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A new fast algorithm for blind MA-system identification based on higher order cumulants

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Abstract

In this paper, a new approach to blind Moving Average (MA) system identification is presented. It is derived from a specific algorithm for fast blind equalization (EVA, EigenVector Algorithm) published recently. The novel approach leads to a generalized eigenvalue problem and is thus denoted as EVI (EigenVector Identification). It will be shown that EVI allows a proper system (channel) estimation on the basis of a relatively small block of data samples. Another crucial property of EVI is its robustness against a system order overfit (this is in contrast to other blind algorithms known so far). Finally the EVI performance will be evaluated in presence of additive gaussian noise; it will be demonstrated by simulation results that the degradation of the channel estimate is very small even with low signal to noise ratios.

1 Introduction

In In recent years, the application of higher order statistics (cumulants) to signal processing problems has attracted much attention since many of these problems cannot be solved by means of conventional 2nd-order statistics ([1, 2, 3]). One of these problems is blind system identification. As is well-known, higher order cumulants of the received signal contain the complete channel information including phase characteristics provided that the source signal is non-gaussian.

Some important ideas on blind MA-system identification were introduced by Giannakis and Mendel [4]. They suggest to solve a set of nonlinear equations involving the 2nd-order covariance and the 3rd- (4th-) order diagonal cumulant sequences in a Least-Squares sense. Some singularity problems occuring with this approach can be overcome by the "Modified Least-Squares" solution [5] or by "Cumulant zero matching" [6] . However, the approaches cited so far do suffer from an extremely large number of data samples necessary for a proper estimation of the cumulants.

This paper is to present an alternative approach based on a closed-form eigenvector solution to blind *equalization*. The algorithm, termed EVA (EigenVector Algorithm) was published recently [7, 8]. Now, we exploit the fact that the equalization problem is closely related to system identification: if the inverse system is known, the system itself is obvious. It will be shown that the EVA solution can be modified so as to identify the system impulse response. This results in a generalized eigenvalue problem (EVI, EigenVector Identification) involving an autocovariance matrix and a specific 4th-order cross-cumulant matrix.

In the following section, a brief review of the eigenvector algorithm for blind equalization (EVA) will be given for convenience. Section 3 presents the novel eigenvector solution to blind MA-system identification (EVI). In section 4 the performance of this algorithm will be illustrated by some simulation results. These examples will demonstrate that the number of data samples necessary for a proper blind channel estimation can be reduced by a factor of about 100 compared with Giannakis' and Mendel's Least-Squares approach. Finally, the influence of additive (gaussian) noise on the EVI solution will be investigated in section 5. It is well-known that the higher order cumulants are not affected by gaussian noise whereas the 2nd-order autocovariance samples do suffer from such noise. Since both 2nd- and 4th-order statistics are involved in the EVI solution additive gaussian noise does influence the result. However, it will be demonstrated by simulation results that this influence is very small indeed due to advantageous properties of the EVI algorithm.

2 The EVA approach to blind equalization

The eigenvector approach is based on the system configuration shown in fig. 1. The transmitted data $d(k)$ are regarded as non-gaussian, independent, identically distributed (i.i.d.) random variables. The channel is described by the impulse response $h(k)$. In the receiver, the linear equalizer with the impulse response $e(k) = e(0), \ldots, e(l)$ as well as a certain reference system is introduced. For the time being, the impulse response $f(k) = f(0), \ldots, f(l)$ of the latter is supposed to have been fixed arbitrarily.

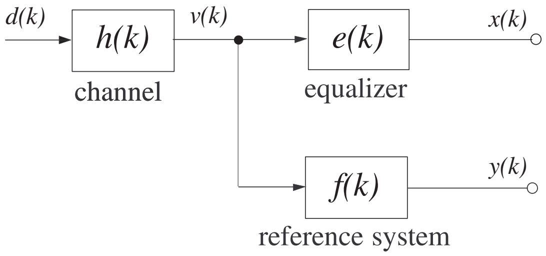


Figure 1: Symbol rate model of a data transmission system including equalizer and reference system

The closed-form solution requires a specific cross-cumulant matrix $\mathbf{C}_4^{yv}$ involving 4th-order cross-cumulants between the samples of the received signal $v(k)$ and the reference systems output $y(k)$. The $(l+1) \times (l+1)$ cross-cumulant matrix is defined as

$$\mathbf{C}_4^{yv} = E\{|y(k)|^2 \mathbf{v}\mathbf{v}^*\} - E\{|y(k)|^2\}E\{\mathbf{v}\mathbf{v}^*\} - E\{\mathbf{v}y(k)\}E\{\mathbf{v}^* y^*(k)\} - E\{\mathbf{v}y^*(k)\}E\{\mathbf{v}^* y(k)\}, \tag{1}$$

where the vectors

$$\mathbf{v}^* = [v(k), v(k-1), \ldots, v(k-l)] \qquad \text{(transjugated form)} \tag{2}$$

$$\mathbf{v} = [v^*(k), v^*(k-1), \ldots, v^*(k-l)]^T \tag{3}$$

are introduced; $E\{\cdot\}$ describes statistical expectation.

Using the autocorrelation matrix

$$\mathbf{R}_{vv} = E\{\mathbf{v}\mathbf{v}^*\} \tag{4}$$

and the vector of the equalizer impulse response

$$\mathbf{e} = [e(0), \ldots, e(l)]^T \tag{5}$$

we finally get the following criterion for a proper adjustment of the equalizer coefficients (refer to [7] for details):

$$\text{maximize } |\mathbf{e}^* \mathbf{C}_4^{yvv} \mathbf{e}| \ \text{ on condition that } \ \mathbf{e}^* \mathbf{R}_{vv} \mathbf{e} = \text{ const.} \tag{6}$$

This maximization problem can be solved uniquely by the generalized eigenvector problem

$$\mathbf{C}_4^{yv} \mathbf{e}_{EVA} = \lambda \mathbf{R}_{vv} \mathbf{e}_{EVA}; \text{ choose } |\lambda| = max\{|\lambda_1|, \ldots, |\lambda_l|\} \qquad \text{(EVA).} \tag{7}$$

Let $w(k)$ be the joint impulse response [1]

$$w(k) = h(k) * f(k).$$

To guarantee the uniqueness of the solution, $w(k)$ must contain a single maximum value

$$\begin{array}{rcll} |w(k)| & = & max\{|w(k)|\} = w_m & \text{if } k = k_m \\ |w(k)| & < & w_m & else. \end{array} \tag{8}$$

In [7], it was shown that the EVA solution converges to the optimum MMSE (Minimum Mean-Square Error) result on the ideal condition

$$|w(k)| = |w(k_m)| \delta(k - k_m).$$

Otherwise, a solution very close to the MMSE result is achieved provided that condition (8) holds.
Of course, equation (8) can not be guaranteed since the channel impulse response $h(k)$ is unknown. In order to avoid an "unlucky guess" of the reference system $f(k)$, an iterative update procedure for the reference system was suggested in [8]: the reference system is loaded with the interim equalizer impulse response calculated in the previous step. Thus, after some iterations (based on the same block of received data), both the reference system and the equalizer contain the same impulse response being very close to the MMSE result. An improved convergence rate can be obtained by a stepwise increase of the equalizer order l during the iteration process.

3 Eigenvector algorithm for blind system identification (EVI)

3.1 The fundamental solution

In the previous section, we have presented a fast algorithm for blind equalization based on a closed-form eigenvector solution. It is obvious that the problem of equalization is closely related to system identification. Consequently, we try to derive a unique expression for blind identification from the fundamental EVA equation (7)

$$\mathbf{e}_{EVA} = \frac{1}{\lambda} \mathbf{R}_{vv}^{-1} \mathbf{C}_4^{yv} \mathbf{e}_{EVA}. \tag{9}$$

It has already been mentioned that the EVA result is very close to the ideal MMSE solution expressed by the well-known normal equation

[1] $*$ denotes the convolution operation

$$\mathbf{e}_{MSE} = \mathbf{R}_{vv}^{-1}\mathbf{r}_{dv} \approx \mathbf{e}_{EVA}. \tag{10}$$

Here, the vector $\mathbf{r}_{dv}$ describes the cross-correlation between the transmitted data $d(k)$ and the received signal $v(k)$

$$\mathbf{r}_{dv} = E\{\mathbf{v} \cdot d(k-k_0)\}, \tag{11}$$

where k_0 denotes a delay of the equalized data which can be used to optimize the equalization result. Of course, the vector $\mathbf{r}_{dv}$ is unknown with *blind* equalization. However, a comparison of eqn.(10) with (9) shows that the expression $\mathbf{C}_4^{yv}\mathbf{e}_{EVA}$ obviously plays the role of the cross-correlation vector. On the other hand, the cross-correlation is linked with the channel impulse resonse. For zero-mean uncorrelated input data $d(k)$ with variance σ_d^2 we get

$$\mathbf{r}_{vd} = \sigma_d^2 \cdot \mathbf{h} \approx \sigma_d^2\,\mathbf{h}_{EVI} = \frac{1}{\lambda}\mathbf{C}_4^{yv}\mathbf{e}_{EVA}. \tag{12}$$

with

$$\mathbf{h} = [h^*(k_0), h^*(k_0-1), \ldots, h^*(0), 0, \ldots, 0]^T. \tag{13}$$

Choosing a sufficiently long delay value k_0, $\mathbf{h}$ contains the complete channel impulse response (conjugate complex) in reverse order.

Now, the EVA equation (9) is multiplied by $\mathbf{C}_4^{yv}$ from the lefthand side

$$\mathbf{C}_4^{yv}\mathbf{e}_{EVA} = \frac{1}{\lambda}\mathbf{C}_4^{yv}\mathbf{R}_{vv}^{-1}\mathbf{C}_4^{yv}\mathbf{e}_{EVA}. \tag{14}$$

A comparison with (12) finally leads to

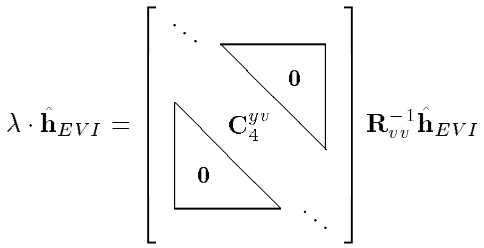
$$\lambda\mathbf{h}_{EVI} = \mathbf{C}_4^{yv}\mathbf{R}_{vv}^{-1}\mathbf{h}_{EVI} \qquad \text{(EVI)}. \tag{15}$$

This can be regarded as a unique formulation of blind MA-sytem identification. Again, an eigenvector problem has to be solved. The matrices involved can be estimated by the receiver without the use of the transmitted data. For the calculation of $\mathbf{C}_4^{yv}$, however, an appropriate reference system $f(k)$ must be updated. This can be done by some EVA pre-iterations. For these pre-iterations, the equalizer can be fixed at relatively small orders since just a suitable reference impulse response (with one predominant coefficient) has to be obtained rather than perfect equalization.

3.2 Solution with reduced matrix dimensions

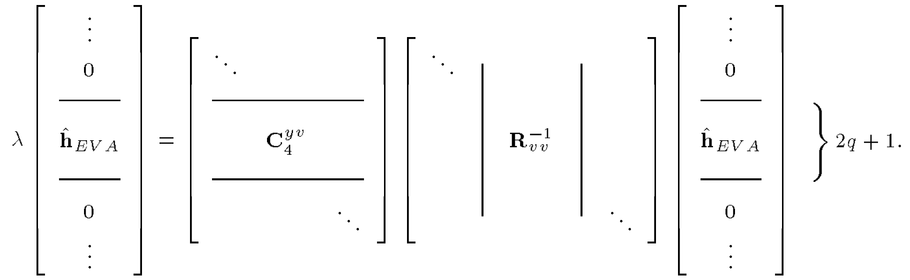
Due to the fact that the EVI equation (15) was derived from the EVA solution, the equalization problem is involved in so far as the dimension of the matrix $\mathbf{C}_4^{yv}\mathbf{R}_{vv}^{-1}$ depends on the equalizer order l: they are $(l+1) \times (l+1)$. If critical channel configurations (with zeros near the unit circle) are under consideration, this leads to a high computational complexity. This is due to the fact that we implicitly use the inverse system for MA-system identification. Consequently, the large resulting vector $\mathbf{h}_{EVI}$ contains just few significant elements (depending on the order of the channel) whereas all the others should be zero. In the sequel, it will be examined whether the matrix dimensions in (15) can be reduced without deteriorating the estimation performance of the system impulse response $h_{EVI}(k)$.

Since the transmitted data $d(k)$ are regarded as i.i.d. random variables, the range of the non-zero cross-cumulants is restricted to the channel order q. Thus, the $(l+1) \times (l+1)$ cumulant matrix contains non-zero values in a given range of diagonals, only. This can be illustrated by the following representation of eqn. (15).



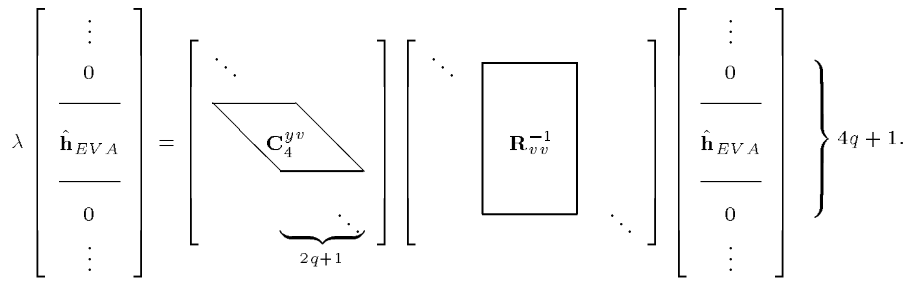
(16)

Furthermore, the reference system used with the EVA determines the delay k_0 introduced in (15): as the non-zero values of the estimated channel impulse response are located within an interval of $2q+1$ elements of the vector $\mathbf{h}_{EVI}$, the rank of the $(l+1)\times(l+1)$ matrix $\mathbf{C}_4^{yv}$ is reduced to 2q+1. Eq. (15) can thus be illustrated as



(17)

It should be taken into account that the vertical position of the relevant part of $\mathbf{C}_4^{yv}$ (marked by 2 horizontal lines) depends on the delay k_0 introduced by the reference system. For simplicity, let us assume the specific delay $k_0 = 2q+1$. Without loss of generality we get the following configuration, then



(18)

Just the indicated areas of support of the autocorrelation and cumulant matrices are relevant for a proper solution of eqn.(15). For this reason, the complexity of the eigenvector problem can be reduced significantly. On the other hand, the problem of the inversion of the $(l+1)\times(l+1)$ autocorrelation matrix remains although only a smaller $(4q+1)\times(2q+1)$ submatrix of $\mathbf{R}_{vv}^{-1}$ is needed in the eigenvector problem.

Further considerations [2] show that the inversion of the original autocorrelation matrix can be avoided. At first, a set of linear equations

$$\tilde{\mathbf{R}}_{vv}\mathbf{r}_i = \mathbf{i} \tag{19}$$

is solved, where

$$\mathbf{i} = [\underbrace{0,\ldots,0}_{2q}, 1, \underbrace{0,\ldots,0}_{2q}]^T \tag{20}$$

[2] A detailed analysis of this problem will be given in [9]

and $\tilde{\mathbf{R}}_{vv}$ is the $(4q+1) \times (4q+1)$ autocorrelation matrix of the received signal. Note that this matrix is always non-singular, even if there are channel zeros on the unit circle. Consequently, eq (19) can be solved without problems. Defining a $(4q+1) \times (2q+1)$ matrix $\tilde{\mathbf{R}}_i$ based on the elements of $\mathbf{r}_i$

$$\tilde{\mathbf{R}}_i := \begin{bmatrix} r_i(-q) & \dots & r_i(-2q) & & 0 \\ \vdots & \ddots & & \ddots & \\ r_i(q) & & \ddots & & r_i(-2q) \\ \vdots & \ddots & & \ddots & \vdots \\ r_i(2q) & & \ddots & & r_i(-q) \\ & \ddots & & \ddots & \vdots \\ 0 & & r_i(2q) & \dots & r_i(q) \end{bmatrix} \tag{21}$$

the eigenvector problem (15) can be put as

$$\lambda \mathbf{h}_{EVI} = \tilde{\mathbf{C}}_4^{yv} \tilde{\mathbf{R}}_i \mathbf{h}_{EVI} \qquad \text{(EVI)} \tag{22}$$

with

$$\begin{aligned} dim(\tilde{\mathbf{C}}_4^{yv}) &= (2q+1) \times (4q+1) \\ dim(\tilde{\mathbf{R}}_i) &= (4q+1) \times (2q+1). \end{aligned}$$

Now, the dimensions of both matrices involved in this equation are determined by the channel order q rather than the order of the inverse system l. Summarily, the overall EVI algorithm is composed of the following steps:

- EVA iteration for the update of an appropriate reference system. The equalizer order l can be relatively small since a suitable reference system has to be obtained rather than perfect equalization.
- Set up the matrices $\tilde{\mathbf{C}}_4^{yv}$ (submatrix of the final EVA result $\mathbf{C}_4^{yv}$) and $\tilde{\mathbf{R}}_i$ (after the inversion of $(4q+1) \times (4q+1)$ autocorrelation matrix).
- Solve the eigenvector problem (22).
- Choose the eigenvector corresponding to the maximum magnitude eigenvalue.
- Pick the conjugate complex reverse vector as an estimate of the channel impulse response.

4 Simulation results

4.1 Convergence behavior without additive noise

First, the performance of blind identification algorithms will be demonstrated by means of two specific channel configurations. The channel zeros and the magnitude of the (complex valued) impulse responses are shown in fig. 2. Note that the first example (fig. 2a) is rather unfavorable for EVI since two channel zeros are close to the unit circle and two coefficients of the impulse response are equal in magnitude. The former channel property would require

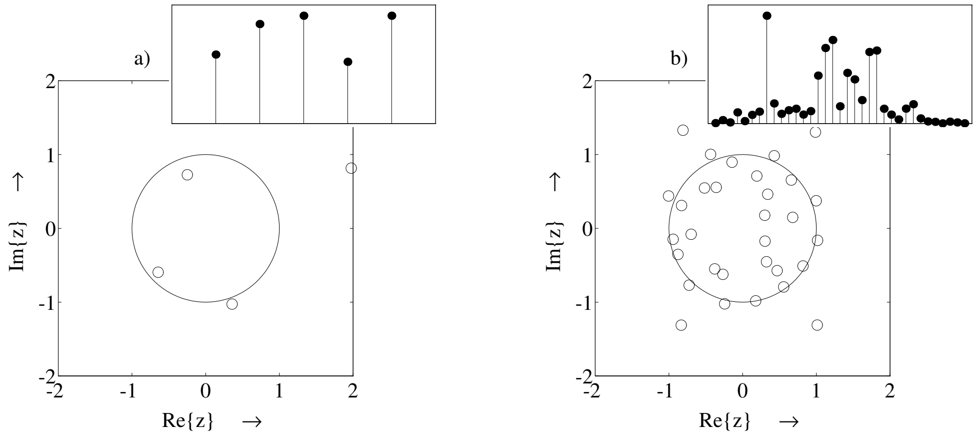


Figure 2: Channel impulse response and zero configuration

a) Example 1 b) Example 2: bad urban

a large equalizer order for a proper equalization (nevertheless, we select an equalizer of order $l = 16$, only, for the pre-iterations which is sufficient for an adequate update of the reference system). On account of the latter property, the optimization of the reference system requires a relatively high number of pre-iterations (six in this example).

On the basis of channel 1, both EVI and Giannakis' and Mendel's Least-Squares approach are compared in fig. 3. The mean value (solid line) $\pm$ the standard deviation (dotted line) of the coefficients of 25 estimated impulse responses (Monte Carlo runs) are given in terms of the block length L. This example illustrates that EVI is faster by a factor of 100, although a *system order overfit of* 4 was assumed in the case of EVI, only (estimated order: $\hat{q} = 8$, true order: $q = 4$). Consequently, four out of nine coefficients estimated by EVI should be zero (see fig. 3).

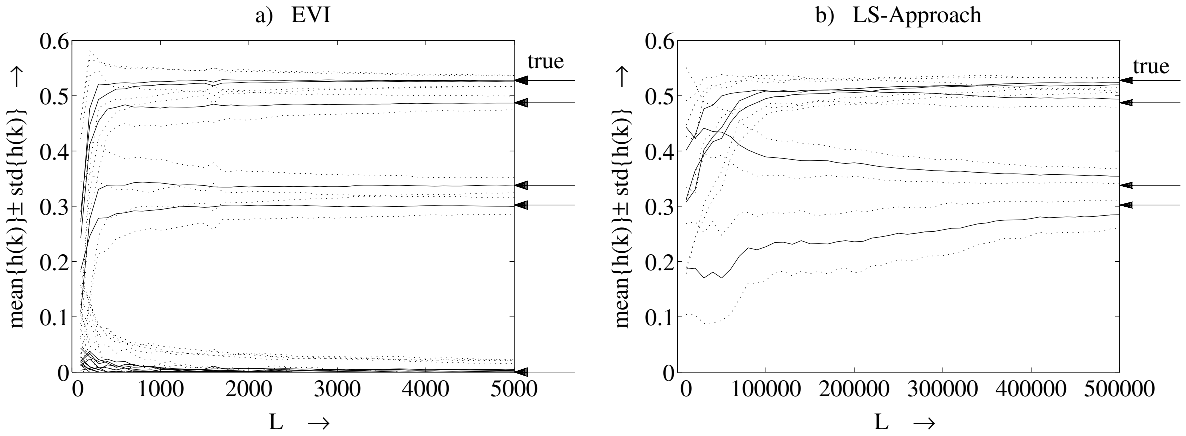


Figure 3: Estimation of the 4th-order channel from fig. 2 a

With the second example, we apply a frequency selective multipath channel in a bad urban environment. Fig. 2b shows the magnitude of the impulse response and the zero configuration. To illustrate the convergence behavior of the identification algorithms we use the alternative representation displayed in fig. 4. The estimated values of the impulse response are shown in the complex plane at different block lengths. On the right hand side, the true values are given. The corresponding standard deviation is plotted beneath. EVI leads to satisfactory results with block

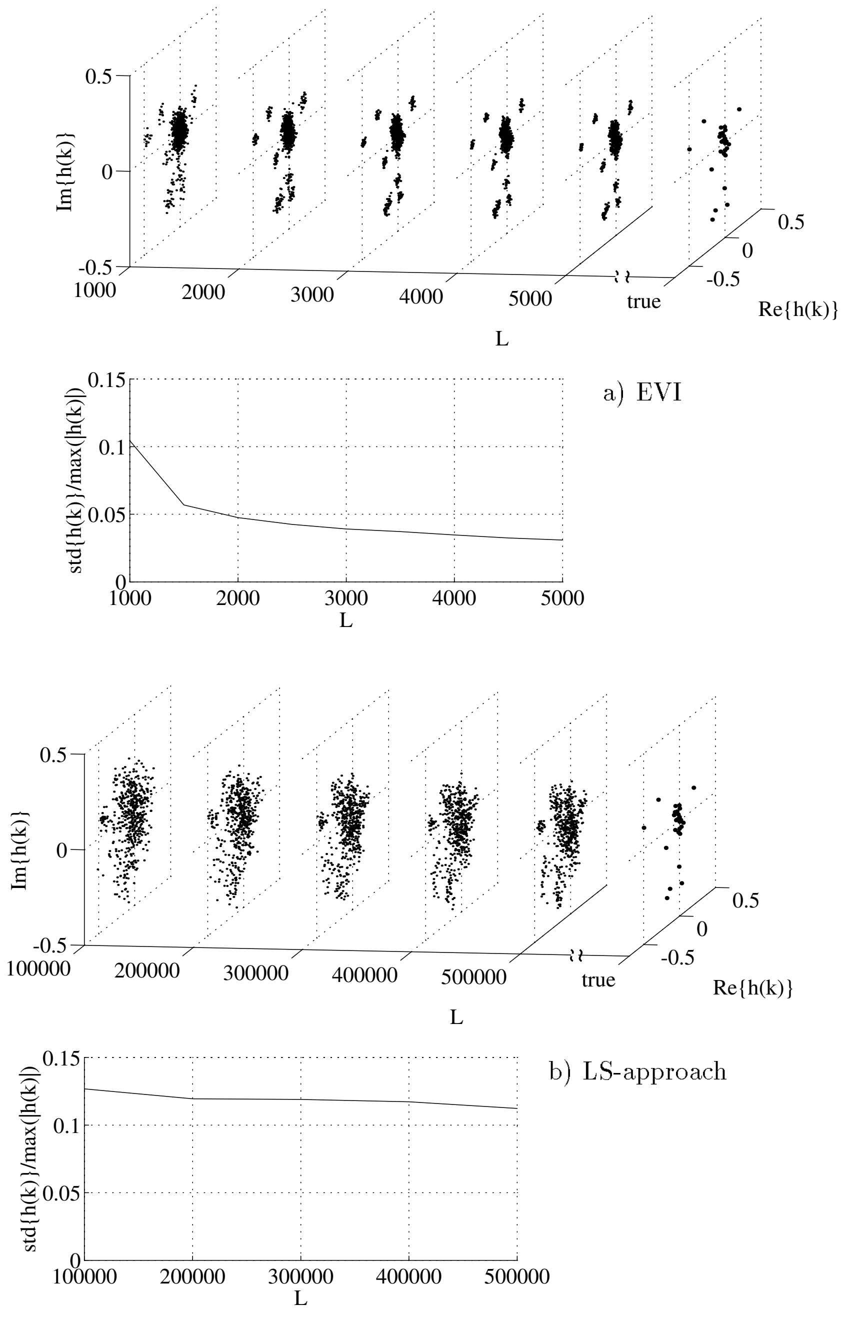


Figure 4: Estimation of a multipath channel (see fig. 2 b)

lengths as small as $L = 4000$ samples whereas the convergence of the LS approach is rather poor, even for a block length as large as 500000.

The third example considered in this section is a 1st-order all-pass channel. Although this is a recursive system, the impulse response can be approximated by a finite length impulse. Note that the effective length of the all-pass impulse response depends on the pole's magnitude $|z_\infty|$. In our example we pick $z_\infty = 0.8e^{j\pi/4}$ which allows an impulse response approximation by an order 16 MA model. Figs. 5a and d display the magnitude of the impulse responses estimated on the basis of 500 and 2000 data samples, respectively; the mean values $\pm$ standard deviations are indicated. Figs. 5b,e and 5c,f show the corresponding magnitude of the frequency response and the group delay. Again, the mean values $\pm$ the standard deviations are marked (solid and dotted lines, respectively). The true values are indicated by dashed lines.

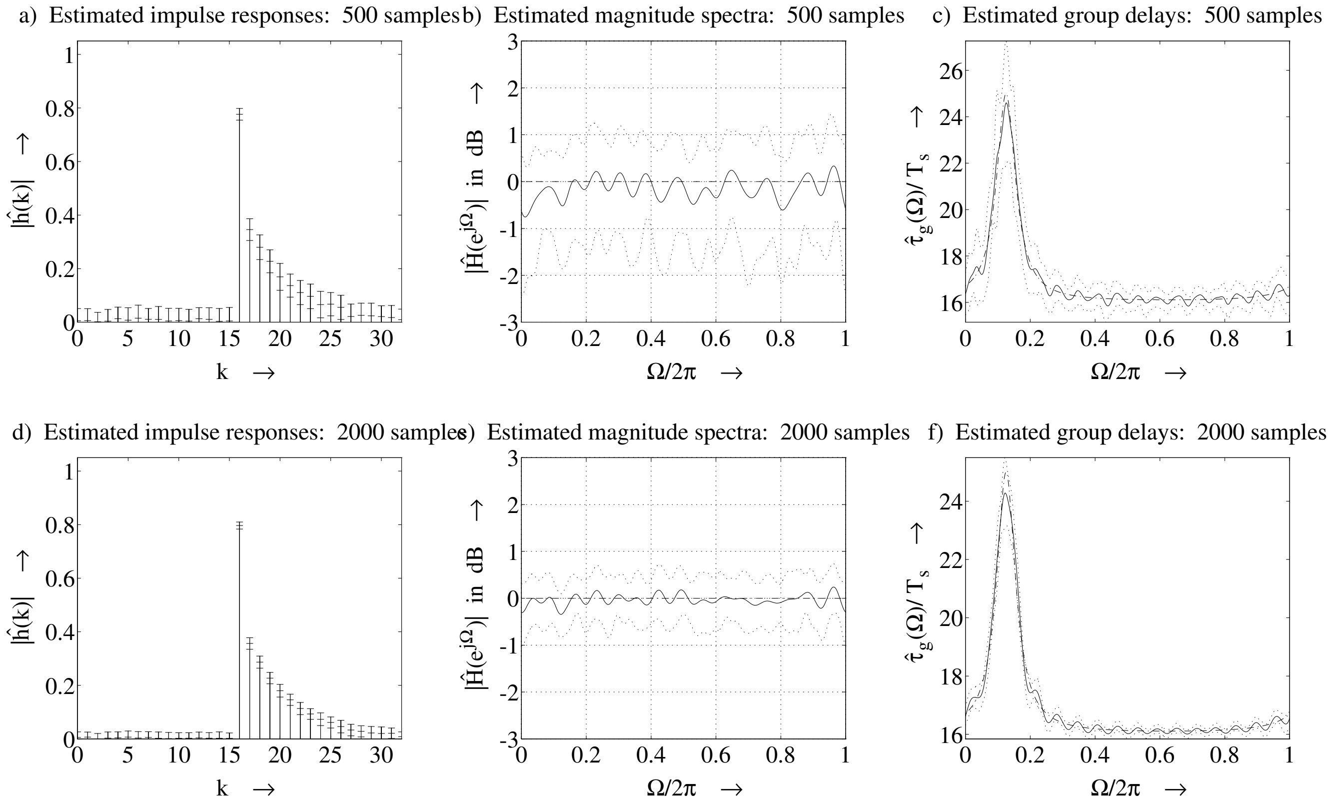


Figure 5: EVI estimation of a 1st-order all-pass channel

4.2 EVI performance in presence of additive gaussian noise

As opposed to the 2nd-order covariance sequence, 4th-order cumulants are not influenced by gaussian noise. Since the EVI equation (15) uses both the autocovariance matrix $\mathbf{R}_{vv}$ and the cumulant matrix $\mathbf{C}_4^{yv}$, it is obvious that the EVI solution is degraded by gaussian noise (as well as the LS result, by the way). Some examples to illustrate this degradation will be given in the sequel. Firstly, the 4th-order channel according to fig. 2a is considered. Fig. 6 demonstrates the EVI performance as a function of the *noise to signal* (N/S) ratio of the received signal. With the lefthand side figure, the true autocovariance with superimposed noise and the true cumulant samples were used . The result demonstrates the negligible degradation up to $N/S = 0.5$ (-3dB). Ignore, for the moment, the negative values of N/S. The righthand side figure gives the estimation results of 100 Monte-Carlo runs with blocklengths of 5000, each. The middle figure shows the zeros of the estimated channel for S/N = 3dB; note that the zeros are very close to their ideal positions.

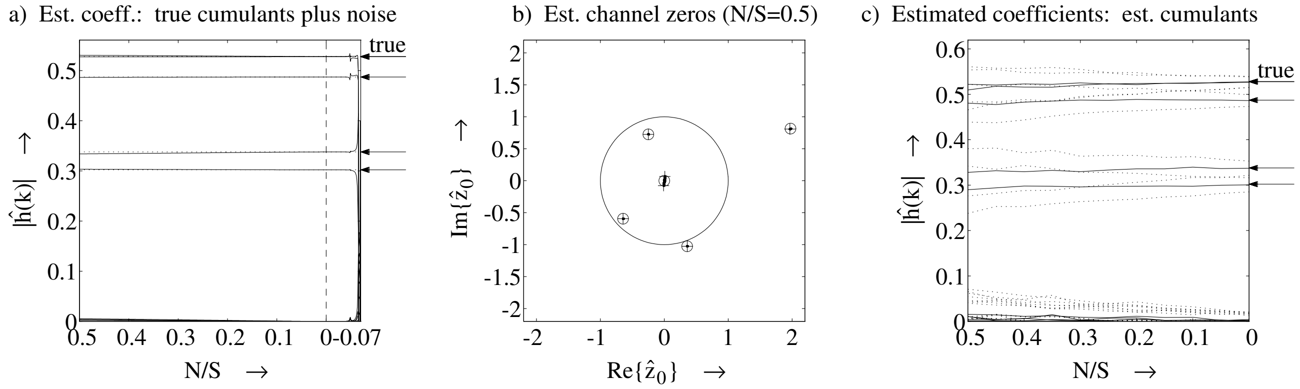


Figure 6: EVI estimation in presence of white gaussian noise (channel fig.2a)

In fig. 7, a fourth example (four channel zeros on the unit circle) is investigated. Here, the influence of the additive noise is increased, although the degradation is still very small. In the middle figure, the trajectories of the estimated zeros are shown for N/S ratios ranging from 0 to 0.5. It is remarkable that the four channel zeros remain on their ideal positions whereas further non-critical zeros are introduced. With the S/N ratio reaching the value 0.5 (3 dB), these zeros approach their final position (marked with "+" signs).

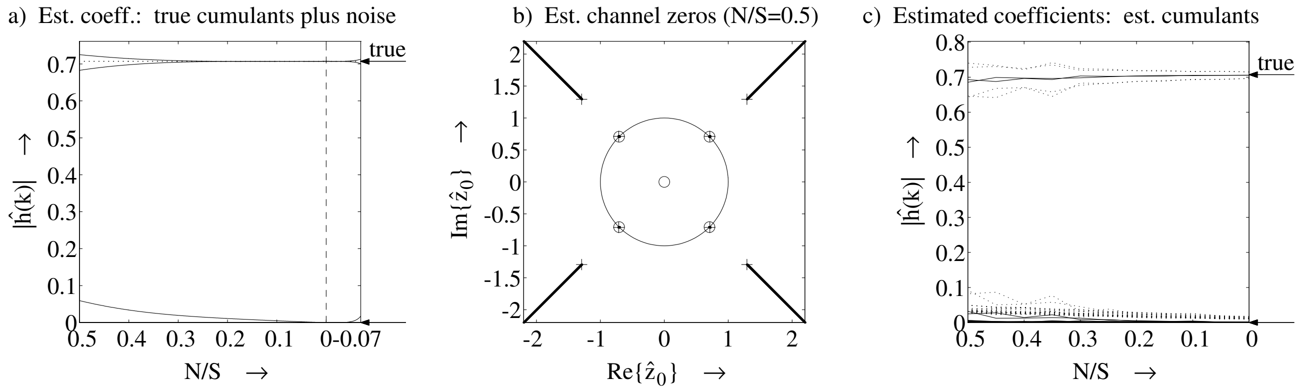


Figure 7: Critical channel estimation in presence of white gaussian noise

This phenomenon can be explained by some more detailed considerations. As shown in [9], the EVI estimation of the channel impulse response can be written as

$$\begin{aligned} h_{EVI}(k) &\approx h(k) * w(k) \\ H_{EVI}(z) &\approx H(z) \cdot W(z), \end{aligned} \tag{23}$$

where $h(k)$ represents the true channel impulse response and $w(k)$ denotes a certain weighting sequence which is a delta impulse in case of an ideal reference system $f(k)$. In presence of additive noise the reference system no longer is ideal and thus $w(k)$ is not a delta impulse any more. Consequently, the system identification result is corrupted. However, eqn.(23) shows that the estimated system contains the true channel zeros plus further zeros introduced by $W(z)$. The influence of the latter is very small because they are located far away from the unit circle and lie on a concentric circle with equidistant positions.

The remaining noise influence can be compensated for provided that the noise autocovariance is known. This autocovariance sequence can be estimated with an alternative method based on 4th-order cumulants ([6]). With this approach, the autocovariance estimates of the received signal do not contain the gaussian noise whereas the conventional autocovariance estimates do. The difference between both yields the autocovariance of the noise.

To compensate for the noise, EVI's matrix $\mathbf{R}_{vv}$ must be rectified. In case of white noise its power has to be subtracted on the main diagonal, only. In order to assess the influence of an *noise overcompensation*, *negative values* of the N/S ratios were considered in the lefthand side images of fig. 6 and fig. 7. Fig. 6 reveals that an overcompensation may cause severe degradations.

5 Conclusions

In this paper, a novel algorithm (EVI) for an efficient blind system identification was introduced. It is based on a specific closed-form solution (EVA) to blind equalization published recently. Since the EVI algorithm implicitly uses the inverse system, the dimensions of the involved matrices may be very large in case of channel zeros near the unit circle. The size of these matrices can be reduced considerably without any effect on the identification result. Various simulations illustrate that the convergence properties of EVI are largely superior to alternative methods known so far (e.g. the Least-Squares approach) especially with system order overfits. Furthermore, the EVI performance in presence of gaussian noise was investigated. It was demonstrated by simulation results that the ideal positions of the channel zeros are hardly affected. Some additional zeros are introduced. Usually, their influence is very small as demonstrated by the above examples. Even for relatively low signal to noise ratios, say 3 dB, this statement remains true.

Further work in the field of blind system identification will focus on blind ARMA system estimation. Presently, it is investigated whether the EVI can be applied to this problem in order to improve the convergence properties of existing algorithms. First results will be presented in [10, 11].

6 Acknowledgement

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