A Blind Adaptive Equalizer Based on a Lattice/All-pass Configuration

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Networks for the blind channel equalization can be designed as a combination of a prewhitening filter (adjusted by means of 2nd-order statistics) and an inverse all-pass system (adjusted by 4th-order statistics). In this paper an appropriate FIR approximation of inverse all-pass systems is presented. On the other hand the inverse all-pass modelling can be avoided if the minimum phase prewhitening filter is replaced by a lattice configuration where the maximum phase (backward prediction error) output is used for decorrelation. In that case conventional recursive all-pass networks can be applied for phase correction. It will be shown that the convergence speed can be considerably increased due to the reduced number of parameters to be identified. Under additive noise conditions a solution near the optimum MSE-solution is obtained.

1 Introduction

Blind Equalization, i.e. adaptive channel correction without use of a trainings sequence or decided data is a new field of research since the recent few years. It is well-known that the traditional 2nd order statistics are "phase blind" in the sense that the transmission path is always interpreted as a pure minimum-phase system (or pure maximum phase, respectively) if only the received signal is used for channel estimation. Consider as an example the classical application of a prediction-error filter as a prewhitening system. The design of this filter is based on the autocorrelation samples of the received signal – the result is a minimum-phase inverse system model.

The fundamental idea to perform a blind adaptive equalization including the phase characteristics is the application of *higher order statistics*. There exist two fundamental problems: 1st channel identification (see [1],[2],[3]) and 2nd inverse channel estimation (see e.g. [4],[5]). The latter problem will be regarded in the present paper.

2 FIR-modelling of inverse systems

The basic idea for the design af a blind equalizer is to make use of all the information that can be derived from 2nd order statistics. Consider, for example, a complex-valued Moving-Average (MA-) channel model with a z-plane zero configuration shown in fig. 1a (mixed phase). If a FIR prediction-error filter is applied pole approximations (denoted as $\otimes$) are introduced as shown in fig. 1b. The resulting pole-zero configuration of the system composed of the channel and the prediction-error filter is demonstrated in fig. 1c. Obviously the resulting system contains a number of all-pass pole/zero pairs which are related to the non-minimum phase channel zeros:

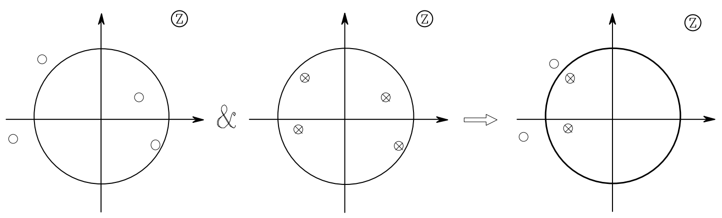


Figure 1: Pole-zero configuration under prewhitening conditions
$\otimes$ approximated pole; $\circ$ zero

A proper equalization requires the additional application of inverse all-pass systems after the prewhitening filter which has to be designed on the basis of higher order statistics. Before the discussion of an appropriate blind adaptive algorithm let us at first discuss the problem of the FIR approximation of poles which is necessary for the prewhitening filter design as well as for inverse all-pass modelling. Consider the 1st order transfer function

$$H_{IIR}(z) = \frac{z}{z - z_\infty}; \qquad |z_\infty| > 1 \tag{1}$$

under the specific condition of the pole outside the unit circle. Eq. (1) can be described by the following series expansion

$$H_{IIR}(z) = -\sum_{k=1}^{\infty} z_\infty^{-k} z^k \tag{2}$$

which converges, since $|z_\infty|^{-1} < 1$. Thus the system can be interpretated as *causal and non-stable* or *non-causal and stable*, equivalently. In the 2nd case we get the impulse response

$$h_{IIR}(k) = z_\infty^k, \qquad k = -1, -2, \ldots \tag{3}$$

which can be approximated by a causal FIR system by time truncation and time shift. This causal impulse response (length ℓ) can be written as

$$h_{FIR}(k) = h_{IIR}(k - \ell) - z_\infty^{-\ell} h_{IIR}(k). \tag{4}$$

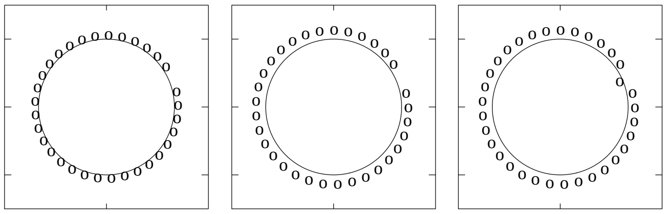
After the z-transform we obtain the transfer function of the approximated pole

$$H_{FIR}(z) = H_{IIR}(z) \ \left[z^{-\ell} - z_\infty^{-\ell}\right], \tag{5}$$

the second term describes ℓ zeros in the z-plane, equidistantly spaced on a circle with the radius $|z_\infty|$. One of these zeros (located at the position of the approximated pole) is canceled by the term $H_{IIR}(z)$. The result of the pole approximation is shown in fig. 2a. This form of approximation is based on a rectangular windowing of the true impulse response. For comparison fig. 2b shows the result of a Least-Squares- (LS-) solution where the zeros are removed from their positions on a circle. Finally fig. 2c shows the FIR approximation of an inverse all-pass system based on a LS design.

3 Phase Correction Networks

As discussed above a proper equalization is obtained by the combination of a prediction error filter and a phase correction FIR-filter. Fig. 3 shows this configuration. In [5] the phase



a) windowing b) LS-solution c) inverse all-pass

Figure 2: Approximation of a pole outside the unit circle
$\ell = 32$

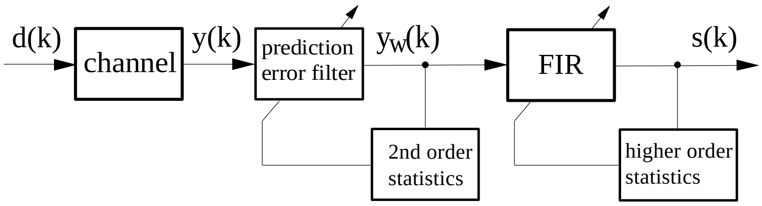


Figure 3: Equalizer with prewhitening filter and phase correction network

correction network was realized by a conventional FIR transversal filter. In the present paper we intend to make use of the specific a-priori knowledge, that the phase correction networks is composed of at most n complex-valued 1st order (inverse) all-pass systems, where n denotes the order of the (MA-) channel. For low-order channels this leads to a great advantage with regard to the adaptive adjustment, since the convergence speed strongly depends on the number of parameter to be adapted: In many applications the approximation degree ℓ of the phase correction network is much higher than the channel order n.

As far as regarded the phase correction network requires the realization of inverse all-pass systems which is possible in the form of the FIR approximation discussed in the last section. On the other hand the inverse all-pass phase correction is a consequence of the minimum-phase prediction-error filter. In contrast we can apply the well-known *lattice prewhitening filter* and use the *backward predictor output* which shows *maximum phase characteristics*. The consequences are illustrated by fig. 4. The phase correction filter can be designed as a cascade of 1st order all-pass systems (instead of inverse) that can be realized by a recursive network with considerably reduced hardware complexity. Fig. 5 shows the resulting equalizer configuration.

4 Adaptive blind equalization

The adaptive design of the prewhitening filter is solved by the power minimization of the forward (and backward) prediction error; of course this is a blind algorithm. A proper phase correction can only be performed under the application of higher order statistics. In [5] a very simple cost function is derived which is based on the specific definition of 4th order cumulants c_4^s:

$$c_4^s = E\{|s(k)|^4\} - 2\left[E\left\{|s(k)|^2\right\}\right]^2 - |E\{s^2(k)\}|^2 \overset{!}{=} max \qquad \text{if} \qquad E\left\{|s(k)|^2\right\} = const; \quad (6)$$

where $s(k)$ denotes the complex envelope of the equalizer output signal and $E\{\cdot\}$ means the expectancy value. The criterion (6) is closely related to the self recovering equalizer introduced

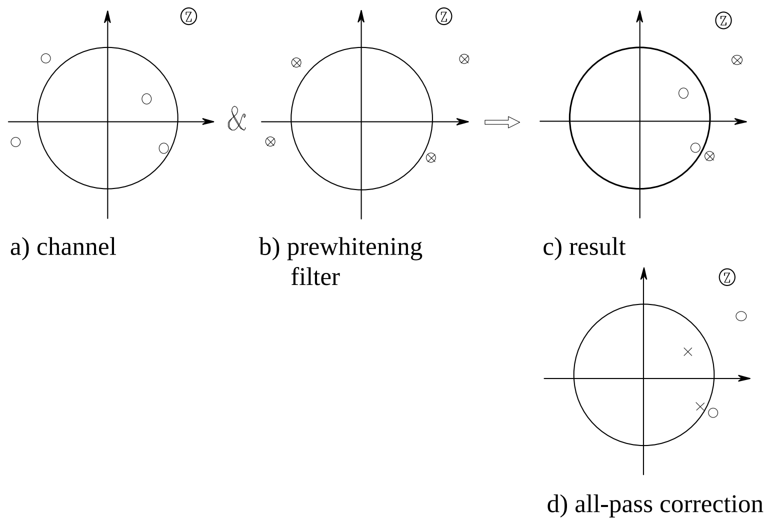


Figure 4: All-pass channel correction using a maximum phase prewhitening filter

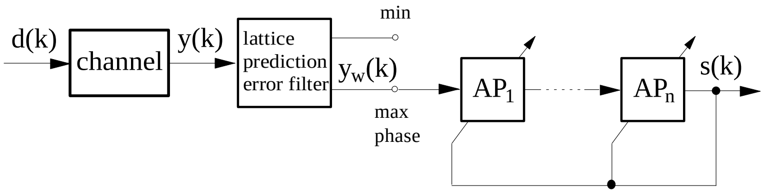


Figure 5: Lattice-all-pass equalizer

by Godard [4].

The cumulant criterion is used in [5] for the adaptive adjustment of the ℓ coefficients of the FIR phase correction filter; in the present paper it is applied for the calculation of the n poles z_{∞_ν} (and the related zeros) of the all-pass network. For the iterative solution the well-known stochastic gradient algorithm is applied. After some steps of calculation we obtain the following update equation:

$$z_{\infty_\nu}(i+1) = z_{\infty_\nu}(i) - \delta \cdot sgn(c_4^d)|s(k)|^2 \left[s^*(k)\frac{\partial s(k)}{\partial z_{\infty_\nu}} + s(k)\frac{\partial s^*(k)}{\partial z_{\infty_\nu}} \right] \quad (7)$$

where δ is a small positive constant value and c_4^d is the 4th-order cumulant of the input data. The symbol $*$ denotes the complex value. In (7) the derivative $\partial s(k)/\partial z_{\infty_\nu}$ is still unknown. The calculation can be carried out in the z-plane (provided that transient effects are suffciently small). Finally we obtain the gradient network shown in fig. 6. The main disadvantage of the adaptive allpass structure explained so far is the high nonlinearity of the cost function (due to the cascade realization of the phase correction filter). This results in secondary minima and thus in convergence problems under unfavourable start conditions. As a more intuitive approach we use the minimum phase solution (which can easily be derived from the lattice filter) as an initial position. Under these conditions a proper adaptive equalization is achieved for sufficiently low channel order. Fig. 7 shows an example for the adaptive equalization of a 4th order mixed phase channel (a). Fig. 7b demonstrates the trajectories of the all-pass poles during the adaption process: Obviously the poles related to the minimum -phase channel remain at their initial positions while the poles related to the maximum-phase channel are removed on the unit circle or into the origin, respectively, where their influence vanishes. For conclusion the convergence behaviour of the lattice/all-pass equalizer is compared to the FIR-solution [5]. As an example we

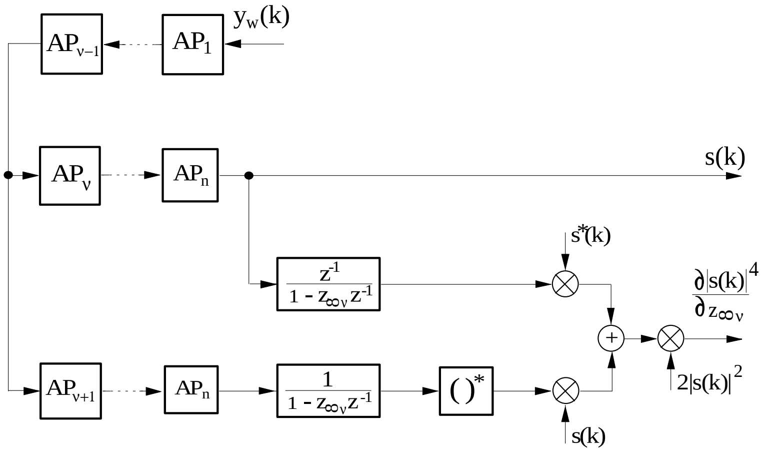
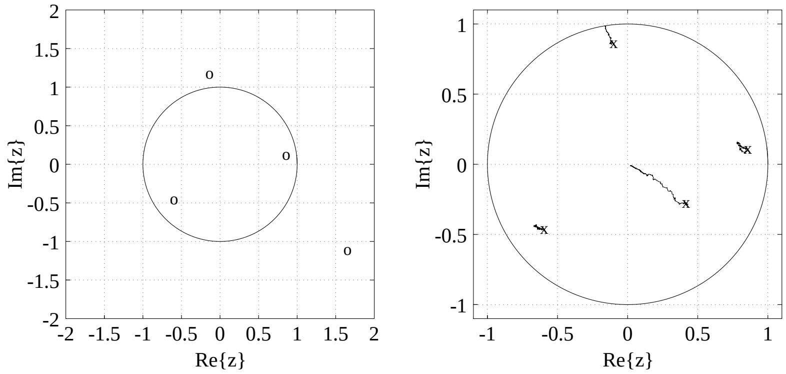


Figure 6: Gradient network for the calculation of $\partial|s(k)|^4/\partial z_{\infty_\nu}$



a) ○ channel zeros

b) × all-pass poles, initial position

Figure 7: Demonstration of the adaptive process

pick the channel characteristics according to fig. 7a. Fig. 8 shows the power of the intersymbol interference (ISI) during the adaption process for different values of δ. It can be seen that the convergence speed of the all-pass structure is superior to the FIR solution due to the small number of parameters, ($n = 4$), in comparision to the $\ell = 32$ FIR filter coefficients.

5 Equalizer performance under additive gaussian noise

In this section the influence of additive noise is considered. It is well-known that the 2nd-order autocorrelation samples are corrupted by additive noise whereas the 4th-order cumulants are insensitive as far as the noise is gaussian. The 1st effect results in a misadjustment of the prewhitening filter; this is illustrated in fig. 9a. Fig. 9b indicates that a proper equalization is not achieved in presence of additive noise. On the other hand an exact modelling of the inverse system (which is related to the so-called "zero-forcing" solution) is not desirable because both – the residual ISI and the power of the additive noise at the equalizer output – have to taken into account. An optimum solution in this sense is the well-known "mean-square error" (MSE) solution. The following examples illustrate that the blind adaptive design of the lattice/all-pass equalizer reveals a solution near the MSE. Fig. 10a shows the resuting power of the error at

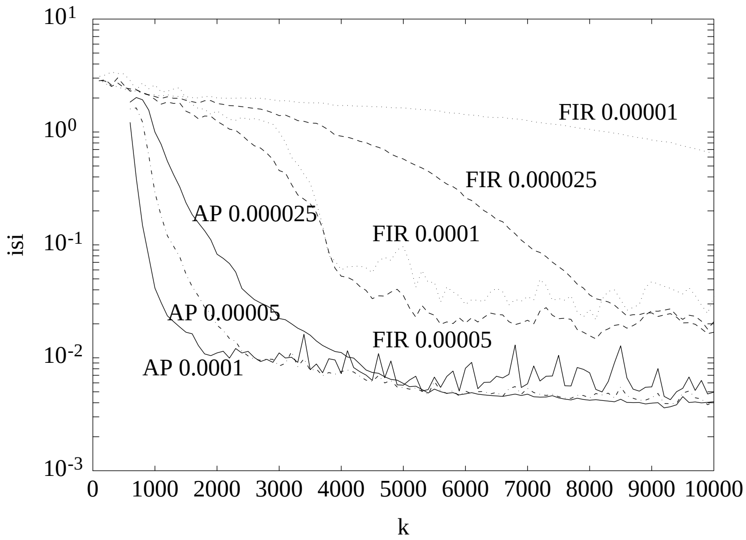


Figure 8: Performance of different blind equalizers (Parameter $\equiv \delta$)

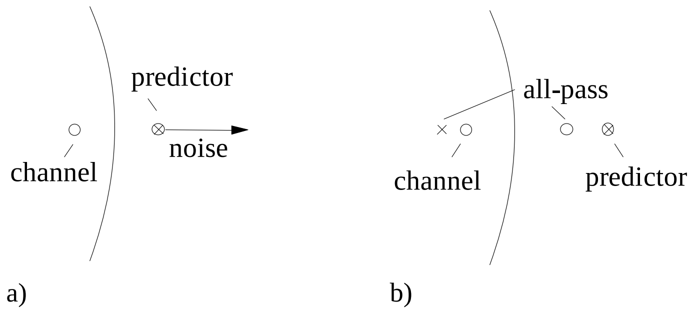


Figure 9: Pole-zero conditions under noise conditions

the equalizer output for zero-forcing, lattice/all-pass and MSE equalization. Fig. 10b gives the related bit error rate as a function of the signal-to-noise ratio. As an example a 1st order channel was used with a zero of $z_0 = 0.9$.

Conclusions

In this paper it was shown that the blind channel equalization problem can be solved efficiently by a certain lattice/all-pass configuration. If the maximum-phase lattice output is used for decorrelation the phase correction network can be designed as a conventional recursive all-pass system. For the blind adaptation a 4th-order scalar cumulant criterion is applied. It was shown that the convergence speed is considerably increased due to the small number of parameters to be identified (all-pass poles). Some simulation results demonstrate that under additive noise conditions a solution near the optimum MSE-solution is obtained.

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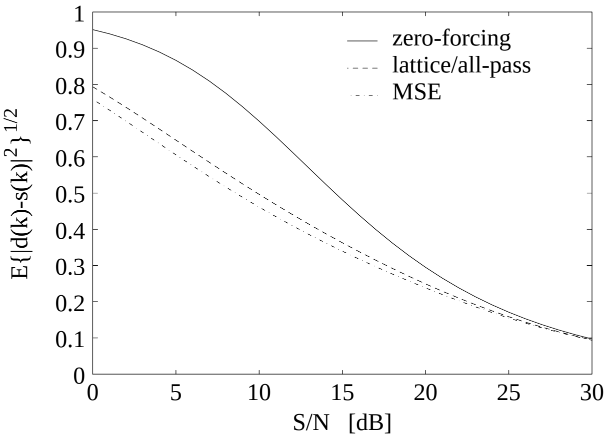
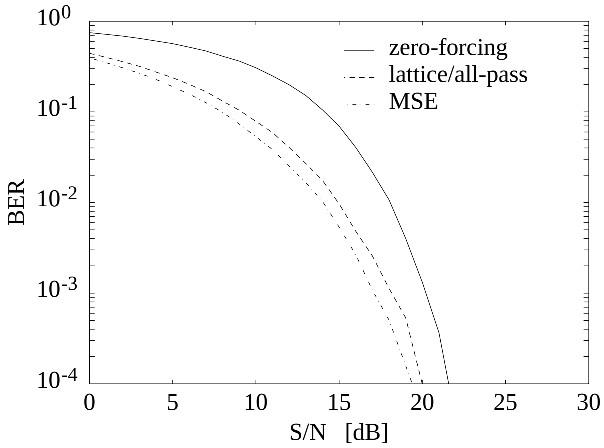


a) power of resulting error



b) bit error rate

Figure 10: Equalizer performance under gaussian noise

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