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Blind Estimation of ARMA Systems

Dieter BOSS and Karl-Dirk KAMMEYER

Hamburg University of Technology, Dept. of Communications, Eissendorfer Str. 40, 21071 Hamburg, Germany,
Tel.: ++(49)-40/7718-2168, Fax: ++(49)-40/7718-2281, E-mail: boss@tu-harburg.de

Abstract[1]**:** In this paper, we present an approach to blind estimation of non-minimum phase ARMA models using fourth order statistics. The algorithm follows a RESIDUAL TIME SERIES (RTS) procedure, which sequentially identifies the AR and MA parts. In step 1 of the RTS concept, the AR estimation is geared to deliver the shortest impulse response of the overall system, so that a new fast approach to MA system identification can be applied in step 2. Investigations have shown that, in order to obtain a given ARMA estimation quality, the Yule-Walker based AR identification requires much more data samples of the channel output signal than does the subsequent MA estimation based on eigenvectors. In an attempt to decrease AR estimation bias and variance at a given blocklength, the STRUCTURED TOTAL LEAST SQUARES (STLS) scheme has proved to achieve better pole estimates of channels with allpasses. However, the STLS technique can not improve on the standard least squares performance, if the cumulant estimation errors are highly correlated, which is true for systems with allpass-free poles close to the unit circle. For these reasons, the ALLPASS SEPARATION (APS) approach has been conceived, which first identifies the allpass-free poles using 2nd order statistics, thereby decorrelating the cumulant estimation errors at the overall system's output. This is why, in a 2nd step, the STLS can be applied successfully. Briefly, for the allpass-free poles, APS benefits from high quality 2nd order estimates while STLS yields the best identification of allpass-poles known to the authors.

1 Introduction

With digital communications, the channel input random processes usually are complex, because they are described by the equivalent baseband representation of digital modulation signals such as PSK or QAM. Although this is neglected quite frequently in literature, we will apply complex cumulants as defined in [1]. Furthermore, digital modulation signals reveal symmetric distributions and thus prohibit the application of 3rd order statistics due to their zero skewness. We will therefore use 2nd and 4th order statistics, exclusively.
The complex discrete channel input random process (r.p.) $d(k)$ is supposed to be stationary, non-Gaussian, zero mean, 2nd and 4th order white, and to have unit power $\sigma_d^2 = 1$.
The z transform of the channel's complex impulse response $h(k)$ is modelled as a causal and stable, possibly *non-minimum phase*, ARMA-(p,q) model

$$H(z) = \frac{B_q(z)}{A_p(z)} \triangleq \frac{b_0 + b_1 z^{-1} + \ldots + b_q z^{-q}}{1 + a_1 z^{-1} + \ldots + a_p z^{-p}}, \tag{1}$$

where the a_i, b_i are the AR and MA coefficients, and p and q are called the AR and MA orders, respectively. The above model is assumed to be free of pole-zero-cancellations and to have unit power amplification. Note that the most critical assumption with a multitude of applications is the time invariance of the transmission channel. This is why we shall try to use as few samples of the channel output r.p. for the estimation of moments as ever possible.
Objective: using samples of the channel output r.p. $x(k)$ only, the aim of blind ARMA identification is to estimate the parameters a_i, b_i, p and q so that the deviation of the estimated impulse response (IR) $\hat{h}(k)$ from the true channel IR $h(k)$ is minimum. Therefore, the criterion for the assessment of an estimated IR will be $\sum_k |\hat{h}(k) - h(k)|^2$.

2 Blind MA identification

In this section, a fast approach to blind MA system estimation will be outlined. As it is based on the eigenvectors of particular cumulant matrices, it is termed **"EVI"** standing for EIGENVECTOR ALGORITHM FOR IDENTIFICATION.
Compared with other algorithms, EVI is a fundamentally novel approach based on a closed-form solution to the problem of blind *equalisation* [2]. The iterative algorithm *"EVA"* derived from this closed-form solution has been presented in [3]; a comprehensive analysis and a generalisation of EVA to include other types of equalisation has been submitted [4]. As far as system *identification* is concerned, EV**I** was presented in [5]. A detailed description has also been submitted [6]. The purpose of this paper is to integrate EVI into a comprehensive algorithm for blind ARMA identification. We will therefore just outline its most distinctive properties.
In contrast to other approaches, such as Giannakis and Mendel's *Least Squares* solution [7] (modified in [1]), the *Cumulant Zero Matching* approach [8] or *Exhaustive Search* algorithms [9], EVI achieves a given estimation variance at considerably reduced blocklengths of the channel output data. Rather than estimating the cumulants on the basis of tens of thousands of samples (which is indispensable on the conditions[2] detailed in sec. 1), this iterative approach requires just some hundred data values [5, 6]. This has to be emphasized, because future work will be directed towards blind estimation of time variant mobile communication channels. EVI is also insensitive with respect to MA order overfits, which lead to singularities in the system of equations

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[2]such as 4th order statistics, complex processes and systems, low (unity) magnitude of kurtosis.

utilised by some approaches (e.g. [7]). Recent investigations have shown that EVI is quite robust with respect to additive white Gaussian noise, too.

3 Blind ARMA identification

In this section, a Residual Time Series (RTS) approach to blind ARMA model identification will be given.

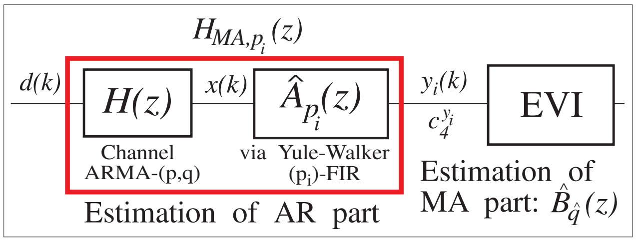

Figure 1: RTS approach

The following three step algorithm is proposed:

S1: Given P values[3] p_i, $i = 1 \ldots P$ of the AR order p, estimate the channel's AR part $A_p(z)$ with the aid of *modified Yule-Walker* equations [10] generalised to 4th order statistics [7]. Realizing the transfer functions of these estimations $\hat{A}_{p_i}(z)$ as FIR-filters, we obtain P overall systems $H_{MA,p_i}(z) = H(z)\,\hat{A}_{p_i}(z)$ (see Fig. 1). The impulse responses $h_{MA,p_i}(k)$ of these systems are called the *"residual time series" (RTS)*.

S2: Select the estimation $\hat{A}_{\hat{p}}(z)$ delivering the *shortest* RTS $h_{MA,\hat{p}}(k)$. This will be accomplished by minimising a higher order cost function. Note that, thereby, the AR order is estimated implicitely: $\hat{p}$.

S3: The selected RTS $h_{MA,\hat{p}}(k)$ ideally being finite (hence its subscript), identify the overall system $H_{MA,\hat{p}}(z)$ with the EVI algorithm described in section 2.

These three steps S1 to S3 will be described in the sequel.

3.1 S1: Blind AR Identification

The AR coefficients $a_1, \ldots a_p$ of the ARMA-(p,q) model $H(z)$ according to eq. (1) can be calculated from the modified 4th order Yule-Walker equation

$$c_4^x(m, n_0, n_1) + \sum_{k=1}^{p} a_k^* \cdot c_4^x(m-k, n_0, n_1) = 0, \qquad (2)$$

where c_4^x represents the 4th order complex cumulant and $m > q + \min\{0, n_0, n_1\}$ with arbitrary integers n_0 and n_1. In a first attempt, we collect p equations (2) by letting $m = q+1 \ldots q+p$ to obtain

$$\begin{bmatrix} c(q,\cdot) & \cdots & c(q+1-p,\cdot) \\ \vdots & \ddots & \vdots \\ c(q+p-1,\cdot) & \cdots & c(q,\cdot) \end{bmatrix} \mathbf{a} = - \begin{bmatrix} c(q+1,\cdot) \\ \vdots \\ c(q+p,\cdot) \end{bmatrix}$$

where $c(\lambda, \cdot)$ has to be replaced with $c_4^x(\lambda, n_0, n_1)$ and $\mathbf{a}$ is defined as $\mathbf{a} = [a_1 \ldots a_p]^*$. Note that the asterisk with a vector or matrix stands for the conjugate complex transpose. With the evident matrix definitions, the above eq. reads

$$\boxed{\mathbf{H}_{n_0,n_1} \cdot \mathbf{a} = -\mathbf{c}_{n_0,n_1}}, \qquad (3)$$

where $\mathbf{H}_{n_0,n_1}$ is a Toeplitz matrix.

The straightforward way to solve for the a_i is to choose $n_0 = n_1 = 0$ and solve eq. (3) by inversion. On account of the symmetry property $c_4(\lambda, 0, 0) = c_4(-\lambda, -\lambda, -\lambda)$, this approach is said to use "diagonal cumulants", only. However, the results gained by this approach generally show a high estimation variance. In order to improve the AR estimation performance, the number of equations can be increased by taking non-diagonal cumulants ($n_0, n_1 \neq 0$) into account. If we concatenate eq. (3) for different values of (n_0, n_1), say $n_0 = n_{01} \ldots n_{0M}$ and $n_1 = 0$, we obtain the overdetermined system of equations

$$\begin{bmatrix} \mathbf{H}_{n_{01},0} \\ \vdots \\ \mathbf{H}_{n_{0M},0} \end{bmatrix} \mathbf{a} = - \begin{bmatrix} \mathbf{c}_{n_{01},0} \\ \vdots \\ \mathbf{c}_{n_{0M},0} \end{bmatrix}$$

or equivalently

$$\boxed{\mathbf{H} \cdot \mathbf{a} = -\mathbf{c}}. \qquad (4)$$

Eq. (4) can the be solved in a least squares sense.

3.2 S2: Selection of the "best" AR estimation

In step S1, we have thus obtained P overall systems

$$H_{MA,p_i}(z) = B_q(z) \cdot \frac{\hat{A}_{p_i}(z)}{A_p(z)}.$$

Ideally, their impulse responses $h_{MA,p_i}(k)$ will be finite. Realistically, however, they will be infinite due to an imperfect AR estimation. Bearing in mind that, the shorter the respective RTS is, the better the estimates of the MA coefficients (c.f. sec. 2) will be, it becomes obvious that we need to select the AR order such that the RTS $h_{MA,p_i}(k)$ has minimum length. Hence, the cost function can be defined as

$$\sum_{k=\gamma}^{\infty} |h_{MA,p_i}(k)|^2 \stackrel{!}{=} \min, \qquad (5)$$

where γ represents a threshold index in time.
Let $w(k)$ be a weighting function defined as

$$w(k) := \sum_{\nu=0}^{k_{max}} |h_{MA,p_i}(k-\gamma-\nu)|^2,$$

which – when multiplied with the squared magnitude of the RTS – suppresses its first γ coefficients. k_{max} is supposed to be chosen large enough such that $|h_{MA,p_i}(k)|^2$ can be regarded negligible beyond the index $\gamma + k_{max}$. Now, considering

$$\begin{aligned} f(p_i, \gamma) &\triangleq \gamma_4^d \sum_{k=0}^{\infty} \left[|h_{MA,p_i}(k)|^2 \cdot w(k) \right] \\ &= \sum_{\nu=0}^{k_{max}} \gamma_4^d \sum_{k=0}^{\infty} |h_{MA,p_i}(k)|^2 |h_{MA,p_i}(k-\gamma-\nu)|^2 \\ &= \sum_{\nu=0}^{k_{max}} c_4^y(-\gamma-\nu,\ 0,\ -\gamma-\nu) \end{aligned}$$

[3] either arbitrarily chosen or selected with some a-priori knowledge.

we conclude that minimising $f(p_i, \gamma)$ can be regarded as equivalent to the requirement in eq. (5). To render this measure independent from the power transfer function of the respective estimation $\hat{A}_{p_i}(z)$, we normalise this function to obtain

$$F(p_i, \gamma) \stackrel{\triangle}{=} \frac{1}{r_{yy}^2(0)} \sum_{\nu=0}^{k_{max}} c_4^y(\gamma+\nu,\ \gamma+\nu,\ 0). \qquad (6)$$

Minimising $F(\hat{p}, \gamma)$ thus delivers the estimated AR order $\hat{p}$.

3.3 S3: ARMA Identification

Having selected the AR estimation $\hat{A}_{\hat{p}}(z)$ with the aid of equation (6), we need to identify the RTS

$$h_{MA,\hat{p}}(k) = \mathcal{Z}^{-1}\left\{H_{MA,\hat{p}}(z)\right\} = \mathcal{Z}^{-1}\left\{H(z)\hat{A}_{\hat{p}}(z)\right\}.$$

As this is FIR (in the ideal case), the EVI approach described in section 2 can be used. Terming its estimation $\hat{B}_{\hat{q}}(z)$, we achieve the final solution to the ARMA estimation problem

$$\hat{h}(k) = \mathcal{Z}^{-1}\left\{\hat{H}(z)\right\}, \quad \text{where} \quad \hat{H}(z) \stackrel{\triangle}{=} \frac{\hat{B}_{\hat{q}}(z)}{\hat{A}_{\hat{p}}(z)}. \qquad (7)$$

Notice the following advantages of this RTS scheme:
(i) Usual applications of the RTS procedure to ARMA system identification attempt to estimate the ARMA parameters p, q, a_i and b_i as exactly as possible. However, in a set-up as described in section 1, this is not really the ultimate goal. We try to approximate the true IR instead.
(ii) Commonly, RTS based algorithms *separately* minimise the variance of the AR and MA estimates. Here, we pursue a unified approach, where the AR estimation is geared to deliver an RTS which is optimum for the subsequent MA identification (i.e. finite). The criterion how to select the "optimum" AR estimation is *NOT* the quality of the approximation of $1/A_p(z)$ by $1/\hat{A}_{p_i}(z)$, but the length of the resulting RTS.
(iii) Remember that badly estimated poles[4] are not relevant in the sense of our overall concept. Such an estimation represents a zero of the z transform of the RTS and will therefore be identified by EVI. The crucial point is that poles with near unity magnitude must be estimated as exactly as possible. Otherwise, the corresponding RTS would be very long and the MA estimation would fail. The algorithm described in step S2 prevents such RTS from being retained.
Given different values of p_i, there are four possible cases:

1. *Critical* underfit: if we underestimate the number of "critical" poles (those with near unity magnitude), we will end up with a long RTS. Such an estimation will *not* be retained in S2 of the approach.
2. *Uncritical* underfit: critical poles will be compensated for. The remaining poles will lead to a (relatively quickly) decaying RTS.
3. No misfit: ideally, the RTS will be finite; however, due to estimation errors, it will be a quickly decaying infinite impulse response.
4. Overfit: additional zeros, which will be identified in step S3. However the RTS will decay quickly.

[4] which occur in case of an order misfit or when the magnitude of a true pole is small.

3.4 Simulation Results

Fig. 2 shows the results of the RTS approach presented here. The input r.p. is QPSK, and the ARMA channels are of order $(p, q) = (3, 1)$ with a zero at $z = 0.4 + 0.6j$. Fig. 2a and b display the true poles as small circles (covered by dots).

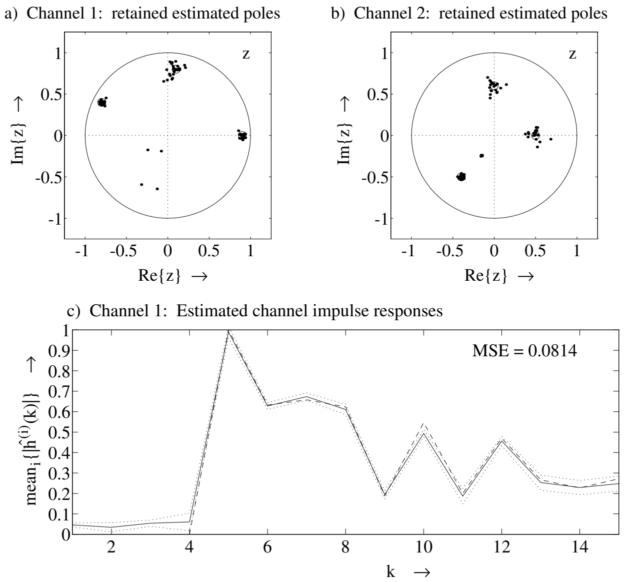

Figure 2: Estimation results of RTS approach

In S1, the AR estimations (25 Monte Carlo runs for each value of $p_i = 1 \ldots 4$) were gained by solving eq. (4) in the least squares sense ($n_{01} = q - p$, $n_{0M} = q$). The zeros of the AR identifications $\hat{A}_{\hat{p}}(z)$ which were retained in S2 are shown as dots (i.e. the retained estimated channel poles). Both eq. (4) and (6) use cumulant estimates based on 30000 samples. Then, in S3, the remaining 25 RTS were estimated by EVI using cumulants and autocorrelation coefficients based on blocks of 1000 data values. For channel 1 (fig. 2a), fig. c shows the mean and std dev. (dotted) of the estimated impulse responses $\hat{h}(k)$ according to eq. (7). For comparison, a shifted version of the true channel IR is given (dashed).

4 Advanced AR estimation techniques

The above simulation results reveal some sort of discrepancy with the RTS approach presented in section 3. While EVI utilises just 1000 samples of the channel output r.p. for a proper MA identification, the Yule-Walker based AR estimation does need as many as 30000. Hence, some effort has been spent on improving AR estimation quality.
So far, equation (4) has been solved in the least squares (LS) sense. However, the standard LS-approach ignores errors in $\mathbf{H}$, because it modifies just the right side vector $\mathbf{c}$ to reduce the rank of eq. (4). Since both $\mathbf{H}$ and $\mathbf{c}$ contain cumulants subject to estimation errors, this is at least questionable.
Approaches that modify both the righthand side vector and the lefthand side matrix to reduce the rank of a linear system of equations are termed *Total Least Squares (TLS)* algorithms [11]. Advanced TLS-techniques, such as the STRUCTURED TOTAL LEAST SQUARES (STLS) scheme [12] permit to account for errors in the matrix while retaining the inherent multiple Toeplitz structure of matrix $\mathbf{H}$.

Further investigations have proven that the performance of the LS-algorithms under consideration (LS, TLS, STLS) do profit from correlations between the cumulant estimation errors. Strong correlations exist in the case of channels with allpass-free poles close to the unit circle. However, with these channels the TLS schemes can not further improve on the standard LS-performance unless the error correlations are known in advance (which is not the case). The only way to further improve AR estimation quality is to resort to 2nd order statistics. On the other hand, with channels containing allpasses, STLS outperforms any other scheme, mainly because the cumulant estimation errors are uncorrelated.

The results mentioned above have led to the development of the two step AllPass Separation (APS) approach:

S1: Estimate the allpass-free poles by solving the 2nd order Yule-Walker eq. obtained by replacing $c_4^x(\lambda, \cdot)$ with $r_{xx}(\lambda)$ in eq. (3). This can be accomplished by matrix inversion or with the LS-approach, as proposed by Cadzow [13]. Here, utilizing the autocorrelation sequence assures excellent pole estimates. Similar to the RTS scheme, the channel output data are fed through the FIR filter with the estimated AR transfer function.

S2: Based on this filter's output signal, estimate the channel poles belonging to allpasses by solving eq. (4) in the STLS sense. As the FIR filter has decorrelated the cumulant estimation errors, STLS yields a remarkably good identification of the allpass-poles.

To work in an optimum fashion, APS requires both the number of allpasses and the total AR order p to be known. These orders can be determined by singular value decomposition of the matrix used in the respective step. The orders are equal to the number of non-zero singular values.

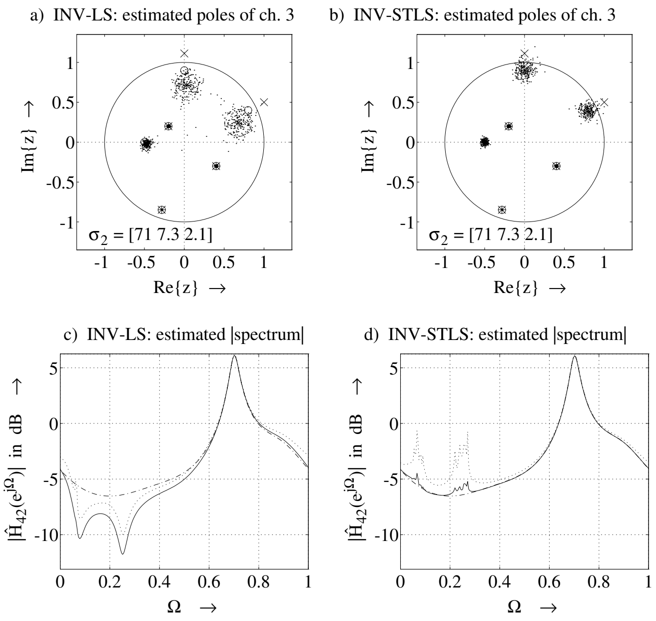

Figure 3: Estimation results of APS approach

Fig. 3 shows the APS estimation results for an ARMA-(6,3) channel having 3 allpass-free poles and a 3rd order allpass. Fig. 3a and b display the pole-zero-plots, where 2 out of 3 zeros belonging to allpass factors are marked with a cross ("×"), the 3rd one is in $z_0 = -2$. True poles are represented by small circles, the pole estimates by dots. The figures 3c and d show the estimated and exact magnitude spectra.

S1: Due to long data blocks (15000 samples), solving the 2nd order Yule-Walker equation by matrix inversion provides us with almost perfect estimations of the 3 allpass-free poles. This can be seen from the figures a and b, where the true ("∘") and the mean estimated ("∗") pole symbols lie on top of each other and no "estimation clouds" are visible.

S2: Here, eq. (4) is solved in the LS (Figures a and c) and STLS (Fig. b and d) sense, respectively. Both the pole plots and the magnitude spectra clearly show that STLS outperforms LS in terms of bias and std deviation. This is due to its application to an ARMA channel containing allpasses.

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