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Statistical Analysis of Cumulant Estimation Errors with regard to an efficient Blind ARMA Identification

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Abstract[1]**:** As many signal processing algorithms in the fields of system identification, equalisation (deconvolution), spectral estimation, signal modelling, image processing etc. utilise estimates of statistical quantities such as the covariance or cumulant sequence, their performance will clearly depend on the statistical properties of these quantities' estimates and their errors. This is why we will investigate the standard deviation and cross-correlation of the covariance and cumulant estimates. As they are often estimated from a finite block of data at the output of an LTI system excited with a stationary white random process, we will link the qualities of the estimation statistics to the properties of the LTI system (channel).
With the problem of blind estimation of possibly mixed phase ARMA transmission channels in mind, this paper is to examine the influence of certain channel properties (pole locations) on the correlation and cumulant estimation statistics at the channel output. Moreover, the effect of the autocorrelation and cumulant estimation statistics on sophisticated least squares approaches (such as the Structured Total Least Squares scheme) to Yule-Walker based AR estimation is studied.

1 Introduction

In this section, we will describe the application, blind ARMA transmission channel identification, that has caused us to investigate the statistical properties of autocorrelation and cumulant estimates. Some results of these investigations are given in sec. 2. In section 3, we return to our application and explain with the results of sec. 2, why some advanced least squares (LS) approaches to blind AR estimations do improve on the standard LS approach with certain classes of channels while they fail to do so with other channels.

1.1 Blind ARMA estimation

The discrete channel input random process (r.p.) $d(k)$ is supposed to be stationary, non-Gaussian, 2nd and 4th order white, and to have zero mean and unit power. It is complex, because with digital communications, it is described by the equivalent baseband representation of digital modulation signals such as PSK or QAM. As such signals generally reveal symmetric distributions and thus prohibit the application of 3rd order statistics due to their zero skewness, 2nd and 4th order statistics will be used here, exclusively.
The z transform of the channel's complex impulse response $h(k)$ is modelled as a causal and stable, possibly *mixed phase*, ARMA-(p,q) model

$$H(z) = \frac{B_q(z)}{A_p(z)} \triangleq \frac{b_0 + b_1 z^{-1} + \ldots + b_q z^{-q}}{1 + a_1 z^{-1} + \ldots + a_p z^{-p}}, \tag{1}$$

where the a_i, b_i are the AR and MA coefficients, and p and q are called the AR and MA orders, respectively. The above model is assumed to be free of pole-zero-cancellations and to have unit power amplification.

Objective: using samples of the channel output random process $x(k)$ only, the aim of blind ARMA identification is to estimate the model's parameters in an "optimum manner", where the criterion for the assessment of a given estimation depends on the specific application.

1.2 Residual Time Series approach

For the blind identification of possibly mixed phase ARMA systems, different Residual Time Series (**RTS**) procedures have been proposed in literature [1, 2]. Although the common basic idea is to separately estimate the AR and MA parameters of the channel model, the algorithms to sequentially identify the denominator and nominator of the rational model do vary. With the RTS scheme proposed in [3, 4], the AR estimation is geared to permit optimum MA identification results. The two identification procedures are:

1. For *AR parameter estimation* of possibly mixed phase ARMA systems, an overdetermined system of 4th order modified Yule-Walker equations is solved in the least squares sense. This will be briefly reviewed in sec. 3.1.
2. To solve the *MA identification problem*, a fast approach based on the eigenvectors of particular cumulant matrices was introduced recently [5]. It is termed **EVI** standing for EigenVector algorithm for Identification. As EVI has been derived from an iterative realisation of a closed-form solution to blind *equalisation* [6, 7, 8], it achieves a given estimation variance at considerably reduced block lengths of the channel output data. Robustness with respect to order overfits and additive white Gaussian noise are further attractive features [5, 9], especially because order overfits often lead to singularities in the system of equations used by some approaches [2, 10].

While EVI offers the remarkable advantage of estimating the MA parameters on the basis of just some hundred samples of the channel output random process, 4th order Yule-Walker based AR identification requires several ten thousands of samples. Therefore, some effort has been spent on eliminating or mitigating this discrepancy between the AR and MA estimations. The following subsection delivers an analysis of the reasons for imperfect AR estimations.

1.3 AR estimation error analysis

If the channel's AR coefficients are estimated on the basis of ACS (autocorrelation sequence) or cumulant estimates by

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the solution of a system of linear equations (mostly overdetermined, Yule-Walker, e.g.), the AR estimation quality is subject to the following influences.

1. The most obvious influences are a non-zero standard deviation and/or bias of the ACS or cumulant estimates.
2. The correlations between the estimates' errors may have a crucial impact on the AR estimation quality, strongly depending on the algorithm used.
3. It depends on the AR estimation algorithm, too, to what extent the error mentioned first is dampened or amplified. The AR estimation performance may well be better than one would expect when considering a given cumulant or ACS estimation quality. The relevant figure of merit is the *condition number* κ of the system of linear equations [11]. It is equal to the maximum relative error amplification.
4. Depending on the specific application, the estimated AR coefficients may just be an interim result on the way to, say, spectral estimation. Therefore, there is an additional transition from a given coefficient estimation performance to the estimation quality of the impulse response, magnitude spectrum, group delay etc.

The former two influences refer to the estimation properties of the ACS and cumulant estimates. They will be examined in terms of certain channel properties in section 2.

Influence 3: apart from the standard least squares (LS) approach, the overdetermined system of 4th order Yule-Walker equations can be solved by sophisticated *Total Least Squares (TLS)* algorithms such as the STRUCTURED TOTAL LEAST SQUARES (STLS) scheme. Furthermore, the ACS can be included into the system of equations to reduce the AR estimation variance. Such approaches will be discussed in sections 3.2 and 3.3. Note that, in order to assess the effectiveness of these approaches, a detailed analysis of the statistical properties of the ACS and cumulant estimates is indispensable. In particular, the STLS approach is efficient if and only if the coefficients of the 4th order Yule-Walker equation (i.e. the cumulant estimates) are subject to equal power estimation errors that are uncorrelated with each other. As mentioned above, these statistical properties will be examined in the following section.

Finally, to depict influence 4, we will assess pole-plots, magnitude spectra and group delays.

2 Influence of channel properties on the ACS & cumulant estimation statistics

The performance of any algorithm utilising statistical quantities (moments, cumulants etc.) of an LTI system's output r.p. will evidently depend on the statistical properties of the relevant estimates, which in turn are influenced by the properties of the LTI system (channel) and the probability density function of the white input r.p.. More precisely, we will investigate in this section, in how far certain channel properties determine the power and correlation of the estimation errors of ACS and 4-th order autocumulant estimates, respectively.

For the following demonstrations, an ARMA system is excited with a stationary i.i.d. QPSK r.p. $d(k)$. On the basis of a given blocklength of 400 realizations $x(k)$ of the resulting steady-state channel output r.p., some ACS and 4-th order cumulants are estimated without bias. From these estimations, the power and the cross-correlation of the estimation errors are calculated. This section's statements are all based on comprehensive simulations executed with 30 different ARMA channels and several blocklengths.

2.1 Error power

Many least squares approaches to solve an overdetermined system of linear equations imply that certain disturbed coefficients of the equations have equal standard deviation values (c.f. section 3.2). With Yule-Walker based AR system identification, e.g., the "coefficients" are autocorrelation or cumulant estimates at the channel output. Hence, we will examine in this subsection, which channel properties determine the autocorrelation and cumulant estimation variances.

Let $c_4^x(\cdot)$ be a 4th order complex cumulant defined as

$$\begin{aligned} c_4^x\,(\lambda_1,\lambda_2,\lambda_3) &\triangleq \\ &E\{x^*(k)x^*(k+\lambda_1)x(k+\lambda_2)x(k+\lambda_3)\} \\ &-E\{x^*(k)x^*(k+\lambda_1)\}\,E\{x(k+\lambda_2)x(k+\lambda_3)\} \\ &-c_2^x(\lambda_2)\,c_2^x(\lambda_3-\lambda_1)-c_2^x(\lambda_3)\,c_2^x(\lambda_2-\lambda_1) \qquad (2) \end{aligned}$$

with the ACS $\quad c_2^x(\lambda)\equiv r_{xx}(\lambda)\triangleq E\{x^*(k)x(k+\lambda)\}$.

Given the ACS and cumulant estimation errors

$$\Delta c_2(\lambda) \;\triangleq\; \hat{c}_2^x(\lambda)-c_2^x(\lambda)\equiv \hat{r}_{xx}(\lambda)-r_{xx}(\lambda) \qquad (3)$$

$$\Delta c_4(\lambda,n_0) \;\triangleq\; \hat{c}_4^x(\lambda,n_0,0)-c_4^x(\lambda,n_0,0) \qquad (4)$$

the MEAN SQUARE ERROR (MSE) is defined as

$$\mathrm{MSE}_n(\lambda)=E\left\{|\Delta c_n(\lambda,\cdot)|^2\right\}\;, \qquad (5)$$

where n stands for the order of the statistics used. As we apply unbiased estimation procedures, MSE is equivalent to the estimation's error power and variance.

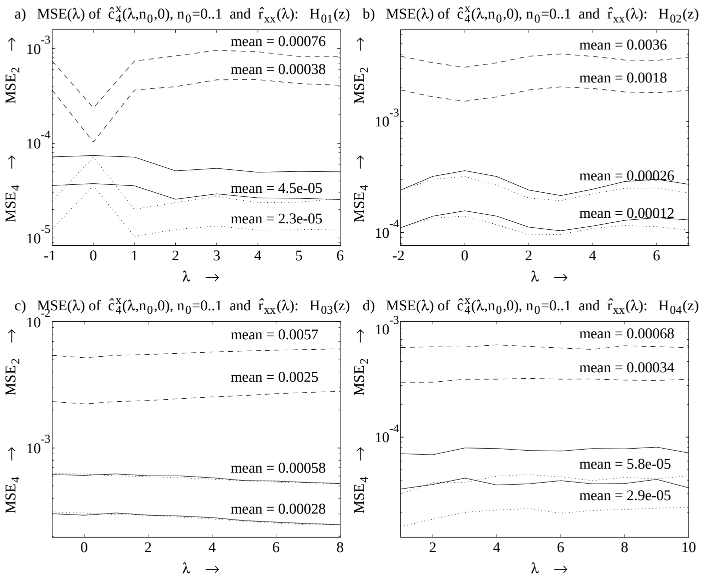

Figure 1: Error power of ACS and cumulant estimates

Fig. 1 displays, on a log scale, $\mathrm{MSE}_4(\lambda)$ (solid lines) on the basis of 15000 and 30000 samples as well as $\mathrm{MSE}_2(\lambda)$ (dashed lines) estimated from 1500 and 3000 channel output samples. Dotted lines correspond to $\mathrm{MSE}_4(\lambda)$ for $n_0 = 1$. As long as the blocklength does not fall below a certain limit (depending on the channel), other blocklengths would lead to further parallel lines. Notice that for the same blocklengths, MSE_2 and MSE_4 would be of the same order of magnitude. However, *relative* or *normalised* values of MSE_4 will exceed those of MSE_2 due to the fact that true autocumulants generally are smaller than true autocorrelation values[2].

[2]except for channels with small magnitude poles, only.

Fig. 1a applies to channel $H_{01}(z)$, which has two medium magnitude poles ($|z_{\infty,1}| = |z_{\infty,2}| = 1/2$). Qualitatively, this figure still holds true, if the channel has additional non-compensating[3] zeros. However, considering channel $H_{02}(z)$ with two poles close to the z-plane's unit circle ($|z_{\infty,1}| \approx |z_{\infty,2}| \approx 0.9$), we can state from fig. 1b, that the error power has increased substantially (by a factor of 5). Essentially, this remains valid even if one critical pole[4] is compensated by a corresponding allpass zero, as shown in fig. 1c with channel $H_{03}(z)$. Only when all critical poles are compensated for, the initial small values of error power are regained, as can be seen from the 3rd order allpass $H_{04}(z)$ in fig. 1d.

Generally, the following statements are valid both for the 2nd and 4th order estimation error power:

- The closer the maximum magnitude pole $z_{\infty,\max}$ of the channel is to the unit circle, the higher are the estimation variances. Allpass poles are not to be taken into account when determining $z_{\infty,\max}$.
- Given a blocklength, MSE can be considered constant with respect to moderate values of λ. In some cases, the real-valued quantities $\hat{r}_{xx}(0)$ (signal power) and $\hat{c}_4^x(0,0,0)$ (kurtosis) play a special rôle (c.f. fig. 1a).

2.2 Cross-correlation between errors

As many approaches to the solution of an overdetermined system of linear equations tacitly assume the estimation errors of the coefficients to be uncorrelated (c.f. section 3.2), the cross-correlation between the autocorrelation or cumulant estimation errors is of some significance. In this sense, for various values of k, we will investigate the correlation

$$r_{n,k}^x(\lambda) \triangleq E\{\Delta c_n^*(k,\cdot)\,\Delta c_n(k+\lambda,\cdot)\} \tag{6}$$

in terms of λ, where $\Delta c_n(k,\cdot)$ is the estimation error according to equations (3) or (4). For each value of k, $r_{n,k}^x(\lambda)$ indicates, in how far the estimation errors at lag k are correlated with those at the "neighbouring" lags $k+\lambda$.

Fig. 2 displays the functions $|r_{n,k}^x(\lambda)|$ for the same systems as in fig. 1. The correlation of the ACS ($n=2$) and cumulant ($n=4$) errors are shown on the left and right side, respectively. With a Yule-Walker based application in mind, the values of k were chosen to be $k = q-(p-1)\dots q$ (solid lines) $\dots q+(p-1)$. Both for the estimation of the ACS and the cumulants, 30000 samples of the channel output r.p. were used. As the graphs for $|r_{4,k}^x(\lambda)|$ do not vary noticeably with modestly changing values of n_0, it has been set to zero.

Figures 2a and b show the error correlation with channel $H_{01}(z)$ (two poles with magnitude 0.5). Significant correlations do occur for very small values of λ, only. If there are, however, two poles near the unit circle, the cross-correlation level is greatly enhanced, as shown in figures 2c and d for channel $H_{02}(z)$. Note the gaps at certain values of λ. Non-compensating zeros would not have any notable impact on these graphs. Compensating one of the critical poles with an allpass zero leads to channel $H_{03}(z)$ with the high error correlation levels given in figures 2e and f. Qualitatively the same graph would be obtained for a channel with a single critical pole. This speaks on behalf of the importance of the equivalent allpass-free system. Only when all poles are compensated with allpass zeros (channel $H_{04}(z)$), we get the nearly "white" graphs given in figures 2g and h.

[3]i.e. distant from the pole (z_∞) and allpass ($1/z_\infty^*$) locations.

[4]poles close to the unit circle will be referred to as "critical".

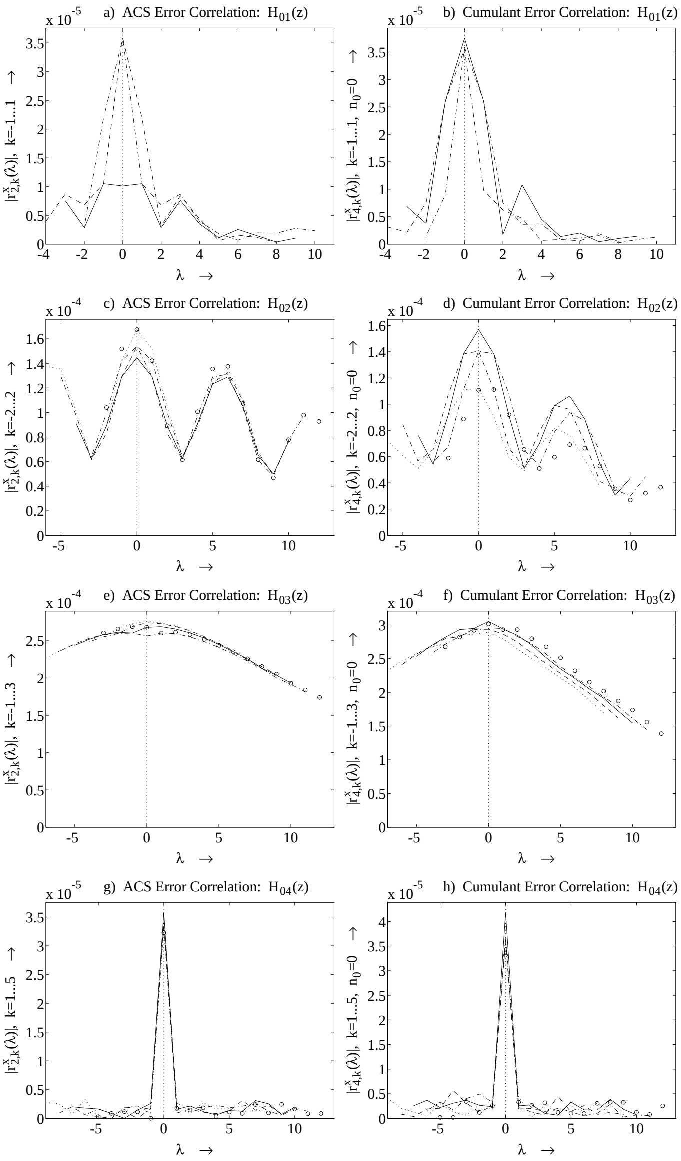

Figure 2: Correlation of the ACS (left) and cumulant (right) estimation errors
—: k=q, – – / – · –: k=q±1, · · · / ◦ ◦: k=q±2.

- Generally, the closer $z_{\infty,\max}$ is to the unit circle, the higher the correlation between the est. errors will be.
- Several critical poles with nearly equal magnitudes lead to a λ-selective behaviour of the error's correlation graph.

Again, allpass poles are not to be taken into account when determining $z_{\infty,\max}$. This means that *both the ACS and cumulant(!) estimation errors are uncorrelated at the output of pure allpasses.* This property will be exploited by the "All-Pass Separation" approach [3], which uses both 2nd and 4th order information to blindly identify ARMA systems.

2.3 Summary

The investigations cited so far suggest to classify the transmission channels into three classes (A to C) of allpass-free systems resulting from the questions:

(1) What is the magnitude of $z_{\infty,\max}$?
(2) Are there several poles with maximum magnitude ?

We will add the classes D and E for the pure allpasses and systems with allpass portions. Table 1 summarizes the main results of this section in terms of the five channel classes.

Channel class: $\max\{\lvert z_{\infty,k}\rvert\}$ is ..	Error power $\mathrm{MSE}_n(\lambda)$:	Error cross-corr. $\lvert r^x_{n,k}(\lambda)\rvert$:
(A) crit., single	high, constant	strongly corr.
(B) crit., multiple	as above	", λ-selective
(C) small	low, constant	weakly corr.
(D) pure allpass	as above	uncorrelated
(E) allpass parts	like equivalent allpass-free ch.	like equivalent allpass-free ch.

Table 1: ACS & cum. estimation error power and correlation

3 Blind AR identification

3.1 Modified Yule-Walker (mYW) eqns

For AR parameter estimation of ARMA systems, the use of 2nd order *modified* Yule-Walker (2mYW) equations [12, 1, 13, 14] is wide-spread, where an additional time delay is introduced into the autocorrelation lags to account for the influence of the model's MA part. This is equivalent to shifting the autocorrelation estimates with lag zero from the main diagonal of the autocorrelation matrix to the q-th diagonal, where q is the MA order.

To enable the estimation of possibly mixed phase ARMA systems, the 2mYW equations have been generalised to higher order statistics [2]. For instance, the following 4th order modified Yule-Walker (4mYW) equation allows the AR coefficients $a_1, \dots a_p$ of the ARMA-(p,q) model $H(z)$ according to eq. (1) to be determined from cumulant estimates:

$$c_4^x(m, n_0, n_1) + \sum_{k=1}^{p} a_k^* \cdot c_4^x(m-k, n_0, n_1) = 0 \qquad \text{for } \; m > q + \min\{0, n_0, n_1\}, \tag{7}$$

where $c_4^x(\cdot)$ is the 4th order complex cumulant defined in eq. (2) and n_0 and n_1 are arbitrary integers.

In a first attempt, we accumulate p equations (7) by letting $m = q+1 \dots q+p$ to obtain

$$\begin{bmatrix} c(q,\cdot) & \cdots & c(q+1-p,\cdot) \\ \vdots & \ddots & \vdots \\ c(q+p-1,\cdot) & \cdots & c(q,\cdot) \end{bmatrix} \mathbf{a} = - \begin{bmatrix} c(q+1,\cdot) \\ \vdots \\ c(q+p,\cdot) \end{bmatrix} \tag{8}$$

where $c(\lambda,\cdot)$ has to be replaced with $c_4^x(\lambda, n_0, n_1)$ and $\mathbf{a}$ represents the AR coefficients' vector $\mathbf{a} = [a_1 \dots a_p]^*$. Note that the asterisk stands for the conj. complex transpose of a vector or matrix. With the evident definitions, eq. (8) reads

$$\boxed{\mathbf{H}_{n_0,n_1} \cdot \mathbf{a} = -\mathbf{c}_{n_0,n_1}}\,, \tag{9}$$

where $\mathbf{H}_{n_0,n_1}$ is a Toeplitz matrix.

The straightforward way to solve for the a_i is to choose $n_0 = n_1 = 0$ and solve equation (9) by inversion. Due to the symmetry property $c_4(\lambda, 0, 0) = c_4(-\lambda, -\lambda, -\lambda)$, this approach is said to use "diagonal cumulants", only. However, the results gained by this approach generally show a high estimation variance. To improve the AR estimation performance, the number of equations can be increased by taking additional information in the form of non-diagonal cumulants ($n_0, n_1 \neq 0$) into account. Concatenating equation (9) for different values of (n_0, n_1), say $n_0 = n_{01} \dots n_{0M}$ and $n_1 = 0$, delivers the overdetermined system

$$\begin{bmatrix} \mathbf{H}_{n_{01},0} \\ \vdots \\ \mathbf{H}_{n_{0M},0} \end{bmatrix} \mathbf{a} = - \begin{bmatrix} \mathbf{c}_{n_{01},0} \\ \vdots \\ \mathbf{c}_{n_{0M},0} \end{bmatrix}, \quad \text{i.e.} \quad \boxed{\mathbf{H} \cdot \mathbf{a} = -\mathbf{c}}. \tag{10}$$

Remarks with regard to 2nd order statistics: replacing $c(\lambda,\cdot)$ in eq. (8) with $r^*_{xx}(\lambda)$ rather than $c_4^x(\lambda, n_0, n_1)$ delivers the well-known system of 2mYW equations. Note that in this case, the orders p and q have to be redefined to be the *equivalent allpass-free orders*. This is to say that in order to apply the 2mYW equations to mixed phase ARMA identification, allpass poles and zeros must *NOT* be taken into account when determining the system orders p and q, respectively. Otherwise, an overestimated equivalent AR order will lead to a singular system of equations, if true ACS values are applied. Although an overestimated MA order would not render the system of equations insolvable, it might considerably degrade the AR estimation performance with certain channels by deteriorating the equation's condition number.

3.2 LS solutions to mYW equations

The overdetermined system (10) of linear 4mYW equations can the be solved in various least squares (LS) senses.

1. **Least Squares (LS):** the standard LS approach modifies the righthand side vector $\mathbf{c}$ so that the system's rank can be reduced by one, or, in other words, so that the modified vector becomes linearly dependent on the colums of $\mathbf{H}$. Thus, the righthand side vector $\mathbf{c}$ is the *quantity to be modified* (QM) by this approach. There is a tacit assumption here that the errors are confined to the righthand side vector, which is *not* true with Yule-Walker eqns, because both $\mathbf{H}$ and $\mathbf{c}$ contain cumulant estimates.
2. **Total Least Squares (TLS):** this algorithm modifies both the elements of $\mathbf{c}$ and $\mathbf{H}$ in order to reduce the system's rank. Hence, the righthand side vector and the matrix together represent the QM. However, if the matrix has some structure, it will generally be distroyed by the correction. Note that this is the case with eq. (10), as $\mathbf{H}$ has a multiple Toeplitz structure.
3. **Structured Total Least Squares (STLS):** with this scheme, the number of the QM's elements can be diminished by the assumption of a certain matrix structure. In the case of equation (10), this is to say that this approach exploits the a-priori information that $\mathbf{H}$ has a multiple Toeplitz structure. Again, both $\mathbf{c}$ and $\mathbf{H}$ represent the QM, limited by some constraints to its structure.
4. **Weighted Least Squares (WLS):** this variant of the LS approach permits to influence its solution by weighting the system's equations. As with the standard LS approach, just $\mathbf{c}$ represents the QM.

The TLS and STLS schemes were developed by S. Van Huffel, J. Vandewalle [15] and B. De Moor [16, 17]. All LS approaches tacitly assume that

- all elements of the QM have equal standard deviation values (however, the weighting has to be taken into account with the WLS approach) and that
- all errors in the QM's elements be uncorrelated (with the STLS scheme, however, just those elements that are not fixed by structural conditions).

From section 2, we can state that the variances of the ACS (and cumulant) estimates can be considered approximately constant with respect to λ. However, their cross-correlation values are not negligible with the channel classes A and B. This should prove unfavourable for the TLS approach, because it assumes the elements of $\mathbf{H}$ to be uncorrelated, although the elements of any diagonal of the submatrices $\mathbf{H}_{n_{0i},0}$ in eq. (10) are equal. The STLS scheme, however, should suffer from the correlations between different diagonals of these submatrices, only. Class D allpass channels with their nearly white ACS and cumulant estimation errors should be in favour of the TLS and STLS approaches.

3.3 Simulations results

Now, the above approaches to blind AR estimation will be applied to the channel classes as defined in sec. 2.3. The algorithms will be referred to by the following abbreviations.

1. **INV:** solution of the non-overdetermined system (8) of 4MYW equations by matrix inversion (pseudo inverse).
2. **LS**-solution to the overdetermined system (10) of 4MYW equations, where $n_{01} = q - p$ and $n_{0M} = q$.
3. **TLS**-solution to the same system of 4MYW equations.
4. **STLS:** iterative STLS solution to the same system.
5. **LS24:** WLS solution to a mixed overdet. system of 4th and 2MYW eqns. For this approach, the non-overdet. system of 2MYW eqns (obtained by replacing $c(\lambda, \cdot)$ in eq. (8) with $r^*_{xx}(\lambda)$) is appended to the overdet. system (10) of 4MYW eqns. The WLS weights are adjusted to ensure equal influences of cumulants and ACS on the solution.
6. **INV2:** for comparison, we include the solution of the non-overdet. sys. (8) of 2MYW eqns by matrix inversion.

For each channel, 200 realizations of the channel output r.p. were generated from a stationary i.i.d. QPSK input r.p.. Unless otherwise stated, blocks of 15000 samples were used to estimate the required cumulants (INV, LS, TLS, STLS) and/or ACS values (LS24, INV2). With these estimates the channel's AR coefficients were determined 200 times according to the above algorithms. The MA and AR orders are supposed to be known. The estimation performance was assessed on the basis of the estimated pole locations, magnitude spectra and group delays.

Class A channels (one max. magnitude pole): according to section 2, class A channels lead to high power cumulant and autocorrelation estimation errors that are strongly correlated. Detailed investigations have also shown, that the condition numbers of the matrices used by the different approaches are quite high[5] for the channel $H_{05}(z)$, so that the errors will be amplified remarkably.
Fig. 3 displays the poles and magnitude spectra[6] estimated by the algorithms mentioned above. In the (3×2) matrix of images, each *row* corresponds to another algorithm. As the INV (INV2) results are very similar to those of the LS (LS24) approach, they have been omitted, here. TLS results are omitted for lack of space. The first (latter) *column* refers to the estimated poles (magnitude spectra). The estimated poles are marked with dots, their mean with an asterisk surrounded by dash-dotted standard deviation circles. True poles are represented by the "o" symbol. The magnitude spectra were computed by $|B(z)/\hat{A}(z)|$, where pole estimates outside the unit circle were disposed of. The mean and the mean±std.dev. graphs are given as solid and dotted lines, resp.; true graphs are indicated by dash-dotted lines.

From the figures, we can state that all algorithms identify the max. magnitude pole with satisfactory values of bias and variance, whereas the less critical poles are estimated with modest quality. STLS can not improve on the LS performance. The only way to improve the estimation variance is by including 2nd order statistics (approaches LS24 and INV2).

[5] $\kappa = 28$ (INV), 29 (LS, TLS, SLTS), 46 (LS24), 47 (INV2).
[6] Group delay estimates are omitted for lack of space.

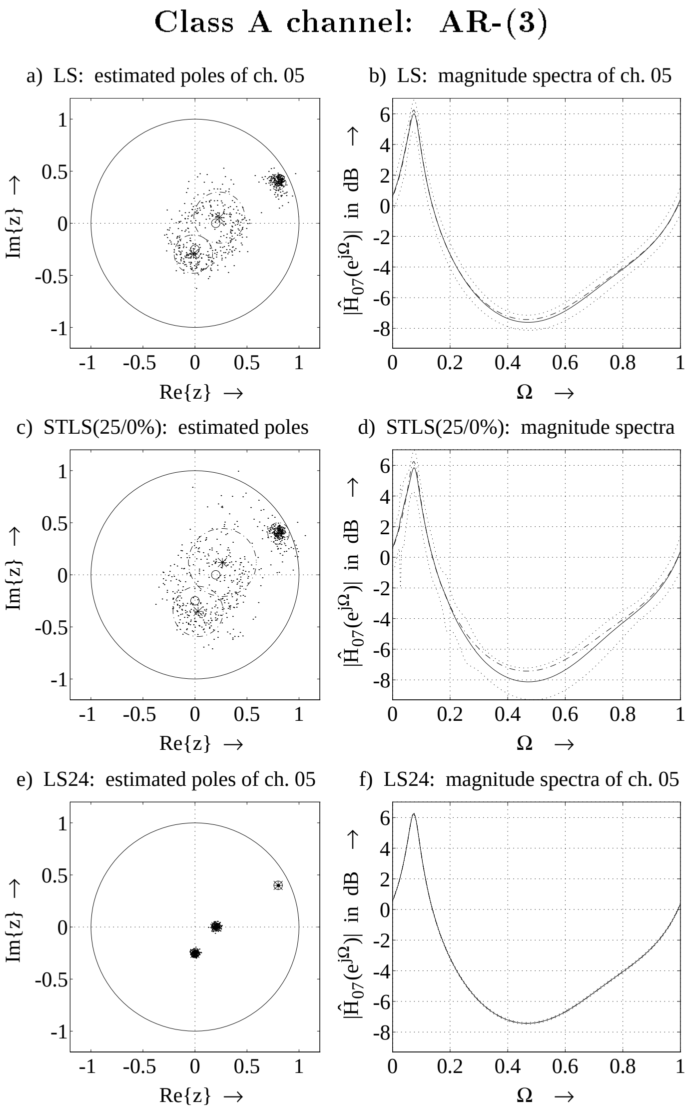

Figure 3: Poleplots and |spectra| of $H_{05}(z)$ (15000 samples)
left : ·/o: est/true poles, ∗: mean, − · −: σ-circles.
right: —: mean, ···: mean±std.dev., − · −: true

Class B channels (several max. magnitude poles): although class B channels also lead to high power cumulant and autocorrelation estimation errors which are strongly correlated (but λ-selective), the approaches considered deliver good (INV, LS, TLS, STLS) to excellent (LS24, INV2) pole estimates for blocklengths of 10000 samples due to good condition numbers (6..7) of the systems of equations. Hence, no simulation results shall be given here.

Class C channels (just small magnitude poles) reveal low estimation errors. Furthermore, the true ACS and cumulant values are on the same level, so that the *relative* MSE are also nearly equal. Therefore, there is no principal disadvantage of using cumulants with this class of channels. With the channel that we have chosen, the condition numbers are ideal (unity). Blocklengths as low as 5000 samples are sufficient to obtain acceptable pole qualities. As the different algorithms do not vary in performance, figures shall be omitted, again.

Classes D and E: pure allpasses have low power autocorrelation and cumulant estimation errors that are nearly white. This is in favour of the STLS scheme, as can be seen from fig. 4 for the 3rd order allpass $H_{04}(z)$, where one zero ("×") lies outside the displayed area of the z-plane. Conditions are moderate (8 to 10).

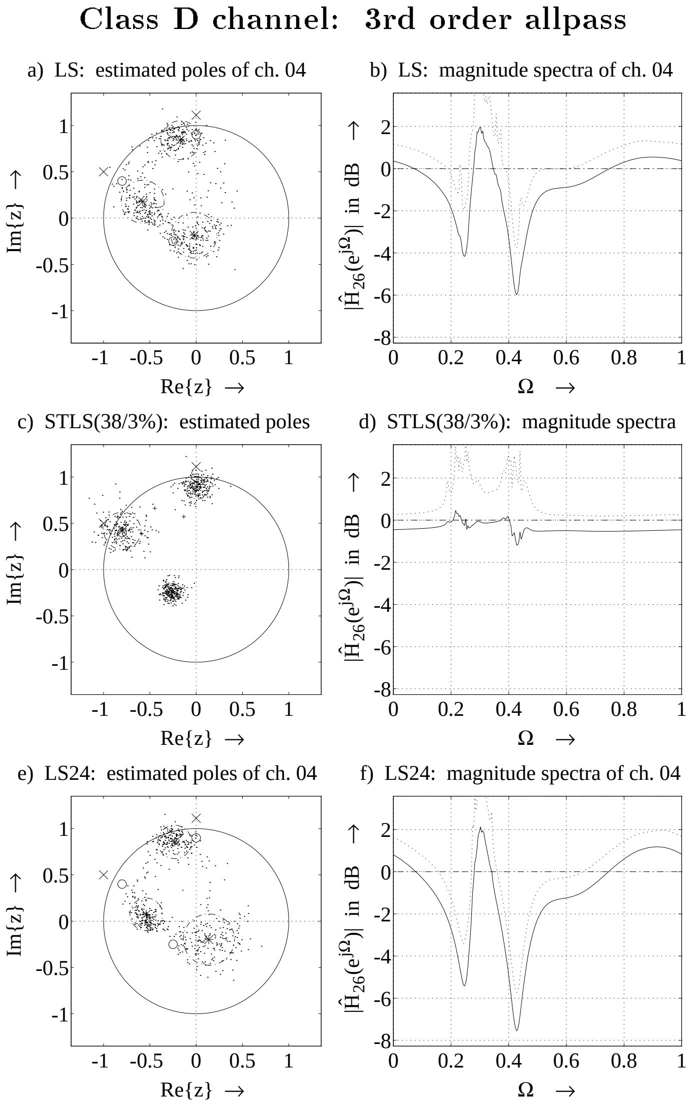

Figure 4: Poleplots and |spectra| of $H_{04}(z)$ (15000 samples)
left : $\cdot$ / $\circ$ / $\times$: est/true poles/zeros, $*$: mean, $-\cdot$: σ-circ.
right: —: mean, $\cdots$: mean+std.dev., $-\cdot-$: true

The simple matrix inversion approach INV suffers very much from the lack of correlations between the cumulant estimates (no figures given). The LS-approach (fig. 4a and b) achieves acceptable levels of variance, but leads to a non-negligible bias. Here, the STLS algorithm (fig. c and d) represents the best choice in terms of standard deviation and variance, because the ACS based approaches lead to high levels of bias (LS24, see fig. e and f). INV2 can not be applied at all, as the true allpass-free orders are zero.
This ranking of the approaches remains true for class E channels (allpass portions), too.

3.4 Conclusion

Critical poles (poles with near unity magnitude) are responsible for strong correlations between the errors of cumulant and ACS estimates, unless the pole is part of an allpass.

- All solutions under consideration (INV, LS, TLS, STLS, WLS) to both 2mYW and 4mYW equations benefit from these correlations, because they lead to compensation effects with the estimation errors.
- Further improvements of AR estimation bias and variance are not possible with the TLS and STLS approaches, because these algorithms implicitly suppose the estimation errors to be uncorrelated. Notice that they could improve the estimation quality if they were given the error correlation matrices, which is not possible without prior knowledge of the channel's properties.

Channels with allpasses lead to uncorrelated autocorrelation *and cumulant* estimation errors.

- Here, the solution by simple matrix inversion as well as by the standard LS-approach suffer from the lack of correlations, since estimation errors do not compensate each other any more.
- On the other hand, the STLS scheme does improve the AR estimation bias and variance considerably.

These results have led to the development of the three step AllPass Separation (APS) approach [3].

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