A Closed Form Solution to Blind Equalization

B. Jelonnek and K.D. Kammeyer
Arbeitsbereich Nachrichtentechnik, Technische Universität Hamburg-Harburg
Eissendorfer Str. 40, D-2100 Hamburg 90, Germany [1]

Abstract

In some recent papers new algorithms for blind adaptive equalization were proposed. These algorithms are based on the stochastic gradient method and thus can be regarded as a "blind" counterpart to the classic LMS- (least mean squares-) algorithms. It is well-known that these algorithms show relatively slow convergence speed. The classic solution to get fast convergence is the RLS- (recursive least squares-) algorithm which makes use of the closed-form solution. The purpose of this paper is to derive a closed-form solution in the sense of blind equalization. It will be shown that the equalizer coefficients can be uniquely derived from the eigenvectors of a specific 4th-order cumulant matrix of the received signal. By means of some examples it will be demonstrated that the eigenvector solution is near the ideal MSE- (mean square error-) solution.

In der letzten Zeit wurden verschiedene neue Algorithmen zur blinden adaptiven Entzerrung vorgeschlagen. Diese Algorithmen basieren auf dem stochastischen Gradientenverfahren, sie können daher als "blinde" Variante des klassischen LMS-Algorithmus betrachtet werden. Bekanntlich weisen diese Algorithmen relativ geringe Konvergenzgeschwindigkeiten auf – die klassische Methode, um zu einem schnellen Konvergenzverhalten zu kommen, ist der RLS-Algorithmus, der von der geschlossenen Lösung ausgeht. Das Ziel der vorliegenden Arbeit besteht in der Formulierung einer geschlossenen Lösung im Sinne der blinden Entzerrung. Es wird gezeigt, daß die Entzerrerkoeffizienten von den Eigenvektoren einer bestimmten Kumulanten-Matrix des Empfangssignals abgeleitet werden können. Anhand einiger Beispiele wird verdeutlicht, daß diese Eigenvektor-Lösung in der Nähe der idealen MSE-Lösung liegt.

[1]This work is supported by the "Deutsche Forschungsgemeinschaft" under project number Ka 841/1.

Introduction

Blind system identification, i.e. channel estimation without the use of training sequences or decided data is a new field of research since some years. It is well-known that traditional 2nd-order statistics are "phase-blind" which means that the phase properties of the channel cannot be derived from the received data. Consider as an example the classic application of a prediction-error filter as a prewhitening system. The design of this filter is based on the autocorrelation samples of the received signal – thus the result is a minimum-phase model of the inverse system. Consequently, higher order statistics have to be applied in order to achieve proper blind channel estimation. In the recent years several approaches were presented. In [1] a certain least-squares solution was proposed which makes use of the so-called diagonal cumulants. Potential singular solutions (which may occur with this algorithm under specific channel conditions) can be avoided by a modified least-squares approach given in [2] or by the so-called "Cumulant Zero Matching" method [3].

A problem closely related to system identification is blind equalization i.e. blind channel correction. The "self-recovering equalizer" introduced by Godard in 1980 was one of the first solutions to this problem [4]. A more general criterion for the blind deconvolution was presented by Shalvi and Weinstein in 1990 [5]. (For the first time this criterion was considered in 1980 by Donoho [6]). This approach maximizes a scalar 4th-order cumulant; for equalizer adjustment the well-known stochastic gradient algorithm is applied. Based on this paper a modified structure of a blind equalizer was suggested in [7] which exploits the fact that a proper phase correction network is an all-pass system: By the combination of a lattice prediction-error filter (maximum phase output) with an all-pass configuration the convergence properties can be improved due to the small number of parameters to be identified.

The blind equalization algorithms mentioned above are based on the stochastic gradient search and thus can be viewed as a certain counterpart to the classic LMS-algorithm. As is well-known these algorithms suffer from a relatively low convergence speed. The classic solution to overcome these problems results in the RLS-algorithm which performs an iterative update of the closed-form solution. The purpose of the present paper is to derive such a closed-form solution in the sense of blind equalization, i.e. a solution which is uniquely defined by the statistical properties of the received data (instead of the equalizer output sequence). It will be shown that the equalizer coefficients can be derived from the eigenvectors of a specific 4th-order cumulant matrix. This method is called "Eigenvector Approach" (EVA). It will be proved that the EVA converges asymptotically (for large equalizer lengths) to the optimum MSE-solution, apart from few "singular" channel configurations. These cases, however, can be avoided by the appropriate choice of the reference system used in this algorithm. Further investigations show that the reference system can be updated iteratively. The resulting algorithm will be published in the near future.

1 Background

The problem of (blind) equalization is illustrated in fig. 1. Using the output sequence $v(k)$ of a linear, time-invariant channel $h(k)$, the transmitted complex sequence $d(k)$ has to be recovered by a complex linear filter $e(k)$, where a certain time-delay k_0 may be allowed. Consequently, the desired impulse response of an equalized channel contains only one nonzero component with unit magnitude [2].

$$\begin{aligned} s(k) &= h(k) * e(k) = \sum_i e(i)\, h(k-i) \\ &\stackrel{!}{=} \delta(k-k_0) \end{aligned} \tag{1.1}$$

Blind deconvolution is achieved, if only the output sequence and some statistical information of the input data are neccessary to adjust the equalizer cofficients. An interesting method to do this based on higher order statistics was proposed in [5]. The input data $d(k)$ are assumed to be independent and identically distributed (i.i.d.) random variables with non-Gaussian probability density function and zero mean. In this case, 2nd- and 4th-order whiteness holds for the random process $d(k)$, which means that the 2nd- and 4th-order cumulants

$$c_2^d(\lambda) = E\{d^*(k)d(k+\lambda)\} = r_{dd}(\lambda) \tag{1.2}$$

and

$$\begin{aligned} c_4^d(\lambda_1,\lambda_2,\lambda_3) = & E\{d^*(k)d^*(k+\lambda_1)d(k+\lambda_2)d(k+\lambda_3)\} \\ & -E\{d^*(k)d^*(k+\lambda_1)\}E\{d(k+\lambda_2)d(k+\lambda_3)\} \\ & -r_{dd}(\lambda_2)\, r_{dd}(\lambda_3-\lambda_1) - r_{dd}(\lambda_3)\, r_{dd}(\lambda_2-\lambda_1) \end{aligned} \tag{1.3}$$

vanish, except for $r_{dd}(0) = \sigma_d^2$ and $c_4^d(0,0,0) = \gamma_4^d$.

On the assumption of a 2nd- and 4th-order white input process the following relations between the cumulants of the output sequence $r_{xx}(0)$ and $c_4^x(0,0,0)$ and $s(k)$ can be derived:

$$r_{xx}(0) = \sigma_d^2 \sum_k |s(k)|^2; \qquad c_4^x(0,0,0) = \gamma_4^d \sum_k |s(k)|^4. \tag{1.4}$$

The inequation of Schwarz

$$\left(\sum_k |s(k)|^2\right)^2 \geq \sum_k |s(k)|^4$$

combines both $r_{xx}(0)$ and $c_4^x(0,0,0)$. Shalvi and Weinstein [5] proved that the only solution to obtain equality is

$$s(k) = s(k_m)\delta(k-k_m).$$

Actually, this impulse response describes a successful channel deconvolution apart from a complex factor. Consequently they proposed the following criterium for blind equalization

$$|c_4^x(0,0,0)| \stackrel{!}{=} max \qquad \text{provided that} \qquad r_{xx}(0) = r_{dd}(0). \tag{1.5}$$

[2] $*$ denotes the convolution operation.

The method to maximize this "constrained" criterium proposed in [5] is spectral prewhitening of the channel output and adopting a stochastic gradient algorithm which maximizes $|c_4^y(0,0,0)|$, using the condition $\sum_k |e(k)|^2 = 1$ for the cofficients of the FIR-equalizer structure. The price paid for the simplicity of this algorithm is twofold: *1st* a high number of parameters are necessary to approximate an all-pass structure (this knowledge can be used to achieve faster convergence properties [7]) and *2nd* the well-known disadvantages of a stochastic gradient implementation consisting of a trade-off between convergence speed and gradient noise.

Stochastic gradient algorithms are widely used in blind deconvolution algorithms to optimize non-linear and non-quadratic criteria. Their counterpart among conventional equalization techniques is the LMS-algorithm, which is known to be inferior in convergence speed and accurancy with respect to the RLS-approach. The RLS-algorithms compute a unique solution of the least-squares criterion $\sum_k |y(k) - d(k)|^2 \stackrel{!}{=} min$.

The purpose of this paper is to develop a class of criteria for blind deconvolution which can be solved uniquely computing an eigenvector of a specific cumulant matrix.

2 Cross-cumulant quality criterion

Generally, the definitions of cumulant sequences are not restricted to auto-cumulants defined in (1.3), but they can be regarded as a special case of 4th-order (cross-) cumulants

$$\begin{aligned} c_4^{x_0,x_1,x_2,x_3}(\lambda_1,\lambda_2,\lambda_3) \;=\; & E\{x_0^*(k)x_1^*(k+\lambda_1)x_2(k+\lambda_2)x_3(k+\lambda_3)\} \\ & -E\{x_0^*(k)x_1^*(k+\lambda_1)\}E\{x_2(k+\lambda_2)x_3(k+\lambda_3)\} \\ & -E\{x_0^*(k)x_2(k+\lambda_2)\}E\{x_1^*(k+\lambda_1)x_3(k+\lambda_3)\} \\ & -E\{x_0^*(k)x_3(k+\lambda_3)\}E\{x_1^*(k+\lambda_1)x_2(k+\lambda_2)\}. \end{aligned} \tag{2.1}$$

Let $d(k)$ be a non-gaussian i.i.d. random process with $E\{d(k)\} = 0$ and $x_i(k)$, $i = 0,\ldots,3$ the output data of four different transmission channels $s_0(k)\ldots s_3(k)$.

$$x_i(k) = \sum_\nu s_i(\nu)d(k-\nu), \quad i = 0\ldots 3$$

Then, using the linearity of the expectation operation the following relation can be derived.

$$c_4^{x_0,x_1,x_2,x_3}(\lambda_1,\lambda_2,\lambda_3) \;=\; \gamma_4^d \sum_\nu s_0^*(\nu)s_1^*(\nu+\lambda_1)s_2(\nu+\lambda_2)s_3(\nu+\lambda_3) \tag{2.2}$$

In the following considerations we choose the specific output data set

$$x_0(k) = x_2(k) = x(k) = \sum_\nu s(\nu)d(k-\nu)$$

and

$$x_1(k) = x_3(k) = y(k) = \sum_\nu w(\nu)d(k-\nu).$$

Thus we obtain by means of (2.2)

$$c_4^{xy}(0,0,0) \quad := \quad c_4^{xyxy}(0,0,0) \quad = \quad \gamma_4^d \sum_\nu |s(\nu)|^2 \, |w(\nu)|^2. \tag{2.3}$$

These cumulants can be used to define a cross-cumulant quality criterion for blind equalization. Consider fig. 2 where a reference system $f(k)$ is introduced. The cross cumulant of the output signals $x(k)$ and $y(k)$ is equal to (2.3) with the definitions (1.1) and

$$w(k) = f(k) * h(k). \tag{2.4}$$

Now consider the inequation

$$\sum_k |s(k)|^2 \, |w(k)|^2 \leq max\{|w(k)|^2\} \sum_\kappa |s(\kappa)|^2. \tag{2.5}$$

Obviously, both expressions are equal if $s(k) = s(k_m)\ \delta(k - k_m)$ (ideal equalization) and $max\{|w(k)|^2\} = |w(k_m)|^2$. Assuming that there exists only one maximum value of $|w(k)|^2$ this solution is unique: The left-hand side of eq. (2.5) can be interpreted as the weighted sum of positive coefficients $|w(k)|^2$. If there is any non-zero factor $|s(k)|^2$ apart from that multiplied by the maximum value of $|w(k)|^2$ the result of this summation will be smaller than its maximum value.

Blind equalization is accomplished if the residual impulse response $s(k)$ of channel and equalizer results in a pure time delay. Apart from a factor, this is fulfilled by eq. (2.5). Eqs. (1.4) and (2.3) contain the required relations between cumulants and transmission channel coefficients. As a consequence, a quality criterion for blind deconvolution is to

$$\text{maximize } |c_4^{xy}(0,0,0)| \quad \text{with respect to } r_{xx}(0) = \sigma_d^2. \tag{2.6}$$

3 Eigenvector Approach (EVA)

The quality criterion we derived in the previous section maximizes the cross-cumulant $|c_4^{xy}(0,0,0)|$, using the condition that the output power of the equalizer is constant. Both, cumulant and autocorrelation sequence contain a quadratic relationship with respect to the coefficients of a FIR equalizer. Therefore, we can find a closed form solution for those coefficients which maximize the quality criterium. The cross-cumulant $c_4^{xy}(0,0,0)$ and power of the equalizer output signal $r_{xx}(0)$ can be uniquely expressed in terms of the equalizer coefficient vector $\mathbf{e}$

$$c_4^{xy}(0,0,0) = \mathbf{e}^* \ \mathbf{C}_4^{yv} \ \mathbf{e} \tag{3.1}$$

$$r_{xx}(0) = \mathbf{e}^* \ \mathbf{R}_{vv} \ \mathbf{e} \tag{3.2}$$

where[3]

$$\mathbf{e}^* = [\ e^*(0) \ldots e^*(q)\ ]\,, \tag{3.3}$$

[3]Note that $c_4^{yv}(\lambda_1, 0, \lambda_3) = [c_4^{yv}(\lambda_3, 0, \lambda_1)]^*$

$$\mathbf{C}_4^{yv} = \begin{bmatrix} c_4^{yv}(0,0,0) & [c_4^{yv}(-1,0,0)]^* & \dots & [c_4^{yv}(-q,0,0)]^* \\ c_4^{yv}(-1,0,0) & c_4^{yv}(-1,0,-1) & \dots & [c_4^{yv}(-q,0,-1)]^* \\ \vdots & \vdots & & \vdots \\ c_4^{yv}(-q,0,0) & c_4^{yv}(-q,0,-1) & \dots & c_4^{yv}(-q,0,-q) \end{bmatrix} \tag{3.4}$$

and R_{vv} denotes the conjugate autocorrelation matrix of the received data.

By means of (3.1) and (3.2) the quality criterion (2.6) can be stated in the compact form

$$\text{maximize } |\mathbf{e}^* \, \mathbf{C}_4^{yv} \, \mathbf{e}| \quad \text{with respect to } \mathbf{e}^* \, \mathbf{R}_{vv} \, \mathbf{e} = \sigma_d^2. \tag{3.5}$$

This maximization problem leads to the classic generalized eigenvector problem

$$\mathbf{C}_4^{yv} \, \mathbf{e} = \lambda \, \mathbf{R}_{vv} \, \mathbf{e}; \text{ choose } |\lambda| = max\{|\lambda_1|, \dots, |\lambda_q|\}. \tag{3.6}$$

Note that neither the cumulant matrix $\mathbf{C}_4^{yv}$ nor the autocorrelation sequence $r_{vv}(k)$ contains the equalizer coefficients or the equalizer output data. The autocorrelation and the cumulant matrices can be estimated directly and the eigenvector problem can be solved uniquely (or iteratively e.g. by means of the power method). *Thus the equalizer coefficients are identified on the basis of the statistical properties of the received signal (apart from a complex factor).* This result can be compared with the closed-form MSE-solution of training sequence based algorithms

$$\mathbf{e} = \mathbf{R}_{vv}^{-1} \, \mathbf{r}_{dv}, \tag{3.7}$$

where the cross-correlation vector $\mathbf{r}_{dx}$ is defined by

$$\mathbf{r}_{dv} = \begin{bmatrix} E\{d(k-k_0)v^*(k)\} \\ \vdots \\ E\{d(k-k_0)v^*(k-q)\} \end{bmatrix}.$$

Eq. (3.7) is very similar to (3.6): For comparision, eq. (3.6) can be rewritten as

$$\mathbf{e} = \mathbf{R}_{vv}^{-1} \, \lambda^{-1} \, \mathbf{C}_4^{yv} \, \mathbf{e}. \tag{3.8}$$

Here the cross correlation vector $\mathbf{r}_{dv}$ of traditional (reference signal based) algorithms is replaced with the term $\lambda^{-1}\mathbf{C}_4^{yv}\mathbf{e}$, i.e. cumulant matrix and the eigenvector of $\mathbf{R}_{vv}^{-1}\mathbf{C}_4^{yv}$ corresponding to the maximum eigenvalue. This information can be derived from the received signal without use of the transmitted data. For illustration let us consider the true cumulants and an asymptotically infinite impulse response of the equalizer. Since

$$E\{d(k-k_0) \, v^*(k-i)\} \quad = \quad \sigma_d^2 h^*(k_0 - i),$$

the cross-correlation vector contains an estimate of the channel coefficients. For $\mathbf{C}_4^{yv}\mathbf{e}$ we get the same result if we assume that the equalizer coefficients $\mathbf{e}$ approximate the inverse system of $h(k)$. The i_1th component is

$$\begin{aligned} \sum_{i_2} c_4^{yv}(-i_1, 0, -i_2) \, e(i_2) &= \sum_{i_2} \gamma_4^d \sum_k |w(k)|^2 \, h^*(k-i_1) \, h(k-i_2) \, e(i_2) \\ &= \gamma_4^d \sum_k |w(k)|^2 \, h^*(k-i_1) \underbrace{\sum_{i_2} \, h(k-i_2) \, e(i_2)}_{\delta(k-k_0)} \\ &= \gamma_4^d |w(k_0)|^2 \, h^*(k_0 - i_1). \end{aligned}$$

Under the ideal condition of infinite length of the equalizer impulse response the cross-cumulant vector and the product $\mathbf{C}_4^{yv}\mathbf{e}$ lead to the same result. An analogous derivation shows that

$$\sum_{i_2} r_{vv}(i_1 - i_2)\, e(i_2) \quad = \quad \sigma_d^2\, h^*(k_0 - i_1).$$

Obviously, the inverse system of $h(k)$ is a solution to the eigenvector problem (3.6). The eigenvector approach guarantees convergence if there is a single maximum eigenvalue $|\lambda|$. (This corresponds to the requirement that $|w(k)|^2$ shows one maximum value only).

So far, it can be stated that on ideal conditions the only requirement for perfect deconvolution is that the linear system $w(k)$ contains one single maximum value $|w(k_m)|^2$; in this case the solution is independent of the other values $|w(k)|^2$, $k \neq k_m$. However, the influence of $w(k)$ increases as conditions become non-ideal (e.g. limited length of the equalizer impulse response or additive noise) . To clarify this fact, we expand the matrix $\mathbf{C}_4^{yv}$ into a weighted sum of vector products (which can easily be derived from its definition, using (2.2)):

$$\mathbf{C}_4^{yv} = \gamma_4^d \sum_k |w(k)|^2 \mathbf{h}_k \mathbf{h}_k^*, \tag{3.9}$$

where $\mathbf{h}_k$ is defined by the k-th subset of the channel impulse response

$$\mathbf{h}_k = [h(k) \ldots h(k-q)]^*.$$

Eq. (3.8) can be rewritten as

$$\begin{aligned} \mathbf{e} &= \mathbf{R}_{vv}^{-1} \frac{1}{\lambda} \mathbf{C}_4^{yv} \mathbf{e} \\ &= \mathbf{R}_{vv}^{-1} \frac{1}{\lambda} \gamma_4^d \sum_k |w(k)|^2 \mathbf{h}_k \mathbf{h}_k^* \mathbf{e} \\ &= \frac{\gamma_4^d}{\lambda} \sum_k |w(k)|^2 s(k) \mathbf{R}_{vv}^{-1} \mathbf{h}_k \\ &= \frac{\gamma_4^d}{\lambda \sigma_d^2} \sum_k |w(k)|^2 s(k) \mathbf{e}_{MSE_k} \end{aligned} \tag{3.10}$$

where the well-known MSE-solution

$$\mathbf{e}_{MSE_k} = \sigma_d^2 \mathbf{R}_{vv}^{-1} \mathbf{h}_k$$

was substituted. Eq. (3.10) demonstrates that the EVA contains a weighted sum of MSE solutions (including different time-delays of the reference data). Thus an ideal MSE result can only be guaranteed if

$$|w(k)|^2 \cdot s(k) = \delta(k - k_0), \tag{3.11}$$

which is fulfilled under two possible conditions:

1st: $|w(k)| = \delta(k - k_0)$, i.e. the output signal of the reference filter is identical to the (delayed) transmitted data. This case describes the application of a training sequence.

2nd: $s(k) = \delta(k - k_0)$ which can only be fulfilled by means of an ideal equalizer with an infinite length of the impulse response.

In realistic configurations the latter condition is met in the sense of a more or less good approximation. As a consequence the EVA will result in an equalizer design *near the MSE-solution*. This is demonstrated by some examples given in the next section.

We conclude the present section with the following statements:

- *A closed form solution for a blind equalizer can be derived from the 2nd-order and 4th-order cumulants of the received signal.*
- *The equalizer coefficients are uniquely defined by an eigenvector problem.*
- *On some specific conditions there is more than one solution. Uniqueness can always be achieved by changing the reference system.*

4 Examples

Let the impulse response of the reference filter be

$$f(k) = \delta(k), \tag{4.1}$$

which means that in (3.8) the *auto-cumulant matrix* of the received signal is used. To show the performance of the EVA with unfavourable conditions the following impulse response is considered:

$$h(k) = \delta(k) - 0.95\,\delta(k-1)$$

Obviously the weighting sequence $w(k)$ contains two coefficients which are approximately equal since in this case $w(k) = h(k)$. Fig. 3b demonstrates that even on these conditions a proper solution is obtained provided that the equalizer order is sufficiently high ($q = 96$). The result for a reduced equalizer order ($q = 32$) shown in fig. 3a demonstrates that the equalization is not perfect due to poor approximation, but the difference between the ideal MSE solution and the EVA result is still relatively small.

As explained above the eigenvector solution will become ambiguous if the reference system (4.1) is used and $h(k)$ contains two (or more) equal magnitude maxima. But this problem can be overcome by the choice of an appropriate alternative reference system. Consider e.g. the impulse response of the reference system

$$f(k) = \delta(k-6) + f(1)\delta(k-7) \tag{4.2}$$

in connection with the channel impulse response

$$h(0) = 0.3, \quad h(1) = h(2) = 1 \quad \text{and} \quad h(3) = 0.5.$$

The power of the intersymbol interference (ISI) after EVA equalization is shown in fig. 4 as a function of $f(1)$. The comparison with the MSE solution demonstrates that in a small range near $f(1) = 0$ severe problems are introduced by the eigenvector approach whereas in a large range the EVA result is near the MSE solution.

This example illustrates that an optimum reference system should be designed such that the weighting sequence $|w(k)|$ contains only one dominant maximum value. Consequently, an ideal solution would be obtained if the ideal equalizer impulse response would be used in the reference system – unfortunately this solution is unknown. For this reason an algorithm was developed in which an iterative updating of the reference system is performed with the aid of the estimated values of the equalizer coefficients. It can be shown that the problems of ambiguity that occur under some specific ill-conditioned channel configurations can be solved. These results will be published in a seperate paper in the near future.

The last example to be considered is based on a realistic radio channel situation. Fig. 5a shows real and imaginary part of the channel impulse response (sampled at baud-rate of 200 kHz) under typical hilly terrain conditions (see e.g. [8]). Fig. 5b shows the received data (8 PSK) in the z-plane without equalization. Large intersymbol interference is introduced which makes a proper decision impossible. Using a data block of $L = 10000$ data samples for the estimation of the cumulant matrix we obtain the equalized signal shown in fig. 5c.

Conclusions

In the present paper a closed-form solution for blind equalization was presented. It was shown that the equalizer coefficients are defined by a generalized eigenvector problem in which the autocorrelation matrix and a specific 4th-order cross-cumulant matrix of the received signal and the output of a given reference system are involved. In most cases the eigenvector solution is approximately equal to the optimum MSE result if the equalizer order is sufficiently large. Under some specific, ill-conditioned channel configurations, however, this solution becomes ambiguous. It was shown that these problems can be overcome by the appropriate choice of the reference system used with the eigenvector approach. The following points have to be considered in future investigations.

- *In order to overcome the problems of ambiguous solutions an algorithm has to be developed which includes an iterative update of the reference system.*
- *The calculation of eigenvectors has to be replaced with iterative methods of low computational complexity – such as the well-known power method which requires only matrix-vector multiplications. This iteration has to be embedded into the iterative algorithm.*
- *The convergence properties of the iterative eigenvector algorithm have to be compared with existing methods based on the stochastic gradient approach. The goal is an adaptation speed comparable to that of the (reference signal based) recursive least squares algorithm.*

Some of the problems mentioned above have already been solved in recent work and will be published in a separate paper in the near future.

References

[1] G. B. Giannakis and J. M. Mendel. Identification of nonminimum phase systems using higher order cumulants. *IEEE Trans. Acoust., Speech, Signal Processing*, 37(3):360–377, 1989.

[2] B. Jelonnek and K. D. Kammeyer. Improved methods for the blind system identification using higher order statistics. *accepted IEEE Trans. on Signal Processing to be published*, 1991.

[3] K. D. Kammeyer and B. Jelonnek. A cumulant zero-matching method for the blind system identification. *International Signal Processing Workshop on Higher Order Statistcs*, 1991. Chamrousse, July 10-12.

[4] D. Godard. Self-recovering equalization and carrier tracking in two-dimensional data communication systems. *IEEE Trans. Acoust., Speech, Signal Processing*, Com-28(11):1867–1875, 1980.

[5] O. Shalvi and G. Weinstein. New criteria for blind deconvolution of nonminimum phase systems (channels). *IEEE Trans. Inform. Theory*, 36(2):312–321, 1990.

[6] D. Donoho. On minimum entropy deconvolution. *Proc. of 2nd Appl. Time Series Symposium*, pages 565–608, 1980.

[7] B. Jelonnek and K. D. Kammeyer. A blind adaptive equalizer based on a lattice/all-pass configuration. *EUSIPCO*, 1992. Brussels, August 25-28.

[8] K. D. Kammeyer, U. Tuisel, H. Schulze, and H. Bochmann. Digital multicarrier-transmission of audio signals over mobile radio channels. *ETT*, 3(3):23–34, 1992.

List of figures

Figures

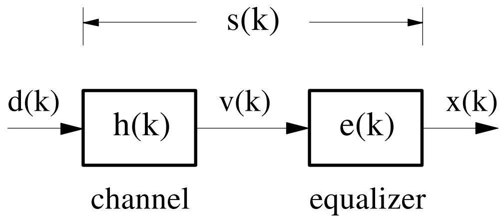


Figure 1: (Blind) equalization

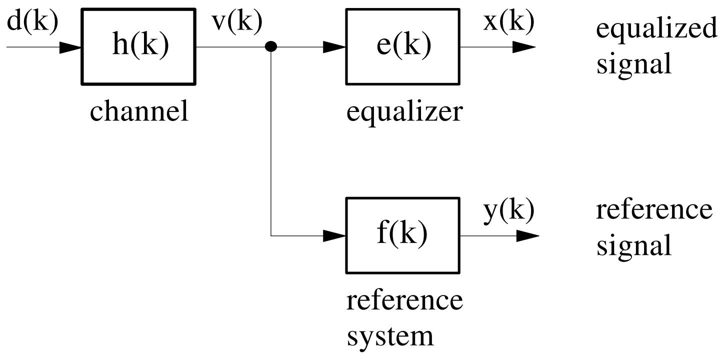
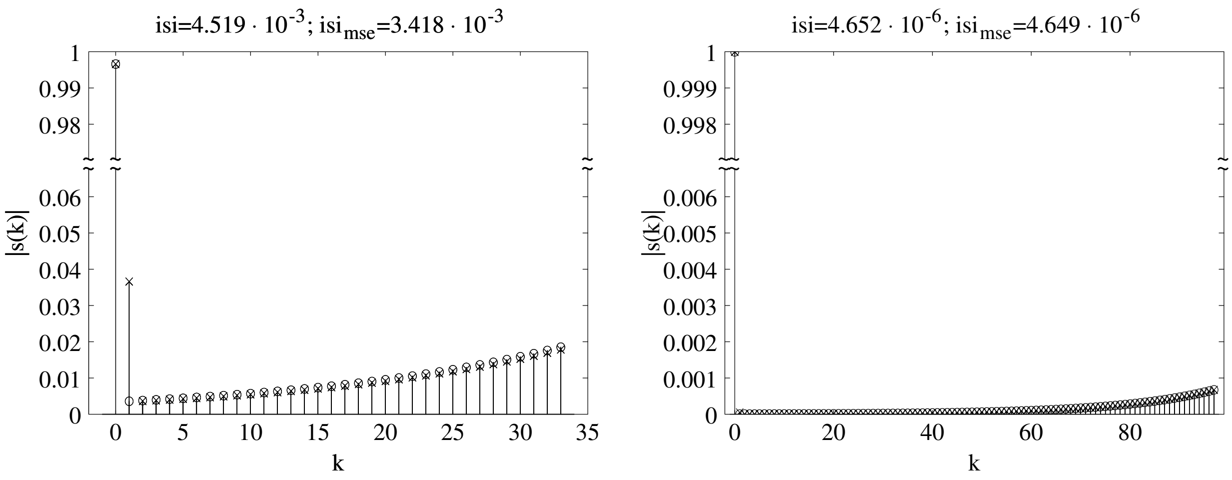


Figure 2: Equalizer and reference system



a) q=32

b) q=96

Figure 3: Channel deconvolution using true cumulants

$h(k) = \delta(k) - 0.95\delta(k-1)$ $\circ \equiv$ MSE-solution

$f(k) = \delta(k)$ $\times \equiv$ EVA-solution

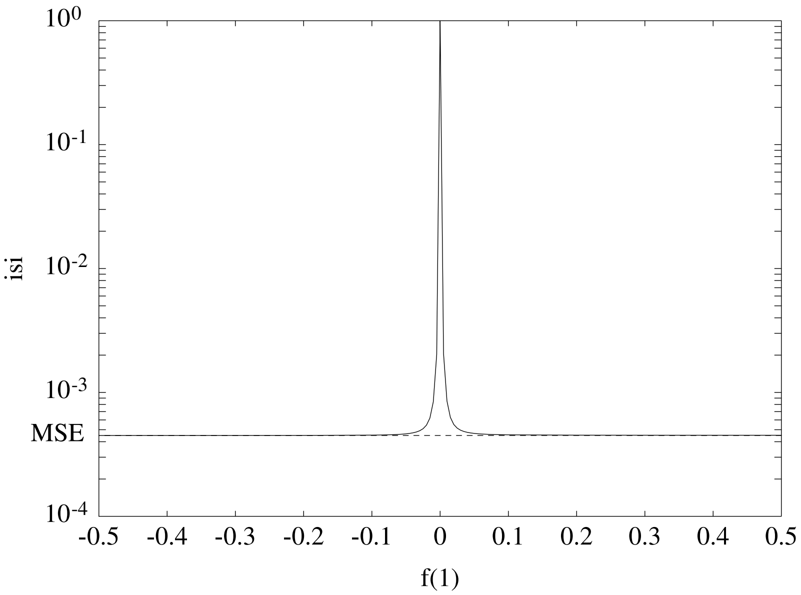
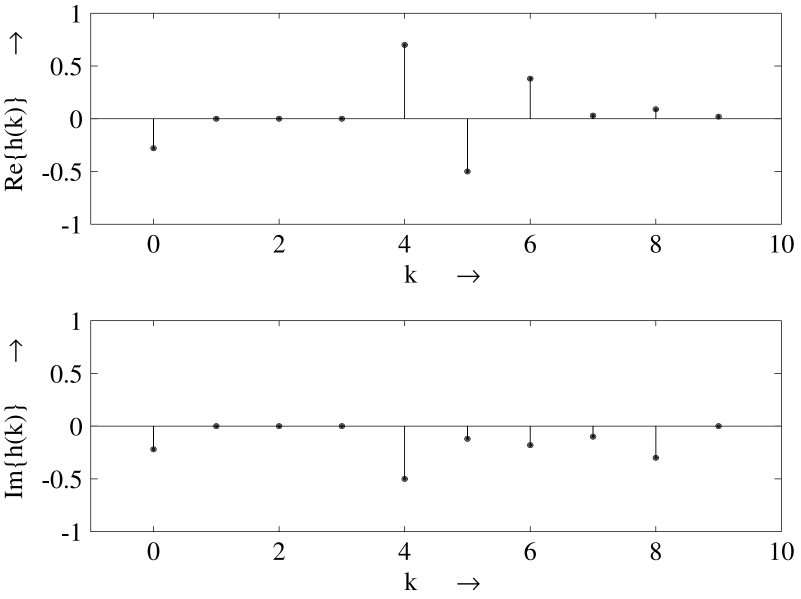
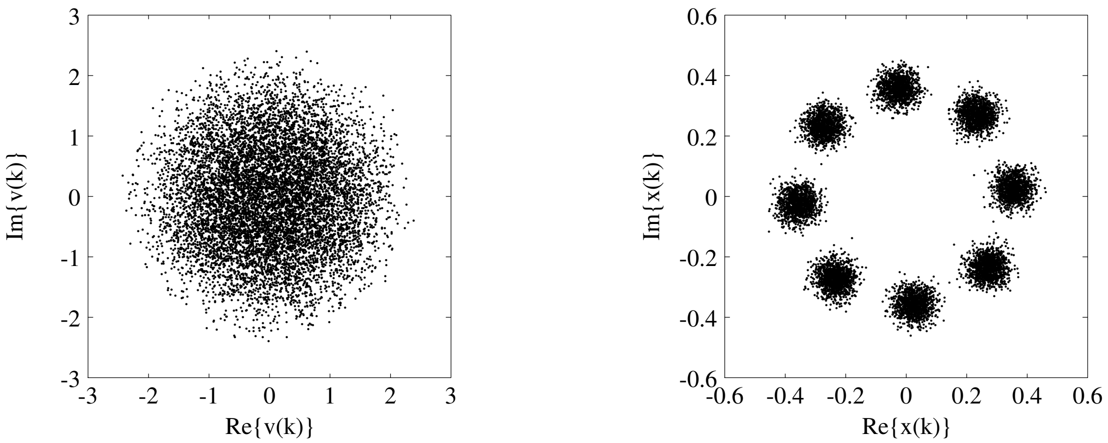


Figure 4: Power of intersymbol interference for different reference systems (channel $h(k) = 0.3\delta(k) + \delta(k-1) + \delta(k-2) + 0.5\delta(k-3)$ reference system $f(k) = \delta(k-6) - f(1)\delta(k-7)$, equalizer order $q = 32$)



a) Radio channel



b) Output sequence of the channel

c) Output sequence of the equalizer

Figure 5: EVA equalization of a hilly terrain radio channel (blocklength $L = 10000$, equalizer order $q = 64$, reference system $f(k) = \delta(k-33)$)