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DECISION-FEEDBACK EIGENVECTOR APPROACH TO BLIND ARMA EQUALIZATION AND IDENTIFICATION

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ABSTRACT[2]**:** The EigenVector Algorithm EVA [1, 2] for the blind *equalization* of a possibly mixed phase ARMA transmission channel has been generalized to multiple parallel channels [3]. As a consequence, EVA can now be applied to various other equalizing structures such as fractional tap spacing [2, sec. 5] or decision-feedback. The purpose of this paper is twofold: First, we will explain how a generalized version of EVA can be applied to the Decision-Feedback (DF) structure in order to blindly equalize ARMA channels. Secondly, we will show that this structure can also be used for an efficient approach to the problem of blind *parameter estimation* of ARMA models. The resulting algorithm, termed DF-EVI standing for Decision-Feedback EigenVector approach to blind Identification, represents an alternative to the sequential "algebraic" AR and MA estimators according to the Residual Time Series procedure [4, 5, 6]. We will investigate DF-EVI's behavior with respect to an overestimation of the channel orders and independent stationary additive Gaussian noise.

1 PROBLEM STATEMENT

The fundamental idea of *blind* approaches to system identification (channel estimation) and deconvolution (channel equalization) is to derive the channel or equalizer characteristics from the received signal only, i.e. *without* resort to training sequences. The complete channel or equalizer characteristics – including phase information – can be obtained by applying 2nd order statistics to the oversampled received signal (i.e. exploiting cyclostationarity) or by applying higher order statistics (HOS) to the received signal sampled at symbol rate. In this paper we use the latter method. Higher order cumulants contain the complete information on the channel's/equalizer's magnitude and phase characteristics provided that the source signal is non-Gaussian. This is the case in many applications, e.g. in digital communications where the source signal distributions are discrete (and known in advance).

On the particular problem of blind *ARMA system* identification and equalization, a large number of algorithms has been proposed. Many approaches are based on third order statistics and real valued signals and systems. Unfortunately, both assumptions do not apply to our main application, which is in digital communications. Extending the existing algorithms to 4th order statistics and complex signals and systems quite frequently results in a poor performance: For a given number of received data samples, they yield high values of estimation variance, or equivalently, they require a very large block of received data samples for a satisfactory estimation of the channel's/equalizer's impulse response. Besides, "knowledge of the system order is of utmost importance" to many identification approaches [7], because they are extremely sensitive to a mis-estimation of the channel order. However, the reliable channel order estimation remains a difficult task, especially in noisy environments.

Consider the system arrangement displayed in Fig. 1. The transmitted data $d(k)$ are regarded as i.i.d. random

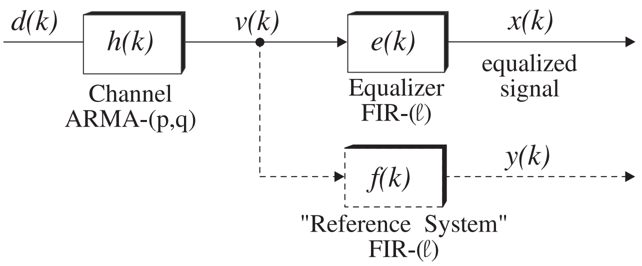

Figure 1: Symbol rate model of a commun. system

variables with known non-Gaussian probability density function (pdf), zero mean and variance σ_d^2. Note

[1] Compressed postscript files of our publications are readily available over Internet via anonymous ftp to 'ftp.et2.tu-harburg.de'.

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that digital modulation signals usually have even pdfs. Let the composite transmission channel be represented by the causal and stable ARMA-(p,q) model with impulse response $h(k)$ and transfer function $H(z) = B_q(z)/A_p(z)$, where $A_p(z)$ and $B_q(z)$ denote the AR and MA parts with orders p and q, respectively. In the receiver, an FIR-(ℓ) equalizer with impulse response $e(k) = e(0), \cdots, e(\ell)$ is introduced. Its output signal, i.e. the equalized signal, is termed $x(k)$. All signals and systems are assumed to be complex-valued due to the equivalent baseband representation of the corresponding bandpass communication system.

EQUALIZATION OBJECTIVE: Given a finite block of the received data sequence $v(k)$ only, adjust the equalizer coefficients $e(k)$ such that the equalized signal $x(k)$ is as close as possible to the delayed transmitted data seq. $d(k-k_0)$ in the MSE sense (mean square error).
IDENTIFICATION OBJECTIVE: Given a finite block of the received data sequence $v(k)$ only, estimate the channel impulse response $h(k)$ or, equivalently, the zeros and poles of $H(z)$ in an optimum manner.

2 APPLICATION OF EVA TO DF-EQUALIZATION

The point of departure for this paper is the iterative EIGENVECTOR ALGORITHM EVA as presented at the 1993 workshop on HOS [2]. There, an FIR-(ℓ) "reference system" $f(k)$ with output signal $y(k)$ was introduced according to the dashed lines in Fig. 1. This one-channel system arrangement has led to a generalized eigenvector problem. In [3], we have derived a modified eigenvector problem for an extended (single input multiple output) system arrangement including M parallel channel/equalizer branches and M reference systems connected in parallel to the equalizers. Fig. 2 displays the case $M = 2$, which will prove to be the case of interest for this paper. It can easily be shown that this two-channel equalization problem leads to the modified generalized eigenvector problem

$$\tilde{\mathbf{C}}_4^{yvv}\,\tilde{\mathbf{e}} = \lambda\,\tilde{\mathbf{R}}_{vv}\,\tilde{\mathbf{e}} \quad \text{with} \quad |\lambda| = \max\{|\lambda_i|\}\ , \tag{1}$$

where the coefficients of both equalizers $e_0(k)$ and $e_1(k)$ are collected in the modified vector

$$\tilde{\mathbf{e}} = [e_0(0), \cdots, e_0(\ell_0),\ \ e_1(0), \cdots, e_1(\ell_1)]^T\ , \tag{2}$$

the $(\ell_0+\ell_1+2)\times(\ell_0+\ell_1+2)$ correlation matrix is defined as[3]

$$\tilde{\mathbf{R}}_{vv} = E\{\tilde{\mathbf{v}}\tilde{\mathbf{v}}^*\} \ \text{ with }\ \tilde{\mathbf{v}}^* = [\mathbf{v}_0^*,\ \mathbf{v}_1^*] \quad \text{and}\quad \mathbf{v}_\mu^* = [v_\mu(k), \cdots, v_\mu(k-\ell_\mu)], \tag{3}$$

[3] $E\{\cdot\}$ denotes statistical expectation; the superscript asterisk stands for the conj. complex transpose of a vector or matrix.

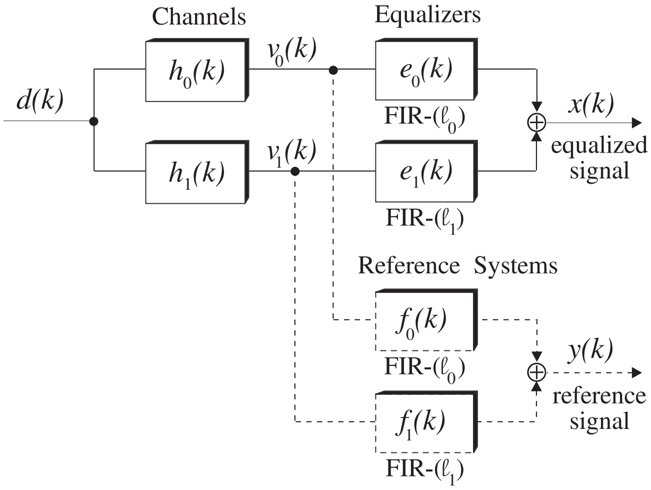

Figure 2: Linear two-channel equalizer structure

and the 4th order[4] cross-cumulant matrix is given by

$$\tilde{\mathbf{C}}_4^{yvv} = \begin{pmatrix} \mathbf{C}_4^{yv_0v_0} & \mathbf{C}_4^{yv_0v_1} \\ \mathbf{C}_4^{yv_1v_0} & \mathbf{C}_4^{yv_1v_1} \end{pmatrix} \tag{4}$$

with the $(\ell_\nu+1)\times(\ell_\mu+1)$ submatrices

$$\begin{aligned}\mathbf{C}_4^{yv_\nu v_\mu} \;=\;& E\{|y(k)|^2\mathbf{v}_\nu\mathbf{v}_\mu^*\} - E\{|y(k)|^2\}E\{\mathbf{v}_\nu\mathbf{v}_\mu^*\}\\ &-E\{y(k)\mathbf{v}_\nu\}E\{y^*(k)\mathbf{v}_\mu^*\}\\ &-E\{y^*(k)\mathbf{v}_\nu\}E\{y(k)\mathbf{v}_\mu^*\}\ .\end{aligned} \tag{5}$$

Analogous to the one-channel EVA [2, sec. 4], the repeated solution of eq. (1) with subsequent replacement of the ref. system's coefficients with those of the updated equalizer yields an efficient iterative algorithm.

Now, we will show that the above two-channel arrangement can be used to describe the DECISION-FEEDBACK (DF) equalizer given in Fig. 3a, where the impulse responses of the feedforward and feedback equalizers are denoted $e_f(0), \cdots, e_f(\ell_f)$ and $e_b(1), \cdots, e_b(\ell_b)$, resp..

Fig. 3a can be redrawn according to Fig. 3b. Ignoring, for a start, the problem of false decisions, i.e. assuming $\hat{d}(k) = d(k)$, we realize from Fig. 3b that the DF-equalizer represents a special case of Fig. 2, where the channels are $h_0(k) = h(k)$ and $h_1(k) = \delta(k-1)$ and the equalizers $e_0(k)$ and $e_1(k)$ correspond to $e_f(k)$ and $-e_b(k)$, respectively. Now, $\tilde{\mathbf{e}}$ in equation (2) and the subvectors $\mathbf{v}_0$ and $\mathbf{v}_1$ in equations (3) and (5) read

$$\begin{aligned}\tilde{\mathbf{e}} &= [e_f(0), \cdots, e_f(\ell_f),\ e_b(1), \cdots, e_b(\ell_b)]^T\ , && (6)\\ \mathbf{v}_0^* &= [v(k), \cdots, v(k-\ell_f)]\ , \qquad \text{and} && (7)\\ \mathbf{v}_1^* &= [d(k-1), \cdots, d(k-\ell_b)]\ . && (8)\end{aligned}$$

[4] Third order cumulants can not be used due to the zero skewness of most digital communication signals.

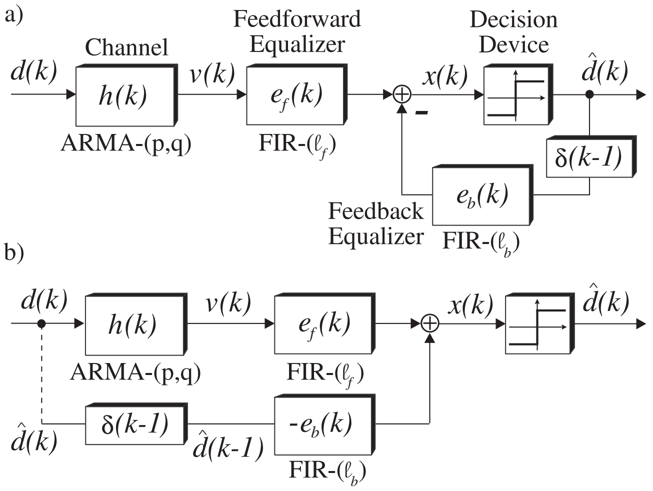

Figure 3: Non-linear (DF) equalizer structure

Thus, the generalized EVA based on equation (1) can be applied to DF-equalization – provided that a sufficiently well equalized and decided received signal $\hat{d}(k)$ is available. This apparent contradiction can be solved by an iterative procedure: In a first step, a linear equalization via the usual (one-channel) EVA is performed with an increased equalizer order $\ell_0 > \ell_f$. In the sequel, perfect non-linear equalization is achieved with the above DF-structure. The resulting five step algorithm will be termed DECISION-FEEDBACK EIGENVECTOR APPROACH to blind equalization (DF-EVA):

- S1: Linear equalization with the iterative one-channel EVA as described in [2] $\Rightarrow$ $e_f(0), \cdots, e_f(\ell_0)$, Decision of the equalized data $\Rightarrow$ $\hat{d}(k)$,
- S2: Reduce the order of the forward equalizer to ℓ_f and introduce the order ℓ_b feedback equalizer,
- S3: Using the decided data $\hat{d}(k-1), \cdots, \hat{d}(k-\ell_b)$, set up $\mathbf{v}_0$ and thus $\tilde{\mathbf{v}}$, estimate $\tilde{\mathbf{R}}_{vv}$ and $\tilde{\mathbf{C}}_4^{yvv}$ and solve the eigenvector problem (1) $\Rightarrow \tilde{\mathbf{e}}$ as in eq. (6),
- S4: Replace the reference systems' coefficients with those of the updated equalizers $\tilde{\mathbf{e}}$,
- S5: Decision of the equalized data $\Rightarrow$ $\hat{d}(k)$, then: Go to step S3.

Obviously, false decisions represent a principal problem with the derivation of this algorithm. In step S1, an estimate $\hat{d}(k)$ of the original data sequence is determined by means of a symbol rate FIR equalizer. On account of imperfect equalization and a possibly high SNR loss due to *linear* equalization, we may get a rather poor initial error rate influencing the following steps. One could argue that, after step S1, the problem is no longer *blind*, so that steps S2 to S5 could be replaced with the (cross-correlation based) MMSE-solution (minimim mean square error). However, the performance of the MMSE-solution based on $\hat{d}(k)$ strongly depends on the initial error rate. As opposed to this, DF-EVA iteratively improves the error rate. Simulation results described in [3] clearly indicate that DF-EVA converges properly even with decision errors. A result near the optimum can be obtained even *without* step S1, if the data blocklength is increased (this is not possible with the MMSE-solution). Thus, DF-EVA turns out to be a very robust algorithm.

3 BLIND ARMA PARAMETER ESTIMATION WITH DF-EVA

NOISELESS CASE: The DF-structure utilized above is capable of perfectly equalizing ARMA systems: Channel poles (zeros of the AR part $A_p(z)$) are cancelled by zeros of the feedforward equalizer $E_f(z)$, so that the resulting overall system

$$H(z) \cdot E_f(z) = \frac{B_q(z)}{A_p(z)} \cdot E_f(z) \qquad (9)$$

corresponds to the channel's MA part $B_q(z)$. Then, the signal on the feedback branch (with the coefficients $e_b(1) \cdots e_b(\ell_b)$) compensates the error signal caused by non-zero values of the overall impulse response $h(k) * e_f(k)$ for $k > 0$. Thus, in the sense of the desired *identification*, the channel's AR parameters are entirely determined by the coefficients of the feedforward equalizer $e_f(k)$, whereas for the MA part, a cross-correlation (at lag zero) between the decision device's input and output signals is needed in addition to the feedback equalizer coefficients $e_b(k), k > 0$. This approach will be called DF-EVI (DECISION-FEEDBACK EIGENVECTOR APPROACH TO BLIND IDENTIFICATION).

For the feedforward equalizer to cancel the channel poles, its order ℓ_f must be equal to the channel's AR order p, which is unknown. If we overestimate the AR order (i.e. $\ell_f = \hat{p} > p$), DF-EVI will estimate additional poles, as can be seen from the following simulation results. For a QPSK transmission and four ARMA-(3,q) channel examples C1 to C4, Fig. 4 displays the poles and zeros estimated by DF-EVI. True channel zeros and poles are represented by circles ("∘") and crosses ("×"), respectively. The channels C1 and C4 are allpass-free ARMA-(3,2) models (Fig. 4a and d), C2 is a 3rd order pure allpass (Fig. b, one zero being outside the displayed portion of the complex plane) and C3 an ARMA-(3,1) system containing a first order allpass (Fig. c). Black half circles and black triangles stand for the estimated zeros and poles, where just 500(!) received data samples were used to estimate the required

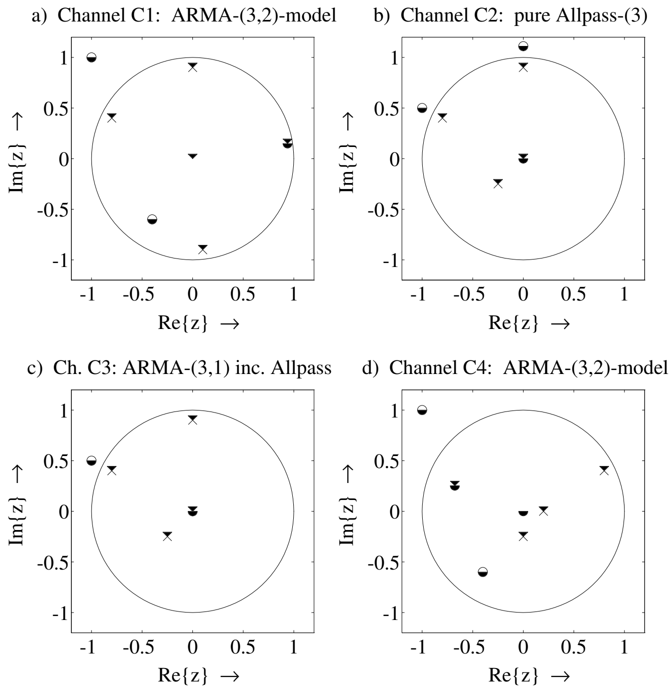

Figure 4: Poles and zeros estimated by DF-EVI (500(!) data samples, noiseless case)
Orders of equalizers: $\ell_f = 6 = 2p$, $\ell_b = 4 > q$,
$\circ/\times$: true zeros/poles,
(black half circle)/(black triangle): est. zeros/poles.

correlations and 4th order cross-cumulants. As can easily be seen from the superimposed symbols, all channel poles and zeros are perfectly identified. However, due to the assumed overestimation of the AR model order ($\ell_f = \hat{p} = 6 = 2p$), three excessive pole estimates emerge in each example. From Fig. 4, these excessive poles either appear at uncritical locations in the complex plane ($|z| = 0$ or ∞) or need to be cancelled by additional zeros of the feedback equalizer (Fig. a and d). Hence, the order ℓ_b of the feedback equalizer must exceed the channel's MA order q by at least the number of additional pole estimates that should be compensated. If this mild requirement is met (which is true in Fig. 4), ideal equalization is guaranteed in the noiseless case. In the sense of an estimation of the channel's spectrum or group delay, DF-EVI is thus *insensitive with respect to channel order overfits.*

NOISY CASE: Considering eq. (1), we can state that independent stationary additive Gaussian noise will affect the correlation matrix $\tilde{\mathbf{R}}_{vv}$ only. Fig. 5 explains the impact of additive white Gaussian noise (AWGN) on DF-EVI's identification quality at signal-to-noise ratios of $S/N = 40$dB (Fig. a to c) and $S/N = 20$dB (Fig. d to i). For the channels C1 to C3 from Fig. 4a to c and 5000 received data samples used to estimate the correlations and 4th order cross-cumulants, the estimated impulse responses are given in the Figures 5a to f. For a total of 10 Monte-Carlo runs, solid and dotted lines indicate the mean and the mean±std.dev. values of the estimated magnitude impulse response; dashed lines represent the magnitude of the true channel impulse response. At $S/N = 40$dB, the impulse responses are estimated without bias and a negligible standard deviation (Fig. 5a to c). However, this is no longer true for the pure allpass channel at $S/N = 20$dB (refer to Fig. 5e). To further clarify this, the poles and zeros estimated in a characteristic Monte-Carlo run at $S/N = 20$dB are displayed in Figures 5g to i. From Fig. 5h can be seen that there is at least one excessive pole estimate outside and close to the unit circle that is not fully cancelled by a zero. This corresponds to a non-causal estimate of the channel impulse response $h(k)$ and therefore leads to the significant mean value of $|\hat{h}(k)|$ for $k < 0$ and increased values of variance in Fig. 5e. Note that the impact of such excessive uncompensated poles outside the unit circle is relevant for the estimated impulse response only if at least one pole is close to the unit circle. In the case of distant poles, the influence on the mean estimated impulse response can be neglected (c.f. Fig. 5d and g). Furthermore, the blocklength of the utilized records of received data samples can be increased to mitigate the effects of high power additive Gaussian noise.

4 CONCLUSION, FURTHER WORK

As opposed to many other 4th order statistics based approaches to blind equalization and identification, the iterative algorithms presented in this paper are capable of equalizing (DF-EVA) and estimating (DF-EVI) a mixed phase ARMA system on the basis of a relatively short record of received data samples. DF-EVI is insensitive with respect to an overestimation of the channel orders. This must be emphasized, because many existing algorithms suffer from ill-conditioned or even singular systems of equations in this case (e.g. [4, 5]). In the noiseless case, DF-EVI estimates the true ARMA parameters without error. We are currently investigating methods to render this approach more robust with respect to additive Gaussian noise. For example, the estimated noise correlation matrix can be subtracted from the correlation matrix $\tilde{\mathbf{R}}_{vv}$ in eq. (1).

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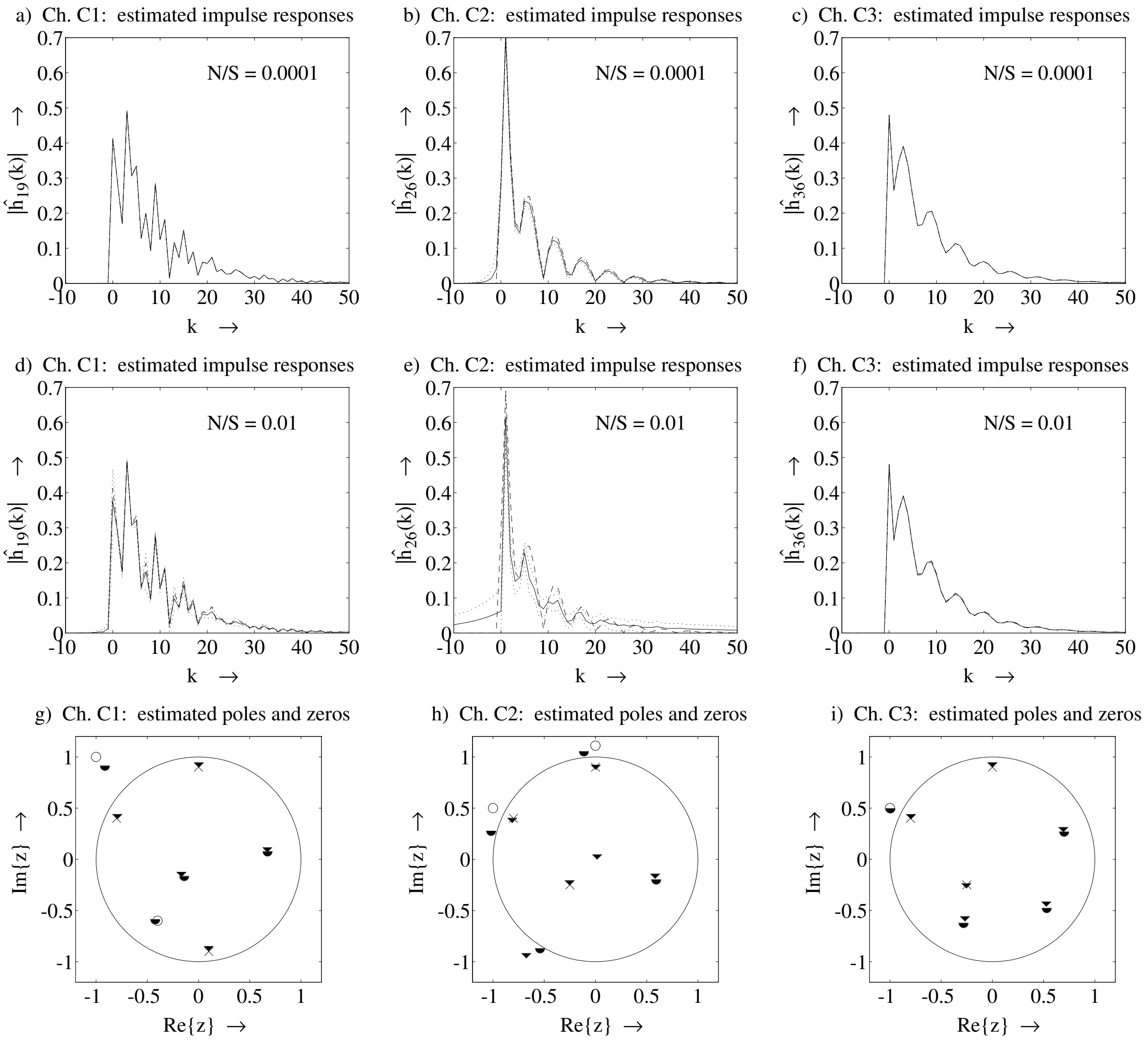

Figure 5: Estimated impulse responses and pole-zero-plots (5000 samples, 10 Monte-Carlo runs) AWGN: $S/N = 40$dB (Fig. a to c) and $S/N = 20$dB (Fig. d to i),
Order of feedforward equalizer: $\ell_f = 6 = 2p$, feedback equalizer: $\ell_b = 4 > q$,
—: mean, ····: mean±std.dev., − −: true impulse response,
o/×: true zeros/poles; (black half circle)/(black triangle): estimated zeros/poles.

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