GLOBECOM '94 CONFERENCE, SAN FRANCISCO, NOV 27 – DEC 1, 1994

Viterbi Decoding of Convolutional Codes with Reliability Information for a Noncoherent RAKE-Receiver in a CDMA-Environment

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Abstract

In this paper the generation and the use of reliability information for the decoding process of convolutionally encoded data in the context of a CDMA-environment is addressed. Special emphasis lies on the noncoherent detection of M-ary orthogonal symbols with a RAKE-receiver. Although symbol by symbol decisions are carried out, it is possible to characterize the reliability of the individual bits with specific soft information. This can be exploited successfully by a Viterbi decoder. A significant improvement of the bit error performance is achieved, even on low signal to noise conditions.

1 Introduction

In the context of a CDMA-transmission system, an M-ary modulation scheme with transmitted symbols chosen from a set of orthogonal waveforms (e.g. Walsh-functions) is a much promising technique [1]. This inner modulation procedure has to be combined with an outer pseudo noise (pn) spreading procedure to determine the CDMA character.
In a white gaussian noise environment, the optimum noncoherent receiver for the inner modulation scheme calculates the correlations of the received signal with all hypotheses for the transmitted signal. Then, it opts for the symbol with the maximum magnitude correlation value [5]. This symbol decision is transformed into bits by a corresponding look-up table.

A major drawback of this approach is the fact that hard decided bits appear at the receiver output due to the underlying symbol decisions. This is very significant, when channel coding, e.g. by means of a convolutional code, is introduced into the system. A way out arises by the use of more sophisticated receiver concepts, which use some environmental observations (about the channel state e.g.) in order to produce some additional information indicating whether or not the decided bits are reliable. This reliability information is then fed to the Viterbi decoder, which uses it to weighten the path metrics [2].

We will start with the derivation of the maximum-likelihood (ML) criteria for the detection of binary data on the AWGN-channel and the binary symmetric channel (BSC). This will provide us with the necessary insight into the nature of reliability information and we will see how to adopt a conventional Viterbi decoder (nonquantized input) to operate on hard bits and reliability information. This will be possible with a neglegligible increase in computational effort and with no modifications to the design of the Viterbi decoder. A key issue of this paper is the question 'how to get reliability information from the receiver'. We will investigate the detection process of a noncoherent RAKE-receiver for a CDMA-system (using M-ary orthogonal modulation) which applies square law combining of the correlation results of several received paths.
The paper will conclude with some simulation results, which underline the efficiency of the described method.

2 ML-detection of convolutionally encoded data for the AWGN-channel

In vector representation $\mathbf{b}$ shall describe the encoded signal with values $\mathrm{b}_\nu \in \{-1, 1\}$, $\boldsymbol{s}$ the received signal and $\boldsymbol{r}$ the noise.

$$\boldsymbol{s} \;=\; \mathbf{b} + \boldsymbol{r} \tag{1}$$

We intend to maximize the probability $P(\boldsymbol{b}\,|\,\boldsymbol{s})$, that $\boldsymbol{b}$ is the encoded bit sequence, when $\boldsymbol{s}$ is the observed signal. On the assumption of a statistically independent noise process we obtain the following form of the ML-criterion [5].

$$P(\boldsymbol{b}\,|\,\boldsymbol{s}) \;\sim\; \Phi_{ML'} \;=\; \prod_{\nu} p(s_\nu\,|\,b_\nu) \;\stackrel{!}{=}\; \text{Max} \tag{2}$$

For binary signals $b_\nu \in \{-1, +1\}$ an equivalent ML-criterion is given by [2]

$$\Phi_{ML} \;=\; \sum_{\nu} b_\nu \,\ln \frac{p(s_\nu\,|\,b_\nu = +1)}{p(s_\nu\,|\,b_\nu = -1)} \;\stackrel{!}{=}\; \text{Max} \tag{3}$$

For convenience, the equivalence of Φ_{ML} and $\Phi_{ML'}$ is confirmed in the appendix.
In case of an additive white gaussian noise channel we have

$$\Phi_{ML} \;=\; \sum_{\nu\in\Omega} b_\nu \,\ln \frac{e^{-\frac{(s_\nu-1)^2}{2\,\sigma_r^2}}}{e^{-\frac{(s_\nu+1)^2}{2\,\sigma_r^2}}} \quad . \tag{4}$$

This yields the equivalent ML-criterion

$$\Phi_{ML}^{AWGN} \;=\; \sum_{\nu\in\Omega} b_\nu\, s_\nu \quad \stackrel{!}{=} \quad \mathrm{Max} \quad , \tag{5}$$

which can be evaluated for each possible hypothesis for the transmitted vector $\boldsymbol{b}$ using the efficient Viterbi algorithm.

3 ML-criterion for the binary symmetric channel

Now we assume that the decoder input signal z_ν consists of hard decided bits due to the involved detection process of the receiver. In addition, some information about the probability of a correct bit decision should be available. Then, the binary symmetric channel, as depicted in figure 1, is an appropriate model.

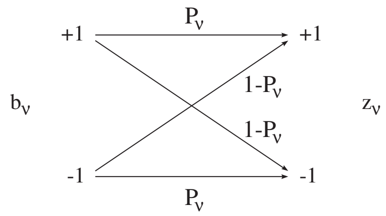

Figure 1: Binary symmetric channel

The ML-criterion (eq. 3) becomes now (see e.g. [3])

$$\Phi_{ML}^{BSC} \;=\; \sum_{\nu\in\Omega} b_\nu\, z_\nu \,\ln \frac{P_\nu}{1-P_\nu} \quad \stackrel{!}{=} \quad \mathrm{Max} \quad . \tag{6}$$

This relation exploits the fact that the probability density $p(z_\nu|\, b_\nu)$ is discrete: $p(z_\nu = b_\nu) = P_\nu$, $p(z_\nu \neq b_\nu) = 1 - P_\nu$. P_ν should vary with the time index ν, otherwise it would not influence the ML-detection. The quantity $L_\nu := \ln P_\nu/(1-P_\nu)$ can be interpreted as reliability indicator for the decided code bit z_ν. For $P_\nu \to 1$ the value L_ν becomes large and the product $z_\nu\, L_\nu$ indicates a reliable bit z_ν. This is in strict analogy to binary antipodal signaling. The similar structure of equations 5 and 6 makes it quite easy to supply the conventional Viterbi decoder (Euclidean metric) with the decided bits and reliability information as well. Therefore we simply identify $z_\nu\, L_\nu$ with the input signal s_ν. Figure 2 depicts the related structure, where $\hat{d}_\kappa$ represents the decoded bit stream.

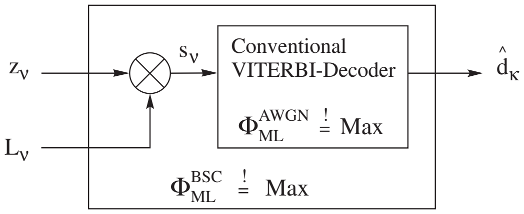

Figure 2: Soft-bit Viterbi-Decoding

4 Reliability information for an M-ary orthogonal modulation scheme

Figure 3 illustrates the transmitter of our CDMA system. The user specific pseudo noise sequence is complex valued, where real and imaginary parts stem from independent real pn-sequences $\in \{-1, 1\}$ (quadrature spreading). The orthogonal waveforms may be Walsh-functions. The j-th Walsh-symbol is described by a vector $\boldsymbol{w}_j$ with components from $\{-1, +1\}$. First, the receiver has to despread the incoming signal with the user specific pn-sequence and then to correlate with all hypotheses of transmitted symbols $\boldsymbol{w}_j$ (Hadamard-transform [4]) on each echo delay considered. For the sake of simplicity, we will combine the processes of orthogonal modulation and pn-spreading. The transmitted symbols are denoted as $\boldsymbol{h}_j,\ j = 1, \ldots, M-1$. The symbol $\boldsymbol{h}_i$ represents the currently transmitted symbol. The μ-th echo of the received signal can be written as

$$\boldsymbol{s}^{(\mu)} \;=\; c_\mu\, \boldsymbol{h}_i + \boldsymbol{r}^{(\mu)} \quad , \tag{7}$$

where c_μ denotes the corresponding channel coefficient and the noise signal $\boldsymbol{r}^{(\mu)}$ results from the superposition of several other user signals and delayed versions of the own signal. Due to the nearly orthogonal nature of all pn-sequences and due to the superposition of many users, the components of vector $\boldsymbol{r}$ can be understood as independent complex white gaussian noise processes with a variance of approximately $2\, N_U$, where N_U is proportional to the number of active users and the factor 2 is due to the quadrature spreading. For one correlation time (echo delay μ) we obtain a vector $\boldsymbol{V}^{(\mu)}$ of correlation results with M components:

$$V_j^{(\mu)} = |\,\boldsymbol{h}_j^* \boldsymbol{s}^{(\mu)}|^2 = \begin{cases} |\,c_\mu\, E_h + \boldsymbol{h}_j^* \boldsymbol{r}^{(\mu)}|^2 \ , & j = i \\ |\,\boldsymbol{h}_j^* \boldsymbol{r}^{(\mu)}|^2 & , \ j \neq i \end{cases} \tag{8}$$

By the use of the squared magnitude the influence of the unknown phase of the channel weight c_μ is eliminated. $E_h = \boldsymbol{h}_j^*\, \boldsymbol{h}_j$ denotes the equal energy of any transmitted symbol $\boldsymbol{h}_j$. The problem of combining several decision vectors $\boldsymbol{V}^{(\mu)}$ in an optimum manner (in order to

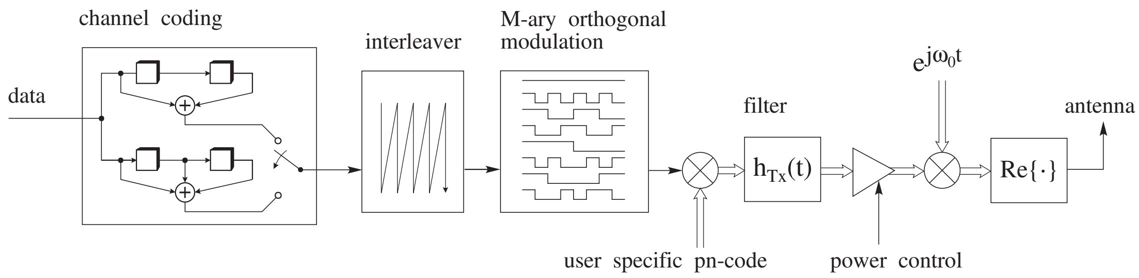

Figure 3: CDMA-transmitter

exploit the energy of echo signals efficiently) is not trivial [7]. The easiest and pragmatic way is square law combining: $\boldsymbol{V} = \sum_{\mu} \boldsymbol{V}^{(\mu)}$ [5]. Since we pursue this approach we have introduced the squared absolute values for the decision components in eq. 8. Due to the orthogonality of the symbols $\boldsymbol{h}_j$ the complex values $\boldsymbol{h}_j^* \, \boldsymbol{r}^{(\mu)}$ are independent gaussian noise processes with variance $E_h \cdot 2N_U$. The maximum likelihood receiver detects the value with maximum magnitude of the decision components V_j. The index of this component indicates the most likely transmitted symbol [5].

Usually, the decision for one specific symbol is not significant for each of the underlying bits. Assume, for the moment, that the the correlation components V_0 and V_{32} in an 64-ary modulation scheme have significantly large magnitudes wheras the remaining components are negligible. The corresponding bit patterns 000000 and 100000 for the symbols $\boldsymbol{h}_0$ and $\boldsymbol{h}_{32}$ only differ in one position. The last 5 bits are reliable, independent of a decision for one of both symbols, while for the first bit no reliable decision can be made. This example illustrates the idea of bitwise reliability information which has to be calculated during the symbol decision process.

The probabilities for the hypotheses of transmitted symbols can be obtained from the decision components V_j. Using Bayes law, the conditional probability, that $\boldsymbol{h}_j$ is the transmitted symbol, under the observation of $\boldsymbol{V}$, can be expressed as

$$P(\boldsymbol{h}_j|\boldsymbol{V}) \;=\; \frac{P(\boldsymbol{h}_j)}{P(\boldsymbol{V})} \cdot p(\boldsymbol{V}|\,\boldsymbol{h}_j) \quad . \tag{9}$$

Equiprobable symbols $\boldsymbol{h}_j$ are assumed. Due to the statistical independence of the decision components V_j we get

$$p(\boldsymbol{V}|\,\boldsymbol{h}_j) \;=\; q(V_0)\cdot \,\ldots\, \cdot Q(V_j)\cdot \,\ldots\, \cdot q(V_{M-1}) \tag{10}$$

with probability density functions

$$q(V_j) \;=\; \frac{1}{(2\,\sigma^2)^L\,(L-1)!}\; V_j^{L-1}\, e^{-\frac{V_j}{2\,\sigma^2}} \tag{11}$$

$$Q(V_j) = \frac{1}{2\,\sigma^2}\Big(\frac{V_j}{\varrho^2}\Big)^{\frac{(L-1)}{2}} e^{-\frac{V_j+\varrho^2}{2\,\sigma^2}} \cdot I_{L-1}\Big(\frac{\varrho\sqrt{V_j}}{\sigma^2}\Big) \tag{12}$$

$$\text{where} \qquad \sigma^2 \;=\; E_h\, N_U \quad . \tag{13}$$

The density $q(V_j)$ represents a central chi-square distribution with $2L$ degrees of freedom. $Q(V_j)$ stands for the corresponding noncentral chi-square distribution. L denotes the number of echo components, which are square law combined. This means we consider an L-path RAKE-receiver. The resulting decision vector $\boldsymbol{V}$ has been built from the additon of L echos, i.e. decision vectors $\boldsymbol{V}^{(\mu)}$. The parameter ϱ^2 describes the noncentrality of $Q(V_j)$.

$$\varrho^2 \;=\; \sum_{\mu=0}^{L-1} (E_h\,|c_\mu|)^2 \quad , \tag{14}$$

where the c_μ are the (complex) channel weights of the echo paths. σ^2 denotes the variance of the real or resp. the imaginary part of the related complex white gaussian noise processes $\boldsymbol{h}_j^*\boldsymbol{r}$. I_{L-1} is the $(L-1)$th order modified Bessel function of the first kind.
In a practical receiver design we obtain an estimate for the noise power σ^2 by averaging $M-1$ decision components $\hat{\sigma}^2 = \frac{1}{2\,L\,(M-1)} \sum_{k=0,\;k\neq i}^{M-1} V_k$ (including L echo paths). For an appropriately large value of M, this estimation becomes quite accurate. The estimation of the received signal energy $E_C := \sum_{\mu=0}^{L-1} |c_\mu|^2$ can be obtained from a short time average with respect to the values $\max_j V_j$ over a few consecutively transmitted symbols and has to be performed, for the operation of closed loop power control, anyway.
We summarize all probabilities $P(\boldsymbol{h}_j|\boldsymbol{V})$ in the following table; M equals 64 in this example.

$$\begin{aligned} P(000000\,|\,\boldsymbol{V}) &= Q(V_0)\,/\,q(V_0)\cdot\alpha \\ P(000001\,|\,\boldsymbol{V}) &= Q(V_1)\,/\,q(V_1)\cdot\alpha \\ &\vdots \\ P(111111\,|\,\boldsymbol{V}) &= Q(V_{M-1})\,/\,q(V_{M-1})\cdot\alpha \quad . \end{aligned} \tag{15}$$

Here, the terms which effect all hypotheses equally have been collected within the constant α. Besides we have substituted the symbol's name $\boldsymbol{h}_j$ with its corresponding bit pattern, which is given by the index j in binary representation. The constant factor

$$\alpha = \frac{1/M}{P(\boldsymbol{V})} \prod_{k=0}^{M-1} q(V_k) \tag{16}$$

is unknown and may vary from symbol period to symbol period. However, it is not necessary to evaluate α explicitely, as it can be determined indirectly by means of

$$\sum_{j=0}^{M-1} P(\boldsymbol{h}_j|\boldsymbol{V}) \;=\; \alpha \cdot \sum_{j=0}^{M-1} \frac{Q(V_j)}{q(V_j)} \;=\; 1 \quad . \tag{17}$$

This equation represents the union of all mutually exclusive elementary statistical events to make up the completely sure event with probability one.
The last step in our derivation is the "columnwise" interpretation of eq. 15. The symbol probabilities have to be transferred to the individual bits for each column separately. Adding all probabilities belonging to a $'1'$ in the first column, e.g., results in an individual probability of the first bit (which shall be denoted b_1 for the moment) being a $'1'$.

$$P(b_1 = 1) \;=\; \sum_{j=32}^{63} P(\boldsymbol{h}_j|\boldsymbol{V}) \tag{18}$$

For the other columns (or bits equivalently) belonging to the observed correlation vector $\boldsymbol{V}$, similar expressions have to be evaluated.
In section 3 this bit specific probability information was denoted as P_ν. It will be used by means of

$$L_\nu \;=\; \ln \frac{P_\nu}{1 - P_\nu} \tag{19}$$

as reliability indicator (metric) in the Viterbi decoding process. In additon, a conversion of the bits from $\{0, 1\}$ to $\{-1, +1\}$ has to be considered for compatibility reasons.
Obviously, the derivation of the probabilities $P(\boldsymbol{h}_j|\boldsymbol{V})$ depends on the receiver design used, but the described principles remain valid in general.
It should be pointed out that the above metric L_ν is optimum in the sense of ML decoding, as long as the ML demand is assessed between a given receiver structure (determining the construction of the observation vector $\boldsymbol{V}$) and the decided bits. Especially the use of the BSC channel model, which helps to understand the involved principles, does not affect the ML demand. As the proof of this is mathematically tedious, it will not be presented here.

5 Results

As an example we present some simulation results for a noncoherent RAKE-receiver in an assynchronous CDMA uplink connection: The following parameters have been chosen to meet the QUALCOMM-CDMA proposal [6]. A 64-ary orthogonal modulation scheme with Walsh-functions is used. The bit rate is $R_0 = 9.6\,kbit/s$, the convolutional code is of rate 1/3 with constraint length 9. The chip frequency f_c equals $1.2288\,MHz$ (processing gain $P_G = f_c/R_0 = 128$). A bad urban (BU) mobile radio channel and a vehicle speed of $60\,km/h$ (carrier frequency at $950\,MHz$) is assumed. The receiver exploits $L = 6$ diversity paths from one or two antennas by square-law combining. We have utilized a closed-loop power control which tackles the problem of moderately slow fading (not fast Rayleigh fading). The interference of N_U other users is modeled as white gaussian noise of appropriate strength N_0. Figure 4 illustrates the gain in bit error performance, which can be achieved by the bitwise use of soft information. The solid lines represent the performance with the described optimum metric on the assumption of an ideal estimation of the channel energy. The dashed lines take into account the effects of a real channel energy estimation and a 3 bit fixed range quantization (optimized for $\bar{E}_b/N_0 = 11\,dB$ with one antenna) for the reliability information. The degradation in performance is quite small.

The dash-dotted lines have been found for a suboptimum metric described in [8]. The dotted lines stem from hard bit decoding (reliability information not used).

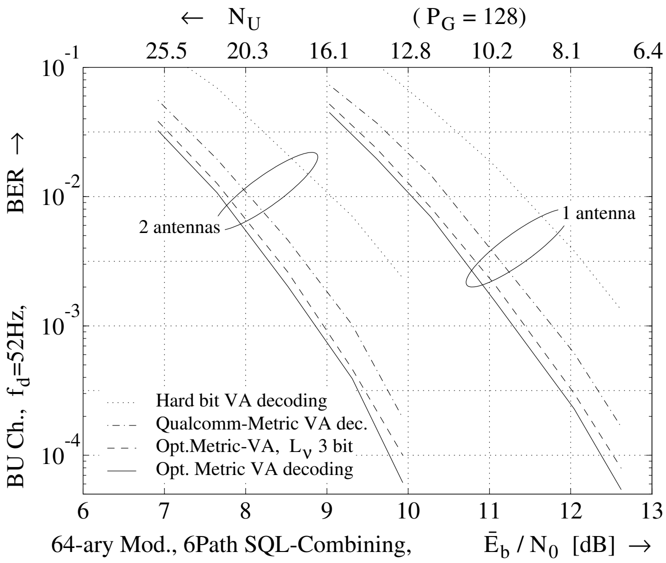

Figure 4: Bit error rates

6 Discussion

The concept developed here delivers bitwise reliability information although an M-ary symbol based modulation scheme is involved. It is maximum likelihood and allows to simulate bounds for the achievable BER performance for a given receiver design in a specified environment. The computational effort for the bitspecific reliability information can be significant. So an M-dimensional funcion $g_{opt} : R^M \rightarrow R : \boldsymbol{V} \rightarrow L_{\nu,opt}$ has to be considered, which makes the use of look-up tables difficult.
The alternative suboptimum metric proposed in [8] uses the maximum correlation component $\max(\boldsymbol{V})$, only. Therefore, a one dimensional function $g_{sub} : R \rightarrow R : \max(\boldsymbol{V}) \rightarrow L_{\nu,sub}$ is used, which is well suited to be represented in tabular form.
In our example the advantage of the optimum metric decoding is in the order of a factor 1/2 in BER performance corresponding to a gain of $1/3\ dB$ in SNR.

Acknowledgement

This work was funded by the Robert Bosch Communication Research Institute Hildesheim, Germany.

Appendix

The equivalence of the both ML-criteria Φ_{ML} (eq. 3) and $\Phi_{ML'}$ (eq. 2) shall be verified. We start with the classical form of the ML-criterion. A comparison of two arbitrary paths A and B can be restricted to time indices $\nu \in \Omega_2$, $\Omega = \Omega_1 \cup \Omega_2$, on which the encoded bits of the both paths differ from each other. We obtain

$$\Phi_{ML}^{(A)} = \prod_{\nu \in \Omega} p(z_\nu \,|\, b_\nu^A) \; \gtrless \; \prod_{\nu \in \Omega} p(z_\nu \,|\, b_\nu^B) \; = \Phi_{ML}^{(B)} \qquad (20)$$

$$\Longleftrightarrow \quad \Phi'_{\Omega_1} \cdot \prod_{\nu \in \Omega_2} p(z_\nu | \, b_\nu) \; \gtrless \; \Phi'_{\Omega_1} \cdot \prod_{\nu \in \Omega_2} p(z_\nu | \, \bar{b}_\nu) \; , \qquad (21)$$

where Φ'_{Ω_1} denotes the path metrics of joint path sections and $\bar{b}_\nu$ is the complement of b_ν.
Since all terms are positive the following equivalence holds:

$$\left[\prod_{\nu \in \Omega_2} p(z_\nu \,|\, b_\nu) \right]^2 \; \gtrless \; \left[\prod_{\nu \in \Omega_2} p(z_\nu \,|\, \bar{b}_\nu) \right]^2 \qquad (22)$$

$$\prod_{\nu \in \Omega_2} \frac{p(z_\nu \,|\, b_\nu)}{p(z_\nu \,|\, \bar{b}_\nu)} \; \gtrless \; \prod_{\nu \in \Omega_2} \frac{p(z_\nu \,|\, \bar{b}_\nu)}{p(z_\nu \,|\, b_\nu)} \qquad (23)$$

Multiplying both sides with an expression for the jointly passed path sections of the form

$$\Phi_{\Omega_1} \; := \prod_{\nu \in \Omega_1} \frac{p(z_\nu \,|\, b_\nu^{A,B})}{p(z_\nu \,|\, \bar{b}_\nu^{A,B})} \qquad (24)$$

yields

$$\prod_{\nu \in \Omega} \frac{p(z_\nu \,|\, b_\nu^A)}{p(z_\nu \,|\, \bar{b}_\nu^A)} \; \gtrless \; \prod_{\nu \in \Omega} \frac{p(z_\nu \,|\, b_\nu^B)}{p(z_\nu \,|\, \bar{b}_\nu^B)} \quad . \qquad (25)$$

Note that the originally existing relation between the probability densities of both paths remains unchanged with the presented modifications. This leads to the alternative ML-criterion

$$\Psi_{ML} \; := \prod_{\nu \in \Omega} \frac{p(z_\nu \,|\, b_\nu)}{p(z_\nu \,|\, \bar{b}_\nu)} \; \stackrel{!}{=} \; \mathrm{Max} \qquad (26)$$

or, using the natural logarithm,

$$\Phi_{ML} \; := \sum_{\nu \in \Omega} \ln \frac{p(z_\nu \,|\, b_\nu)}{p(z_\nu \,|\, \bar{b}_\nu)} \; \stackrel{!}{=} \; \mathrm{Max} \quad . \qquad (27)$$

The latter formulation can be modified slightly for $b_\nu \in \{-1, +1\}$, which leads to the compact form [2]:

$$\Phi_{ML} \; = \sum_{\nu \in \Omega} b_\nu \, \ln \frac{p(z_\nu \,|\, b_\nu = +1)}{p(z_\nu \,|\, b_\nu = -1)} \; \stackrel{!}{=} \; \mathrm{Max} \qquad (28)$$

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