring of zeros on the unit circle rather than the 3 zeros in the origin. On the other hand, TXK and EVI yield results comparable to those in Fig. 4b and c (as indicated in sec. 3).

[6] For channel C1 in section 3, a few hundred samples are sufficient.

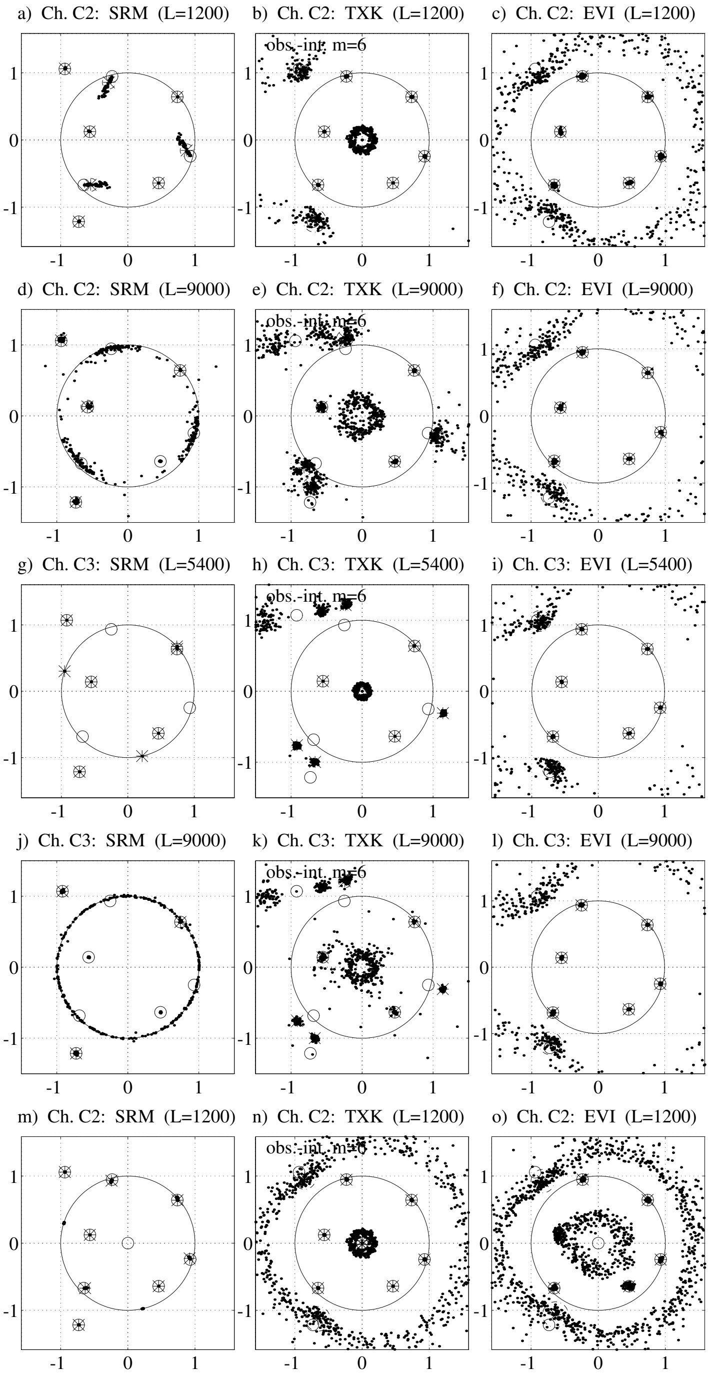

Figure 4: Estimation performance of SRM, TXK and EVI

We conclude with the estimation of a realistic ($q = 34$) *bad urban* communication ch. from 1200 samples ($M = 2$). Fig. 5 shows the true (dashed lines) and estimated magn. spectra (in dB) in terms of norm. frequency. The mean±std.dev. values are represented by solid and dotted lines, respectively.

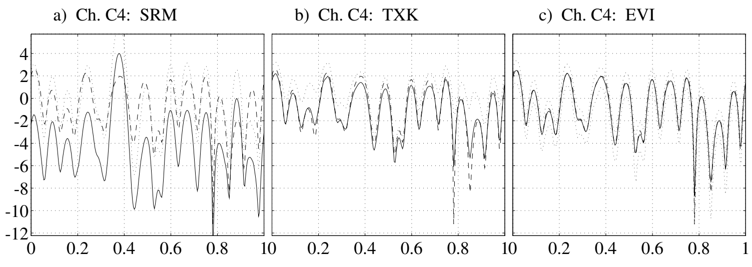

Figure 5: Estimation of a bad urban comm. channel

Although all approaches successfully identify the zeros close to the unit circle, SRM undermines its result by adding many spurious zeros on the unit circle. Both TXK and EVI yield reliable estimates, but EVI performs slightly better in terms of bias and variance. Note that TXK benefits from the relatively long min. distance between subchannel zeros (0.085). Thus, in contrast to the wide-spread opinion that class A approaches using 4th order statistics require too many samples for a satisfactory performance, Fig. 5 reveals that EVI is capable of identifying this frequency selective multipath channel from blocklengths as required by class B algorithms.

5 CONCLUSION & FURTHER WORK

In summary, we can state that neither SRM nor TXK are capable of estimating channels with closely spaced subchannel zeros. However, this ability may be crucial when it comes to the identification of communication channels. It is also clear from the above simulation results that HOS-based algorithms do *not* principally require blocklengths higher than those required by approaches based on 2nd order statistics.

In future, we will examine the influence of coloured noise. We will also investigate how the different channel estimators can be used for MLSE in the receiver of a dig. comm. system.

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