EIGENVECTOR ALGORITHM FOR BLIND EQUALIZATION

B. Jelonnek and K.D. Kammeyer

Technische Universität Hamburg-Harburg, Arbeitsbereich Nachrichtentechnik
Eißendorfer Str. 40 D-2100 Hamburg 90 Germany
E-mail: Jelonnek@tu-harburg.dbp.de

ABSTRACT[1]

This paper introduces a new algorithm for blind equalization which uses a set of cost functions. Each of them guarantees a closed form solution of the equalization problem and approximates the ideal MSE (mean square error) solution. On the basis of an iterative process the best approximation is selected. Application of this algorithm is not limited to linear equalizers operating at symbol rate. As a possible generalization to include other areas (such as system identification, decision-feedback equalization or fractionally spaced equalization), an extension to fractional tap space equalizers is outlined.

1 INTRODUCTION

In order to equalize non-minimum phase channels in communications systems, training sequences are usually applied. As a matter of principle, this knowledge, however, is not necessary. "Blind" equalization algorithms therefore deduce information about linear channel distortions from the received signal only.

To an increasing extent, recent papers on blind equalization use higher order statistics (HOS). Unfortunately, the field of potential applications is limited by the speed of convergence which is small compared to non-blind algorithms. Frequently, this is due to the use of a stochastic gradient approach required to minimize a strongly non-linear cost function (cf. e.g. [1, 3, 4]).

An alternative approach to blind equalization shall be proposed in this paper. It assumes a linear equalizer and a reference system which is independent of it. On the basis of HOS, a cost function will be derived for a large class of reference systems (section 2), which leads to a closed form solution of the equalization problem in the guise of an eigenvector problem (section 3).

As the choice of the reference system determines both the speed of convergence and the minimum intersymbol interference (ISI) to be achieved, an adaptive version of the above algorithm periodically updates the reference system (section 4). For constant magnitude input signals, convergence speed will approach the one of the recursive least squares (RLS) algorithm. This will be shown by simulation results.

Generalization of the approach is straightforward. It will be performed in section 5 with the aid of a fractionally spaced equalizer.

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2 CROSS-CUMULANT QUALITY CRITERION

Let us consider the problem of (blind) equalization which is shown in the upper part of fig. 1. The input sequence $d(k)$ is transmitted through a linear, time invariant transmission channel $h(k)$. It is corrupted by additive gaussian noise. The purpose of the equalizer $e(k)$ is to recover the input data $d(k)$.

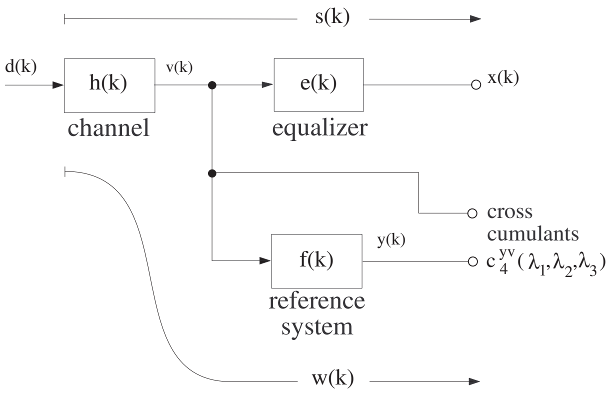

Figure 1: System configuration for blind equalization

Blind equalization requires some knowledge about the nature of the data sequence. Let the input data be independent and identically distributed (i.i.d.) random variables with non-Gaussian probability density function and zero mean. In this case the 2nd and 4th order cumulants of the input process vanish except for $r_{dd}(0) = \sigma_d^2$ and $c_4^{dd}(0,0,0) = \gamma_4^d$. For this reason there is a simple relation between the 4th order cross-cumulants of the equalizer output, $x(k)$, and the output $y(k)$ of the reference system $f(k)$, which will be regarded as an arbitrary system with fixed coefficients for the time being. With the definitions [2]

$$s(k) = h(k) * e(k) \ = \ \sum_{\nu} h(\nu)\, e(k-\nu)$$

and

$$w(k) = h(k) * f(k) \ = \ \sum_{\nu} h(\nu)\, f(k-\nu)$$

we get a unique expression for the following cross-cumulant sequence[3]

$$\begin{aligned} c_4^{xy}(\lambda_1, \lambda_2, \lambda_3) \\ = \ & E\{x^*(k)y^*(k+\lambda_1)x(k+\lambda_2)y(k+\lambda_3)\} \\ & -E\{x^*(k)y^*(k+\lambda_1)\}E\{x(k+\lambda_2)y(k+\lambda_3)\} \\ & -E\{x^*(k)x(k+\lambda_2)\}E\{y^*(k+\lambda_1)y(k+\lambda_3)\} \\ & -E\{x^*(k)y(k+\lambda_3)\}E\{y^*(k+\lambda_1)x(k+\lambda_2)\} \\ = \ & \gamma_4^d \sum_{\nu} s^*(\nu)w^*(\nu+\lambda_1)s(\nu+\lambda_2)w(\nu+\lambda_3), \quad (1) \end{aligned}$$

which leads to the specific expression

$$c_4^{xy}(0,0,0) = \gamma_4^d \sum_{\nu} |s(\nu)|^2\, |w(\nu)|^2. \qquad (2)$$

With a negligible influence of additive noise, the autocorrelation sequence can be expressed as

$$r_{xx}(\lambda) = E\{x^*(k)x(k+\lambda)\} = \sigma_d^2 \sum_{\nu} s^*(\nu)s(\nu+\lambda_1) \qquad (3)$$

and thus

$$r_{xx}(0) = \sigma_d^2 \sum_{\nu} |s(\nu)|^2. \qquad (4)$$

Equations (2) and (4) are related by the inequation

$$\sum_{\nu} |s(\nu)|^2\, |w(\nu)|^2 \leq max\{|w(\nu)|^2\} \sum_{\mu} |s(\mu)|^2. \quad (5)$$

Suppose the reference system $f(k)$ generates a single maximum value $|w(k_m)|^2$ – which is true for the overwhelming majority of all possible reference systems. Equality of both expressions in (5) will hold if and only if

$$s(k) = s(k_m)\, \delta(k-k_m). \qquad (6)$$

Apart from a factor this solution is equal to the desired combined impulse response of the equalizer and the transmission system. It is uniquely determined by the maximum value of the left-hand side of eq. (5) provided that the right-hand side can be considered constant. Hence, a quality criterion for blind equalization is to

$$\text{maximize } |c_4^{xy}(0,0,0)| \quad \text{provided that } r_{xx}(0) = \sigma_d^2. \qquad (7)$$

The only requirement for perfect deconvolution is that the impulse response $w(k)$ must contain a single maximum coefficient only.

[2] where "$*$" denotes the convolution operation

[3] x^* denotes the conjugate value of x

3 EIGENVECTOR APPROACH

An outstanding example of a reference system is $f(k) = e(k)$ leading to $y(k) = x(k)$. The resulting quality criterion for blind deconvolution has been proposed by Shalvi and Weinstein [4]. With a proper adjustment of the equalizer, a single maximum value of $|w(k)|^2 = |s(k)|^2$ is guaranteed. Unfortunately, this criterion depends, in a non-quadratic way, on the equalizer coefficients. This requires some kind of stochastic gradient algorithm to be used. The shortcomings of these algorithms are two-fold: first, the resulting convergence speed strongly depends on a step-size parameter; secondly, gradient noise is inevitable.

As opposed to this approach, a closed form solution of eq. (7) can be found. This is possible because both the autocorrelation coefficient $r_{xx}(0)$ and the cross-cumulant $c_4^{xy}(0,0,0)$ are quadratic in terms of the coefficients of a FIR equalizer:

$$r_{xx}(0) = \mathbf{e}^*\, \mathbf{R}_{vv}\, \mathbf{e} \qquad (8)$$

$$c_4^{xy}(0,0,0) = \mathbf{e}^*\, \mathbf{C}_4^{yv}\, \mathbf{e} \qquad (9)$$

where $\mathbf{e}$, $\mathbf{R}_{vv}$ and $\mathbf{C}_4^{yv}$ are defined as [4]

$$\mathbf{e}^* = [\ e^*(0) \ldots e^*(q)\ ]\,,$$

$$\mathbf{R}_{vv} = \begin{bmatrix} r_{vv}(0) & r_{vv}^*(1) & \ldots & r_{vv}^*(q) \\ r_{vv}(1) & r_{vv}(0) & \ldots & r_{vv}^*(q-1) \\ \vdots & \vdots & & \vdots \\ r_{vv}(q) & r_{vv}(q-1) & \ldots & r_{vv}(0) \end{bmatrix}$$

and

$$\mathbf{C}_4^{yv} = \begin{bmatrix} c_4^{yv}(0,0,0) & [c_4^{yv}(-1,0,0)]^* & \ldots & [c_4^{yv}(-q,0,0)]^* \\ c_4^{yv}(-1,0,0) & c_4^{yv}(-1,0,-1) & \ldots & [c_4^{yv}(-q,0,-1)]^* \\ \vdots & \vdots & & \vdots \\ c_4^{yv}(-q,0,0) & c_4^{yv}(-q,0,-1) & \ldots & c_4^{yv}(-q,0,-q) \end{bmatrix}$$

Consequently, eq. (7) may be rewritten as to

$$\text{maximize } |\mathbf{e}^*\, \mathbf{C}_4^{yv}\, \mathbf{e}| \quad \text{provided that } \mathbf{e}^*\, \mathbf{R}_{vv}\, \mathbf{e} = \sigma_d^2. \qquad (10)$$

The solution to this problem is the eigenvector of

$$\mathbf{C}_4^{yv}\, \mathbf{e} = \lambda\, \mathbf{R}_{vv}\, \mathbf{e} \qquad (11)$$

corresponding to the eigenvalue with maximum magnitude, called λ_{max}. Moreover, a gain control algorithm guaranteeing

$$\mathbf{e}^*\, \mathbf{R}_{vv}\, \mathbf{e} = \sigma_d^2 \qquad (12)$$

has to be applied. From now on, eq. (11) will be termed *eigenvector approach* (EVA). It should be emphasized that, in (11), neither the autocorrelation matrix

[4] q denotes the order of the equalizer
$\mathbf{e}^*$ is a transposed vector with conjugate elements

$\mathbf{R}_{vv}$ nor the cross-cumulant matrix $\mathbf{C}_4^{yv}$ uses the output data of the equalizer. Therefore it is possible to use a two-step algorithm: After the estimation of $\mathbf{R}_{vv}$ and $\mathbf{C}_4^{yv}$ the generalized eigenvector problem (11) is solved. In effect, the equalizer coefficients are estimated on the basis of statistical properties of the received signal.

This method is quite similar to the MSE-solution, where the elements of both the autocorrelation matrix $\mathbf{R}_{vv}$ and a cross correlation vector $\mathbf{r}_{dv}$ are computed prior to the solution of a set of equations.

4 ADAPTIVE IMPLEMENTATION

Up to now it can be stated that, under ideal conditions (i.e. infinite length of the equalizer impulse response and absence of additive noise), the only requirement for perfect equalization is that $w(k)$ must show a single maximum value $|w(k_m)|^2$. This does not affect the other values of $|w(k)|^2$, $k \neq k_m$. Yet, the influence of these "other values" increases as conditions become non-ideal. In this case some properties of EVA can be derived:

- *The equalizer coefficients may be rewritten as a weighted sum of MSE-solutions.* An expansion of $\mathbf{C}_4^{yv}$ in (11) leads to the expression (see [2] for the derivation)

$$\mathbf{e} \;=\; \frac{\gamma_4^d}{\lambda_{max}\sigma_d^2} \sum_k |w(k)|^2 s(k) \mathbf{e}_{MSE_k} \quad (13)$$

  where $\mathbf{e}_{MSE_k}$ describes the well-known MSE-solution

$$\mathbf{e}_{MSE_k} = \sigma_d^2 \mathbf{R}_{vv}^{-1} \mathbf{h}_k$$

  and $\mathbf{h}_k$ denotes the k-th subset of the channel impulse response

$$\mathbf{h}_k = [h(k) \ldots h(k-q)]^*.$$

  Note that the EVA solution corresponds to the MSE solution for

$$|w(k)|^2 \cdot s(k) = \delta(k - k_m) \quad (14)$$

  If, however, eq. (14) does not hold, EVA computes a weighted sum of MSE solutions according to (13). Nevertheless, a good approximation of the MSE solution is achieved in many cases, even if high values of $|w(k)|$ for $k \neq k_m$ result in a relatively high departure from the corresponding coefficient of $s_{EVA}(k) = h(k) * e_{EVA}(k)$. This behavior is demonstrated in fig. 2. The underlying channel transfer function is

$$\begin{aligned} H(z) = {} & 0.1\,(2 + j + (2-3j)z^{-1} - (2-5j)z^{-2} \\ & +(9-2j)z^{-3} - (5-8j)z^{-4} + (4+2j)z^{-5} \\ & +(1-5j)z^{-6} + 2jz^{-7}), \end{aligned} \quad (15)$$

  and the reference system is $f(k) = \delta(k - k_0)$. Obviously, this is a unfavorably conditioned reference system as $|w(3)| \approx |w(4)|$. True cumulants are assumed. In fig. 2a a small order equalizer ($q = 8$) prevents proper EVA equalization. Thus the difference between the MSE and the EVA equalizer is relatively high. This is particularly true for $|s(7)|$ which is related to $|w(3)|$. An increase in the number of equalizer coefficients ($q = 32$) permits EVA to achieve quasi-MSE equalization, as shown in fig. 2b.

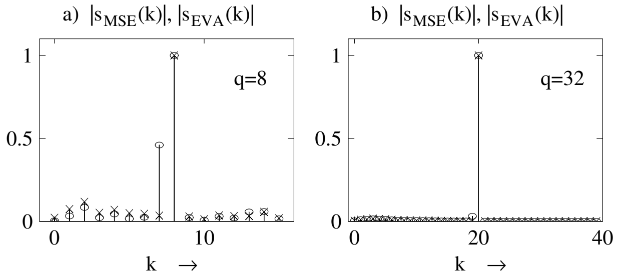

Figure 2: Deconvolution for different equalizer orders
$\circ \equiv$ EVA-solution $\quad \times \equiv$ MSE-solution

- *The EVA solution $s_{EVA}(k) = h(k) * e_{EVA}(k)$ approximates a linear phase system, if $w(k)$ is symmetric with respect to $k = q/2$ and a sufficiently high equalizer order q has been assumed.*

- *The convergence speed of the EVA solution strongly depends on the reference system (cf. fig. 3).* Its impact will become obvious when considering the channel (15) with two different reference systems leading to $w_1(k) = h(k)$ and $w_2(k) = \delta(k - k_m)$. While, in the former case, EVA does not converge properly during some initial iterations this period vanishes in the latter case.

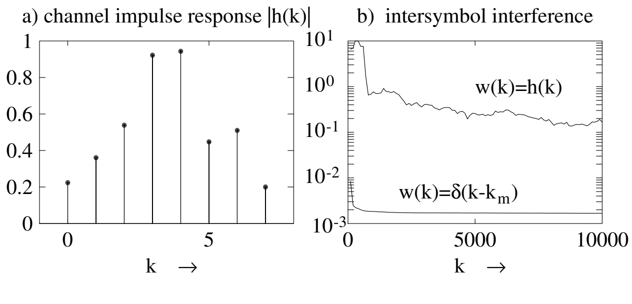

Figure 3: EVA convergence properties using different reference systems

The three items mentioned above demonstrate that optimum convergence at a linear phase total impulse response of channel and equalizer is obtained if the output signal of the reference system is equal to the input sequence sent. However, this knowledge of the input signal does not comply with the aim of blind(!) channel equalization.

Instead, the adaptive algorithm proposed here uses an arbitrary initial reference system $f_0(k)$ and solves EVA for the equalizer coefficients $e_0(k)$. This equalizer represents the inverse channel transfer function better than the initial reference system does. Therefore, the reference system is updated with the equalizer coefficients: $f_1(k) = e_0(k)$. Solving EVA again with $f_1(k)$ delivers a considerably improved equalizer $e_1(k)$.

Notice that the order q_0 of $f_0(k)$ may be inferior to q, as zeros can be added iteratively to the calculated equalizer impulse responses. The reason for this is that $q_0 < q$ reduces the number of parameters to be estimated, which in turn allows for quicker convergence even with short input time series. In summary, the algorithm comprises the following steps:

- calculate a suboptimum equalizer impulse response $e_0(k)$ according to (11) on the basis of a given block of the received data sequence (with the initial reference system set to $f_0(k) = \delta(k - k_0)$)
- load the reference system with this impulse response: $f_1(k) = e_0(k)$
- calculate a new equalizer impulse response $e_1(k)$ on the basis of the same block of the received data sequence; update the reference system once more: $f_2(k) = e_1(k)$
- repeat this procedure iteratively.

The properties of this *adaptive* EVA will be demonstrated in the sequel. The input signal is supposed to be a QPSK-signal comprising 1000 samples, the channel transfer function is as in (15), and the initial reference system shall be chosen to be $f(k) = \delta(k - k_0)$. Fig. 4a shows the initial equalization $s_0(k) = e_0(k) * h(k)$ achieved by EVA. The largest error appears at $|s_0(19)|$ corresponding to $|h(3)|$. Nevertheless, the system $s_1(k)|w_1(k)|^2$ represents a better approximation

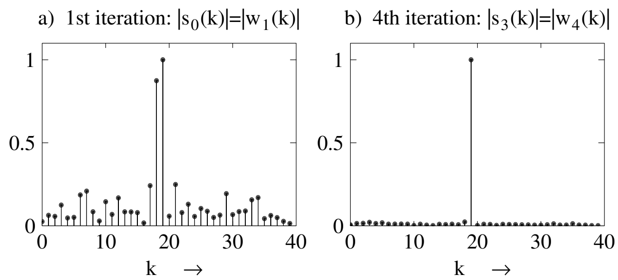

Figure 4: Performance of the iterative EVA

to $\delta(k - k_m)$ than does $s_0(k)|w_0(k)|^2$. This ensures a better approximation of an MSE-solution in the sense of eq. (14). As shown in fig. 4b, three further iteration steps result in a nearly perfect equalization.

Fig. 5 depicts ISI in terms of the length L of the received data block, for different modulation signals and algorithms. With EVA, the initial order 8 of the reference system was applied to the first 4 iterations, whereas the following 4 iterations assume a reference system of order 32. It becomes obvious that EVA approximates the convergence of RLS in the case of QPSK modulation. This property is generally true for input signals with constant magnitude. Different input signals do not reach RLS convergence speed (cf. fig. 5: 16QAM).

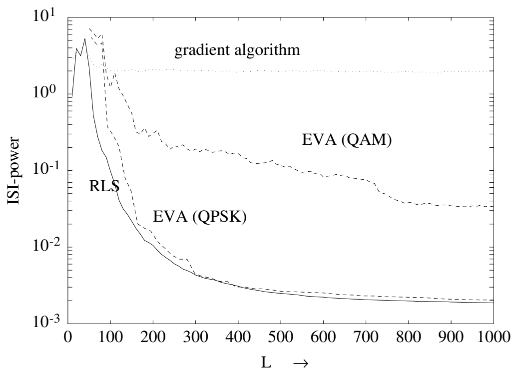

Figure 5: Convergence in terms of block length

Nonetheless, convergence speed would be sufficient to recover satisfactorily the QAM input sequence with as few as 1000 data values (cf. fig. 6a). Performance is enhanced remarkably in the case of a QPSK signal, though (cf. fig. 6b).

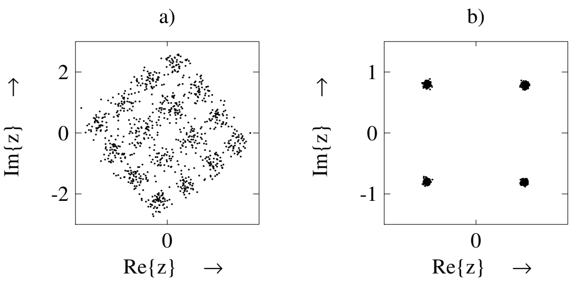

Figure 6: EVA equalization of different modulation signals

5 EVA for fractionally spaced equalizers

Application of EVA is not restricted to linear equalizers operating at symbol rate. Generalization to include areas such as system identification, decision-feedback equalization or fractionally spaced equalization can be easily performed. The crucial prerequisite is a symbol rate channel model permitting the approximation of $\delta(k-k_m)$ for the combined channel/equalizer impulse response $s(k)$.

Fig. 7 shows the system configuration for the example of a fractionally spaced equalizer. In comparison with fig. 2, the only difference is that the channel, the equalizer and the reference system all operate at an integer multiple M of the symbol rate.

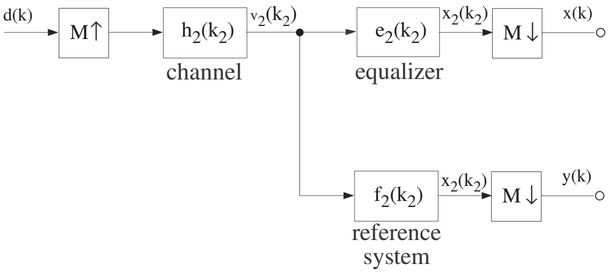

Figure 7: Fractional tap spacing equalizer

The desired relation can be concluded directly from the usual aim of equalization ($x(k)=\delta(k-k_m)$):

$$\begin{aligned} x(k) &= x(k_2)|_{k_2=Mk} = v_2(k_2)*e_2(k_2)|_{k_2=Mk} \\ &= \sum_{\mu=0}^{M-1}\sum_{\nu_\mu} v_2(M\nu_\mu+\mu)e_2(Mk-M\nu_\mu-\mu) \\ &= \sum_{\mu=0}^{M-1} v_\mu(k)*e_\mu(k), \end{aligned}$$

where $v_\mu(k)$ emerges from $d(k)$ by linear filtering:

$$v_\mu(k)=h_2(Mk+\mu)*d(k).$$

Thus $x(k)$ can be regarded as a linear combination of output signals of various subsystems operating at symbol rate. Now, the resulting total impulse response has to be equalized:

$$s(k)=\sum_{\mu=0}^{M-1} h_2(Mk+\mu)*e_\mu(k)=\delta(k-k_m)$$

Since there is an equivalent linear combination for the reference system, we can keep the previously derived equations (2) and (4). Inequation (5) remains valid and so does the cross-cumulant quality criterion (7). The derivations of EVA and its adaptive version are also analoguous to the above sections 3 and 4. The single difference is that $\mathbf{C}_4^{yv}$ and $\mathbf{R}_{vv}$ now link the output signals of several subsystems instead of one system only.

Sampling at M times the symbol rate often leads to an enhanced equalization performance, because aliasing components in the equalizer input signal are suppressed. Fig. 8b displays ISI in terms of blocklength, both for single and double symbol rate sampling. The channel impulse response is given in fig. 8a. In both cases the equalizer order is increased iteratively in steps of 4 to the final order of 32. For each value, 4 EVA iterations are executed. From the figure, we can see that the double symbol rate sampling results in an ISI inferior to the one for the single symbol rate sampling, as soon as a certain threshold block length is exceeded.

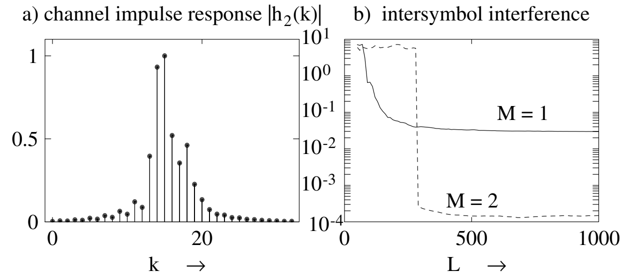

Figure 8: EVA equalization using different equalizers

6 CONCLUSION

An alternative approach to blind equalization was suggested in this paper. It is based on a particular structure of systems including a linear FIR equalizer and a reference system. Founded on this structure, it has been shown that there is a closed form solution of the equalization problem. The initial choice of the reference system determines both the speed of convergence and the minimum ISI to be achieved by the closed form solution. Optimum properties are obtained for a reference system approximating the inverse channel transfer function. This was attempted by the adaptive EVA. For constant magnitude input signals, convergence speed approaches the one of the RLS algorithm. Generalization was performed for a fractionally spaced equalizer.

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