Extraction of Soft Bit Information with the Use of M-ary Modulation

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September 16, 1995

Abstract

We investigate an M-ary modulation scheme and present an extension of the demodulation process. Instead of a simple symbol decision the a-posteriori symbol-probabilities for all hypotheses of transmitted symbols are calculated. Each symbol represents a group of $\log_2(M)$ bits. This symbol probabilities are mapped for each bit individually into a-posteriori probabilities of the bits in the maximum likelihood sense. This allows to assign reliability information to the bits, at the demodulator output, which can be used in subsequent decoding processes. The philosophy of the presented method is analog to the suboptimum method described in [6], which is not maximum likelihood due to some simplifications, but is less complex with respect to the computational effort.

The presented optimum extraction of reliability information allows to determine lower bounds on the performance for a specified system environment including channel coding.

1 Introduction

In a CDMA transmission system several types of coding are combined. Besides the source code, a channel code e.g. by means of a convolutional code is deployed anyway. Furthermore the spreading procedure may include some intelligent type of coding. The M-ary modulation process of the QUALCOMM system [5] should be interpreted in this way.

The concatenation of different coding schemes requires that not only decided bits have to be transferred to subsequent decoding stages but also reliability information to achieve optimum performance. The generation of reliability information for the output bits on the Viterbi decoding process e.g. is described in [3].

Our interest in this paper is focussed to the modulation scheme. For a binary antipodal transmission, which is assumed to be demodulated coherently, the baseband model may be represented simplified by a binary source, which is corrupted by white gaussian noise. Let us assume that the following channel decoder is a Viterbi decoder which has the capability to work on a soft input signal (4 bit quantizaton e.g. or more). This would be an efficient combination.

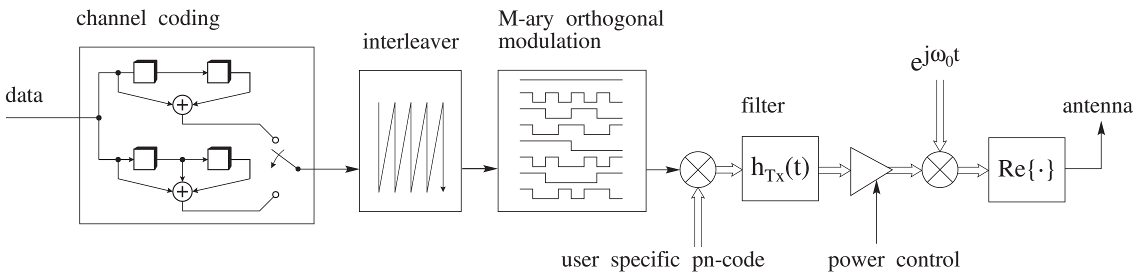

Figure 1: CDMA-transmitter

For an M-ary modulation scheme the situation is quite different. Lets consider a 64-ary orthogonal signal set. Always $6 = \log_2(64)$ bits are mapped into one of the 64 signals. The optimum receiver performs the correlation with all possible signals and may opt for the one with the maximum correlation result. The related group of 6 bits may be delivered to the demodulator's output. Up to this point these bits would not be accompanied by some reliability information.
The advantage in BER performance of M-ary modulation schemes, known for the case of uncoded transmission [4], will be overcome by the loss wich is due to hard bit Viterbi decoding.

2 Reliability information for an M-ary orthogonal modulation scheme

The transmitter of the CDMA system under investigation ist depicted in figure 1.
After the convolutional encoding and interleaving, groups of $\log_2(M)$ bits are mapped into the corresponding orthogonal symbols e.g. Walsh functions. These symbols are spread by the user specific complex pseudo noise sequence (quadrature spreading). The orthogonality of the transmitted symbols is not affected by the spreading. Because of this and to simplify the presentation we don't refer the spreading explicitely in future. The transmitted symbols are denoted as $\boldsymbol{h}_j$, $j = 1, \ldots, M-1$ and shall include the spreading process. The symbol $\boldsymbol{h}_i$ represents the currently transmitted symbol. The μ-th echo of the received signal can be written as

$$\boldsymbol{s}^{(\mu)} \; = \; c_\mu \, \boldsymbol{h}_i + \boldsymbol{r}^{(\mu)} \quad , \tag{1}$$

where c_μ denotes the corresponding channel coefficient and the noise signal $\boldsymbol{r}^{(\mu)}$ results from the superposition of several other user signals and delayed versions of the own signal. In our presentation the receiver is implemented as a noncoherent RAKE receiver. For one correlation time (echo delay μ) we obtain a vector $\boldsymbol{V}^{(\mu)}$ of correlation results with M components:

$$V_j^{(\mu)} = |\, \boldsymbol{h}_j^* \boldsymbol{s}^{(\mu)}|^2 = \begin{cases} |\, c_\mu \, E_h + \boldsymbol{h}_j^* \boldsymbol{r}^{(\mu)}|^2 & , \; j = i \\ |\, \boldsymbol{h}_j^* \boldsymbol{r}^{(\mu)}|^2 & , \; j \neq i \end{cases} \tag{2}$$

By the use of the squared magnitude the influence of the unknown phase of the channel weight c_μ is eliminated. $E_h = \boldsymbol{h}_j^* \, \boldsymbol{h}_j$ denotes the equal energy of any transmitted symbol

$\boldsymbol{h}_j$. The several decision vectors $\boldsymbol{V}^{(\mu)}$, where each stem from one RAKE path, are combined by the summation $\boldsymbol{V} = \sum_{\mu} \boldsymbol{V}^{(\mu)}$ in the presented receiver (square law combining).

The decision for the maximum component of the vector $\boldsymbol{V}$ or for one specific symbol, resp., is not significant for each of the underlying bits. Assume, for the moment, that the the correlation components V_0 and V_{32} in a 64-ary modulation scheme have significantly large magnitudes whereas the remaining components are negligible. The corresponding bit patterns 000000 and 100000 for the symbols $\boldsymbol{h}_0$ and $\boldsymbol{h}_{32}$ only differ in one position. The last 5 bits are reliable, independent of a decision for one of both symbols, while for the first bit no reliable decision can be made. This example illustrates the idea of bitwise reliability information which has to be calculated during the symbol decision process. Figure 2 depicts the described receiver in princple.

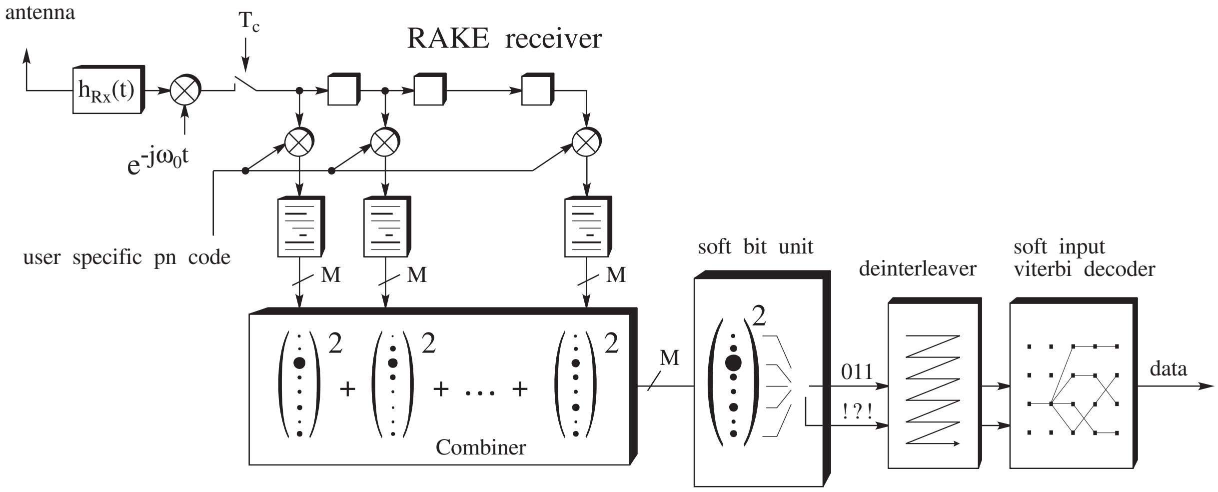

Figure 2: CDMA-receiver

In the "soft bit unit" first the a-posteriori probabilities $P(h_j \mid \boldsymbol{V})$ for all possibly transmitted symbols are calculated from the M decision variables. We assume that the noise process can be approximated by white gaussian noise of appropriate strength. In the context of an incoherent receiver with square law combinig and L-path diversity, we obtain [1]:

$$P(h_j \mid \boldsymbol{V}) \; = Q(V_j) \,/\, q(V_j) \cdot \alpha \;, \tag{3}$$

where $Q(V_j)$ and $q(V_j)$ represent noncentral and central chi-square distributions with 2L degrees of freedom, respectivly, and α stems from a normalisation.

$$q(V_j) \;=\; \frac{1}{(2\,\sigma^2)^L\,(L-1)!}\;\; V_j^{L-1}\, e^{-\frac{V_j}{2\,\sigma^2}} \tag{4}$$

$$Q(V_j) = \frac{1}{2\,\sigma^2}\Big(\frac{V_j}{\varrho^2}\Big)^{\frac{(L-1)}{2}} e^{-\frac{V_j+\varrho^2}{2\,\sigma^2}} \cdot I_{L-1}\Big(\frac{\varrho\sqrt{V_j}}{\sigma^2}\Big) \tag{5}$$

The decision vector $\boldsymbol{V}$ represents the additon of L echos, i.e. the decision vectors $\boldsymbol{V}^{(\mu)}$. The parameter ϱ^2 describes the noncentrality of $Q(V_j)$ and contains the current channel energy.

$$\varrho^2 = \sum_{\mu=0}^{L-1} (E_h\,|c_\mu|)^2 \quad , \tag{6}$$

where the c_μ are the (complex) channel weights of the echo paths. σ^2 denotes the variance of the real or resp. the imaginary part of the related complex white gaussian noise processes $\boldsymbol{h}_j^* \boldsymbol{r}$. I_{L-1} is the $(L-1)$th order modified Bessel function of the first kind.
The constant α has to be calculated for every symbol period individually by

$$\sum_{j=0}^{M-1} P(\boldsymbol{h}_j|\boldsymbol{V}) = \alpha \cdot \sum_{j=0}^{M-1} \frac{Q(V_j)}{q(V_j)} = 1 \quad . \tag{7}$$

This equation represents the union of all mutually exclusive elementary statistical events to make up the completely sure event with probability one.
In a practical receiver design we obtain an estimate for the noise power σ^2 by averaging $M-1$ decision components $\hat{\sigma}^2 = \frac{1}{2\,L\,(M-1)} \sum_{k=0,\ k\neq i}^{M-1} V_k$ (including L echo paths). For an appropriately large value of M, this estimation becomes quite accurate. The estimation of the received signal energy $E_C := \sum_{\mu=0}^{L-1} |c_\mu|^2$ can be obtained from a short time average with respect to the values $\max_j V_j$ over a few consecutively transmitted symbols and has to be performed, for the operation of closed loop power control, anyway.

Now, the symbol a-posteriori probabilities and the corresponding bit patterns are summerized in a table.
The last step in our derivation is the "columnwise" interpretation of such a table. The symbol probabilities have to be transferred to the individual bits for each column separately. Adding all probabilities belonging to a '1' in the first column, e.g., results in an individual probability of the first bit being a '1'.
The following example illustrates the calculation process in detail. For simplicity reasons $M = 4$ has been chosen.

j	$(b_0\ ,\ b_1)$	$P(\boldsymbol{h}_j\,\|\,\boldsymbol{V})$
0	0 0	0.22
1	0 1	0.27
2	1 0	0.25
3	1 1	0.26

The a-posteriori probabilities $P(\boldsymbol{h}_j\,|\,\boldsymbol{V})$ have been chosen arbitrarily. The bitspecific probalilities can be calculated by means of

$$P(b_0 = +1) = 0.25 + 0.26 = 0.51 \tag{8}$$
$$P(b_1 = +1) = 0.27 + 0.26 = 0.53 \tag{9}$$

Note that we only have calculated values $P_\nu := P(b_\nu = +1)$. The corresponding values $P(b_\nu = -1)$ can be calculated by

$$P(b_\nu = -1) = 1 - P(b_\nu = +1) \quad . \tag{10}$$

In the given situation the maximum symbol probability detector would decide the bit pattern (0 , 1) corresponding to the symbol $\boldsymbol{h}_1$, while the individual bit probaliblities show that the combination (1 , 1) would be the better choice. But we don't want to decide any bit at this stage of the receiver.

Instead we use the bit specific probability information P_ν, belonging to the assumption of $b_\nu = +1$ for the demodulator output bit to calculate a reliability information [2]

$$L_\nu \;=\; \ln \frac{P_\nu}{1 - P_\nu} \quad , \tag{11}$$

which is used as reliability indicator (metric) in the Viterbi decoding process .

For this purpose just the soft bit informations L_ν, instead of hard decided bits, have to be supplied to the input of the Viterbi decoder.

It should be pointed out that the above metric L_ν is optimum in the sense of ML decoding, as long as the ML demand is assessed between a given receiver structure (determining the construction of the observation vector $\boldsymbol{V}$) and the decided bits.

3 Results

As an example we present some simulation results for a noncoherent RAKE-receiver in an assynchronous CDMA uplink connection: The following parameters have been chosen to meet the Qualcomm-CDMA proposal [5]. A 64-ary orthogonal modulation scheme with Walsh-functions is used. The bit rate is $R_0 = 9.6\,kbit/s$, the interleaver comprises a $20\,ms$ data frame (approx. 200 bits), the convolutional code is of rate 1/3 with constraint length 9. The chip frequency f_c equals $1.2288\,MHz$ (processing gain $P_G = f_c/R_0 = 128$). A 4 Tap Rayleigh mobile radio channel and a variable vehicle speed (carrier frequency at $950\,MHz$) is assumed. The receiver exploits the $L = 4$ diversity paths from one antenna by square-law combining. We have utilized a closed-loop power control which tackles the problem of (moderately slow) Rayleigh fading (a power control instruction of $\pm 1dB$ is used to adapt the power of the transmitter every $1.25\,ms$). The interference from other users is modeled as white gaussian noise of appropriate strength N_0. The signal to noise ratio $E_b/N_0 \approx 8.3\,dB$ has been chosen to meet a BER performance of 10^{-3}, acceptable for speech transmission.

Figure 3 illustrates the gain in bit error performance in terms of the vehicle speed, which can be achieved by the use of bitwise soft information.

The markers stem from simulations, while the curves represent polynomial approximations based on markers. The solid lines represent the performance with the described optimum metric on the assumption of a realistic estimation of the channel energy. The dash-dotted line has been found for a suboptimum metric described in [6]. The dotted lines stem from hard bit decoding (reliability information not used).

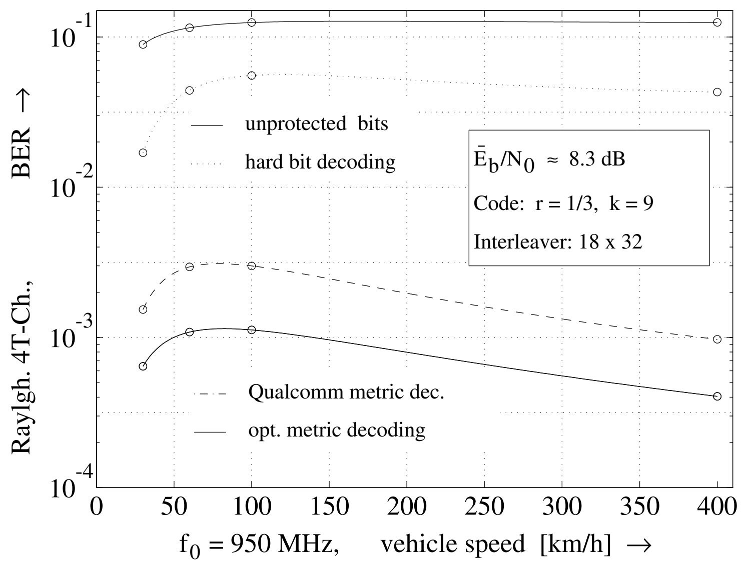

Figure 3: Bit error rates

The curve labeled "unprotected bits" represents the bit error rate, measured on that bit-stream leaving the demodulator output or entering the Viterbi decoder input equivalently.

The optimum metric and the QUALCOMM metric are very superior to the Viterbi decoding with hard decided bits at the demodulator output.
While the optimum metric contains reliability for each bit individually, the QUALCOMM metric is calculated on symbol based reliability for a group of bits. The advantage in BER performance for the optimum metric is obvious. On the other hand the computational effort for the optimum metric is much higher than for the suboptimum strategy.

For low speeds the power control works sufficiently accurate to eleminate the effects of fading, the BER performance becomes low. Without power control the fading length would overcome the interleaver length and the BER performance would be very poor. For higher speeds the efficieny of the power control decreases due to the time delay of the closed loop control mechanism. Nevertheless the fading bursts become shorter and the interleaver becomes more efficient. The combination of power control and interleaver is able to 'equalize' the BER performance over a wide range of vehicle speeds.
To illustrate the efficiency of the power control for low speeds it is instructiv to have a look at the soft bit signal entering the deinterleaver or the Viterbi decoder respectively. Especially for the slowest example presented, e.g. $8\,km/h$ with $E_b/N_0 = 8.75\,dB$, at the first glance, the histogram might stem from an antipodal signal constellation, which is corrupted by gaussian noise. The complete transmission system including the fading channel or the M-ary modulation can't be "seen" from this point of view. For higher speeds the power control begins to fail. The Rayleigh characater begins to dominate the

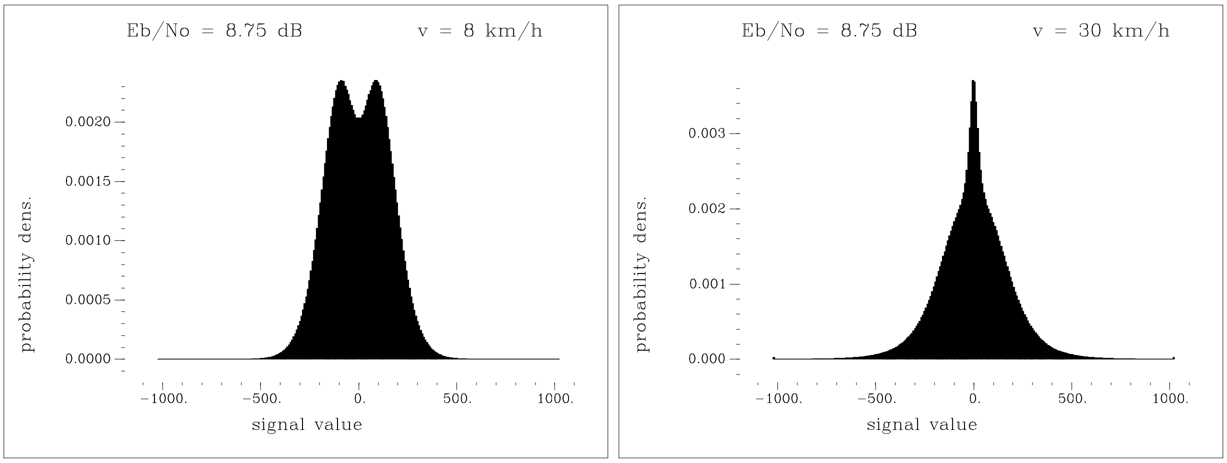

Figure 4: soft bit histograms

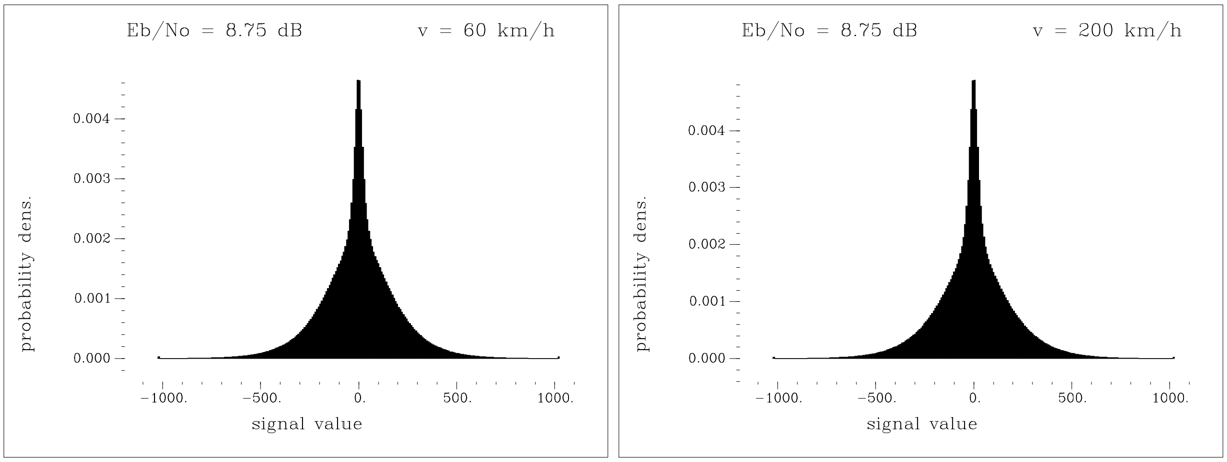

Figure 5: soft bit histograms

histogram of the soft bits. There is no significant difference in the histograms comparing the 60 km/h to the 200 km/h case. The improvement in BER performance becomes now the effect of shorter fading bursts with respect to the fixed interleaver length.

Acknowledgement

This work was funded by the Robert Bosch Communication Research Institute Hildesheim, Germany.

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