Implementation of Polyphase Filterbanks for (Flexible) Multicarrier Systems

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Abstract — **This article will emphasize on how to efficiently implement multicarrier (MC-) systems different from and alternative to OFDM with polyphase filterbanks. It will show that when using polyphase filterbanks this is an incomplex and flexible solution. It will be derived that with polyphase filterbanks an additional MC-system design parameter exists; not only spacings between adjacent subcarriers in integer multiples of the inverse MC-symbol duration are possible but that also *non-integer* multiples of the inverse MC-symbol duration are easily realizable.**

I. INTRODUCTION

With the growing market in mobile communications multicarrier schemes are receiving an increasing amount of attention; mainly because of their capability to handle frequency selective (i.e. mobile radio) channels better than ordinary single carrier systems can.

Figure 1 shows the structure of a MC-system. In the transmitter following a serial to parallel conversion the data symbols are upsampled; generally by the factor $w = N$, with N the number of subcarriers. Afterwards the data is filtered and modulated with the subcarrier's frequency. The values of all subcarriers are then summed up and transmitted as one symbol. The functionality of the receiver is inverse to the one of the transmitter. What distinguishes the different types of MC-systems is the filtering g_k and h_k.

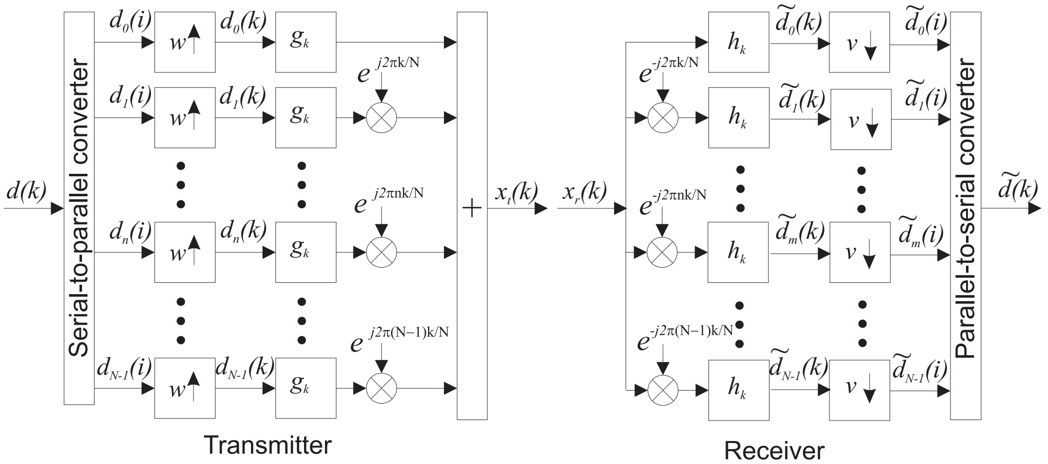

Fig. 1: General structure of MC-system

The multicarrier system of main interest, OFDM, uses rectangular impulses for g_k and h_k. For its implementation the complex structures as shown in Figure 1 can be replaced by an IFFT-unit for the transmitter and an FFT-unit for the receiver. The subcarrier spacing for OFDM is $\Delta F = 1/T_s$ (with ΔF the subcarrier spacing and T_s the MC-symbol duration) unless it is decided to transmit e.g. on only half of the carriers. Nevertheless OFDM always requires that $\Delta F T_s \in \mathbb{N}$. To avoid intersymbol interference an OFDM system generally also comprises the insertion and removal of a guard interval, which increases the OFDM symbol duration to $T_{OFDM} = T_s + T_g$. As no new information is transmitted during the guard interval the bandwidth efficiency is reduced with its insertion.

MC-systems with different impulse shapings for g_k and h_k have been introduced. Some compensate disadvantages of OFDM; e.g.:

- The loss of bandwidth efficiency (e.g. [1, 2]).
- The large out of band radiation (e.g. [3, 4, 5]). Note that within OFDM-systems the symbols are often windowed before transmission to overcome this problem. As will be shown later in Section II.A, this windowing is very similar to using a different impulse shape in the first place.
- Its sensitivity to frequency shifts (e.g. [6, 7]),
- The non-exploiting of channel diversity when channel coding is not applied (e.g. [8, 9]). Adding a CDMA component to the OFDM system is also a possibility to overcome this disadvantage.
- Its performance loss in strongly time variant environments (e.g. [10, 11]).

II. Implementation of Polyphase Filterbanks

A. Transmitter

The MC-transmission signal $x_t(k)$ according to Figure 1 is

$$x_t(k) = \sum_{n=0}^{N-1} \underbrace{\sum_{p=0}^{P-1} g_p\, d_n(k-p)\; e^{j2\pi\frac{kn}{N}}}_{x_c(k)} \qquad \text{with } \frac{P}{w} \in \mathbb{N} \tag{1}$$

and P the number of samples of the filter g_k (which can be several times the symbol duration T_s long). The fact that

$$d_n(k) = \begin{cases} d_n(i) & \text{for } k = iw \\ 0 & \text{else} \end{cases} \tag{2}$$

influences $x_c(k)$ (i.e. the convolution with g_k) as only every w^{th} term of $x_c(k)$ differs from 0. To include this in the calculation the index k can be replaced with $k = iw + \lambda$ with $\lambda = 0, \ldots, w-1$. This changes Equation (2) to

$$d_n(k) = d_n(iw+\lambda) = \begin{cases} d_n(i) & \text{for } \lambda = 0 \\ 0 & \text{else} \end{cases}. \tag{3}$$

If p is equally substituted [12, 13] by the polyfilter base $p = p'w + \lambda$ the convolution term $x_c(k)$ becomes

$$x_c(iw+\lambda) = \sum_{p'=0}^{P/w-1} g_{p'w+\lambda} d_n(iw+\lambda-p'w-\lambda) = \sum_{p'=0}^{P/w-1} g_\lambda(p') d_n(i-p'). \tag{4}$$

For the transmission signal of Equation (1) this leads to

$$\begin{aligned} x_t(iw+\lambda) &= \sum_{n=0}^{N-1} \sum_{p'=0}^{P/w-1} g_\lambda(p') d_n(i-p') e^{j2\pi\frac{n}{N}(iw+\lambda)} \\ &= \sum_{p'=0}^{P/w-1} g_\lambda(p') \underbrace{\sum_{n=0}^{N-1} d_n(i-p') e^{j2\pi\frac{n\lambda}{N}} \underbrace{e^{j2\pi\frac{inw}{N}}}_{\text{cyclic shift}}}_{\text{IFFT-base}} \\ &= N \sum_{p'=0}^{P/w-1} g_\lambda(p') \mathrm{IFFT}_N\{d_n(i-p')\}_{(iw+\lambda)\mathrm{mod}N}. \end{aligned} \tag{5}$$

With ideal subcarrierspacing $\Delta F T_s = w/N = 1.0$ the cyclic shift (represented in Equation (5) by the term "$(iw + \lambda)\mathrm{mod}N$") disappears and the implementation of the transmitter looks as shown in Figure 2.

Following the IFFT each N samples of one MC-symbol are multiplied with the respective N samples of the impulse response g_k. Note that when OFDM is implemented with windowing to suppress the large out of band radiation the transmitter structure looks *very* similar to the one shown in Figure 2 when only one branch is considered, i.e. when $P = N$ or resp. $P = N + G$ in the case of OFDM, with G the number of samples in the guard interval.

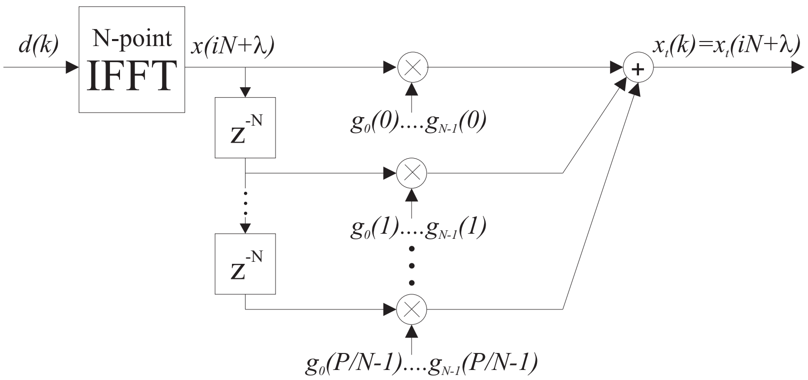

Fig. 2: Structure of polyphase filter MC-transmitter for $w = N$

The possibility to easily implement MC-systems with polyphase filterbanks also when $\Delta FT_s = w/N \notin \mathbb{N}$ (but with $w, N \in \mathbb{N}$) is one of the main advantages of polyphase filterbanks. It means that an additional parameter (the subcarrier spacing) can be exploited to optimally adapt the MC-system to its environment[1]. For OFDM (or any other orthogonal MC-system) $\Delta FT_s \in \mathbb{N}$ has to be maintained and the oversampling w cannot easily be changed.

For the implementation of $\Delta FT_s \notin \mathbb{N}$ the cyclic shift has now to be included in the polyphase filter transmitter as shown in Figure 3. The shift and demultiplexer blocks have the following functionality: Each N input signals $x(iN + \kappa)$ with $\kappa = 0, \ldots, N - 1$ of one MC-symbol i are mapped to w output signals $x(iw + \lambda)$ of the same symbol with $\kappa = (iw + \lambda)\text{mod}N$ and "mod" the modulo operation.

An example: The $\lambda = 4^{\text{th}}$ output sample $x(iw + \lambda)$ of the $i = 5^{\text{th}}$ MC-symbol, when $N = 16$ and $w = 17$, is the $\kappa = 9^{\text{th}}$ input signal $x(iN + \kappa)$ also of the $i = 5^{\text{th}}$ MC-symbol.

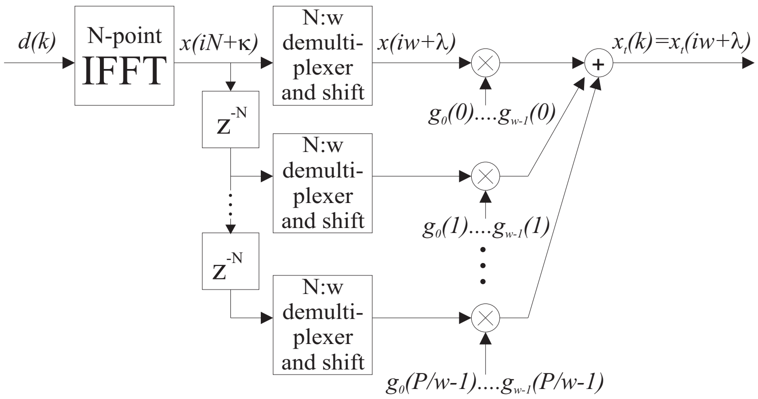

Fig. 3: Polyphase filterbank transmitter for multicarrier systems

B. Receiver

For the receiver the approach is similar to the transmitter one. The recieved signal on subcarrier m before downsampling is

$$\tilde{d}_m(k) = \sum_{q=0}^{Q-1} h_q \, x_r(k-q) \, e^{-j2\pi \frac{(k-q)m}{M}} \qquad \text{with } \frac{Q}{N} \in \mathbb{N} \tag{6}$$

and Q the number of samples of the filter h_k. With the downsampling only every v^{th} sample of the received signal is needed

$$\tilde{d}_m(i) = \tilde{d}_m(k = iv) = \sum_{q=0}^{Q-1} h_q \, x_r(iv - q) \, e^{-j2\pi \frac{(iv-q)m}{N}}. \tag{7}$$

[1]For a given mobile radio environment and symbol rate the main other parameter is the number of subcarriers N [14].

To enable also a less complex implementation a new polyfilter base [13, 12] is introduced which is now $q = Nq' - \lambda$ with $\lambda = 0, \ldots, N-1$. This leads for the received signal of Equation (7) to

$$\begin{aligned}
\tilde{d}_m(i) &= \sum_{\lambda=0}^{N-1} \sum_{q'=0}^{Q/N-1} h_{Nq'-\lambda} x_r(iv - Nq' + \lambda) e^{-j\frac{2\pi(iv-Nq'+\lambda)m}{N}} \\
&= \underbrace{e^{-j2\pi i \frac{vm}{N}}}_{\text{rotation}} \underbrace{\sum_{\lambda=0}^{N-1} \sum_{q'=0}^{Q/N-1} h_\lambda(q') x_r(iv - Nq' + \lambda) e^{-j2\pi\frac{\lambda m}{N}}}_{\text{FFT-base}} \\
&= e^{-j2\pi i \frac{mv}{N}} \mathrm{FFT}_N \left\{ \sum_{q'=0}^{Q/N-1} h_\lambda(q') x_r(iv - Nq' + \lambda) \right\} (m). \qquad (8)
\end{aligned}$$

In case of ideal subcarrier spacing $\Delta F T_s = v/N = 1.0$ the received signal is

$$\tilde{d}_m(i) = \mathrm{FFT}_N \left\{ \sum_{q'=0}^{Q/N-1} h_\lambda(q') x_r((i-q')N + \lambda) \right\} (m). \qquad (9)$$

and the implementation is very straight forward as shown in Figure 4. Here the received signal undergoes a similar sort of windowing procedure as it does in the transmitter (see Figure 2) before the FFT is performed.

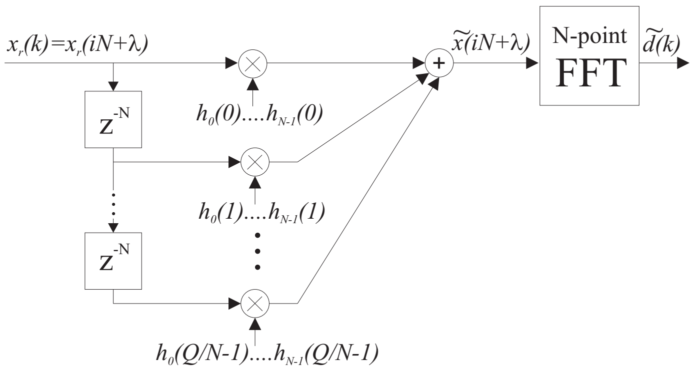

Fig. 4: Polyphase filterbank receiver for MC-systems when $\Delta F T_s = 1.0$

For $v \neq N$ the receiver structure is shown in Figure 5. In this block diagram the rotation with $e^{-j2\pi i \frac{mv}{N}}$ has been implemented as a cyclic shift before the FFT; it merely changes the order of the incoming signals. An incoming signal at the κ^{th} position within one MC-symbol $\tilde{x}(iN + \kappa)$ will thus have the position $\lambda = (iv + \kappa)\mathrm{mod}N$ in the output signal $\tilde{x}(iN + \lambda)$ of the cyclic shift block.

Nevertheless there are two not so obvious aspects to take care of. First, the filter values $h_n(i)$ of Figure 4 are *not* identical with the filter values $h_n(i)$ of Figure 5. In the case of $v \neq N$ there are v values for each symbol duration. This means that the sample $h_n(i)$ of Figure 5 corresponds to the $((iN + n)\mathrm{mod}v)^{\text{th}}$ sample of the $((iN + n)\mathrm{div}v)^{\text{th}}$ symbol duration of the filter[2]. Second, while after each N samples in the first branch $v - N$ samples are deleted these values are nonetheless transferred via the memories to the second branch.

An example: In the i^{th} timeslot the sequence that is (samplewise) multiplied with $(h_0(0), \ldots, h_{N-1}(0))$ in the first branch is $(x_r(iv), \ldots, x_r(iv + N - 1))$. The sequence that is multiplied with $(h_0(1), \ldots, h_{N-1}(1))$ in the second branch is $(x_r(iv - N), \ldots, x_r(iv - N + N - 1) = x_r(iv - 1) = x_r((i-1)v + v - 1)$. This last sample $x_r(iv - 1)$ e.g. has been deleted processed in the first branch (provided $v > N$) in timeslot $i - 1$ but is now further processed in the second branch. In the Q/N^{th} branch $(x_r(iv - (Q/N - 1)N), \ldots, x_r(iv - (Q/N - 1)N + N - 1))$ is samplewise multiplied with $(h_0(Q/N - 1), \ldots h_{N-1}(Q/N - 1))$.

[2]Missing values in the last branch are filled up with zeros.

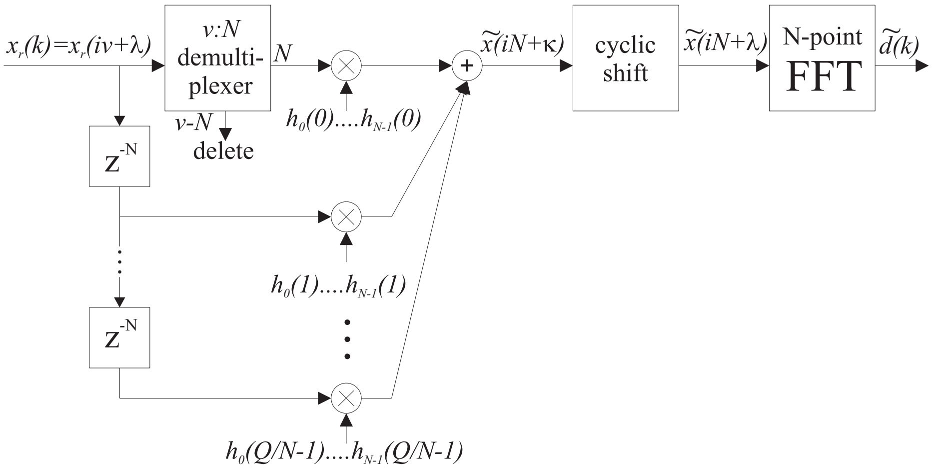

Fig. 5: Polyphase filterbank receiver for multicarrier systems

III. An Application Example

The flexible variation of the subcarrier spacing has been exploited when implementing a multicarrier system with Gaussian impulse responses for g_k and h_k (as done for MCSIS, multicarrier system with soft impulse shaping, [2, 12, 11]). The Gaussian impulses overlap in time and frequency direction (depending on the 3dB-bandwidth parameter). The thus caused intersymbol (ISI) and intercarrier (ICI) interferences influence the performance. In order to be able to apply nevertheless a one-dimensional channel equalization in either time or frequency direction only, some residual interference in the respective other direction has to be coped with. In case of residual interference in frequency direction, the enlargement of the subcarrier spacing can reduce this interference and ameliorate the performance of the system.

How significantly this improves the performance is shown in Figure 6. Even though the bandwidth efficiency is reduced when increasing the subcarrier spacing, a performance gain of more than $1dB$ can be achieved[3] at $BER = 10^{-2}$ (compare the solid and the dashed line).

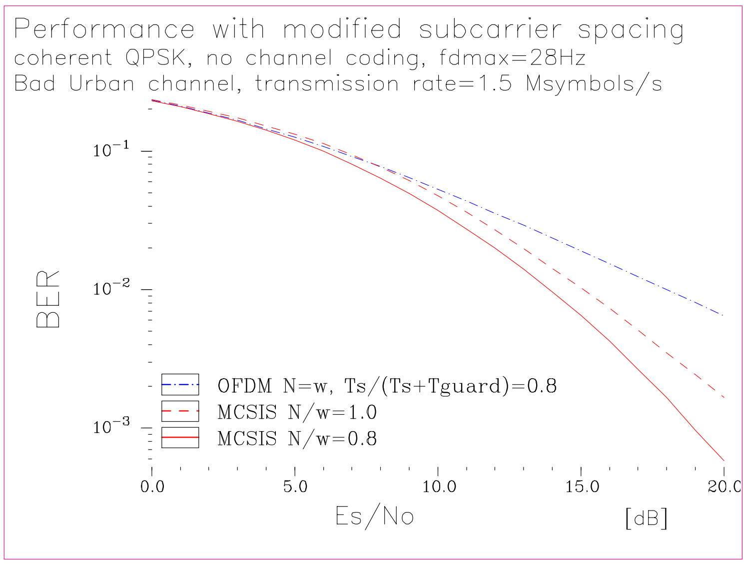

Fig. 6: Performance example for $w = v \neq N$

[3]The performance gain compared with OFDM is due to diversity effects [11].

IV. SUMMARY

It has been shown that also multicarrier systems with non-rectangular impulse shaping can be easily implemented with the help of polyphase filterbanks, whose complexity is comparable to the one of OFDM with windowing. Additionally polyphase filterbanks allow the variation of the subcarrier spacing with a fine granularity. Thus polyphase filterbanks provide an additional parameter to actively influence the interference situation and the performance of a multicarrier system.

V. REFERENCES

[1] B. Hirosaki, "An Orthogonally Multiplexed QAM System Using the Discrete Fourier Transform," *IEEE Trans. on Commun.*, vol. 29, July 1981.

[2] K. Kammeyer, U. Tuisel, H. Schulze, and H. Bochmann, "Digital Multicarrier-Transmission of Audio Signals Over Mobile Radio Channel," *European Trans. on Telecommunications (ETT)*, vol. 3, May-June 1992.

[3] A. Vahlin and N. Holte, "Optimal Finite Duration Pulses for OFDM," in *Proc. of GLOBECOM*, vol. 1, pp. 258–262, Nov. 1994.

[4] R. Haas and J.-C. Belfiore, "Multiple Carrier Transmission with Time-Frequency Well-Localized Impulses," in *Proc. 2nd IEEE Symposium on Communications and Vehicular Technology in the Benelux*, (Louvain-la-Neuve), pp. 187–193, Nov. 1994.

[5] V. Benedetto, G. D'Aria, and L. Scarabosio, "Performance of the COFDM Systems with Waveform Shaping," in *Proc. of ICC*, (Montreal), 1997.

[6] M. Gudmundson and P.-O. Anderson, "Adjacent Channel Interference in an OFDM System," in *Proc. of VTC*, (Atlanta), pp. 918–922, 1996.

[7] D. Dardari, "MCM Systems with Waveform Shaping in Multi-User Environments: Effects of Fading, Interference and Timing Errors," in *Proc. of GLOBECOM*, (Phoenix), 1997.

[8] K. Matheus and K.-D. Kammeyer, "Channel Equalization in Multicarrier Systems with Soft Impulse Shaping," in *ITG-Fachb.: Mobile Kommunikation, European Conf. on Personal Mobile Communication*, Sept. 1997.

[9] S. B. Slimane, "Performance of OFDM Systems with Time-limited Waveforms over Multipath Radio Channels," in *Proc. of GLOBECOM*, 1998.

[10] J. Armstrong, P. Grant, and G. Povey, "Polynomial Cancellation Coding of OFDM to Reduce Intercarrier Interference due to Doppler Spread," in *Proc. of GLOBECOM*, (Sydney), 1998.

[11] K. Matheus, *Generalized Coherent Multicarrier Systems for Mobile Communications*. PhD thesis, Dept. of Telecommunications, University of Bremen (FB-1), Shaker Verlag, Aachen, Germany, Jan. 1999.

[12] U. Tuisel, *Multiträgerkonzepte für die digitale, terrestrische Hörfunkübertragung*. PhD thesis, Hamburg University of Technology, May 1993.

[13] N. Fliege, *Multirate Digital Signal Processing*. Chichester, UK: John Wiley and Sons, 1994.

[14] H. Steendam and M. Moeneclaey, "Optimization of OFDM on Frequency-Selective Time-Selective Fading Channels," in *Proc. of ISSSE*, Sept. 1998.