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Time Truncation of Channel Impulse Responses by Linear Filtering: A Method to Reduce the Complexity of Viterbi Equalization

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Time Truncation of Channel Impulse Responses by Linear Filtering: A Method to Reduce the Complexity of Viterbi Equalization

The hardware complexity of optimum receivers based on Viterbi detection grows exponentially with the increase of the channel impulse response length. In the present paper methods for time truncation by means of linear filtering are discussed. A very simple approach derived from the optimum MMSE-solution to decision feedback equalization will be introduced which requires the calculation of a set of linear equations. It will be compared with Falconer's and Maggee's well-known eigenvector solution under the aspect of the performance of the Viterbi-detector. It will be shown by simulation results that the simple MMSE-solution is superior over the more complex eigenvector approach.

Lineare Filter zur zeitlichen Konzentration von Kanalimpulsantworten: Ein Verfahren zur Aufwandsreduktion bei Viterbi-Entzerrern

Der Realisierungsaufwand für einen optimalen Datenempfänger mit Viterbi-Detektion steigt exponentiell mit der Länge der Kanalimpulsantwort. In der vorliegenden Arbeit werden Methoden zur Verkürzung der Kanalimpulsantwort durch lineare Filter diskutiert. Es wird ein einfaches Verfahren von der optimalen MMSE-Lösung für Entzerrer mit quantisierter Rückführung abgeleitet, das die Berechnung eines linearen Gleichungssystems erfordert. Dieses wird mit der bekannten Eigenvektor-Lösung von Falconer und Magee unter dem Aspekt der Viterbi-Detektion verglichen. Anhand von Simulationsresultaten wird gezeigt, daß die einfache MMSE-Lösung gegenüber dem komplexeren Eigenvektor-Verfahren Vorteile aufweist.

1. Introduction

In 1972, Forney presented an optimum receiver structure for data transmission in presence of intersymbol interference [1]. This receiver consists of a matched filter (which takes the transmitter filter and the channel impulse response into account), a digital symbol-rate decorrelation filter for whitening the coloured noise, and finally a Viterbi-detector. The complete data transmission system can be described by the equivalent symbol-rate impulse response which includes all the filters mentioned [2]. The complexity of the receiver grows exponentially with the length of the equivalent symbol-rate impulse response (due to the exponential increase of Viterbi states). Consequently, in several papers suboptimum receivers with reduced complexity were investigated. For example, Falconer and Magee introduced a linear pre-filter to truncate the length of the equivalent symbol-rate impulse response [3]. In this approach the solution of an eigenvalue problem is necessary. In the present paper an alternative method will be presented which is based on the closed-form MMSE-solution (minimum mean-squared error) for nonlinear decision-feedback equalizers combined with a linear nonrecursive pre-filter (FIR-DF equalizer). This approach requires the solution of a set of linear equations. The coefficients of the FIR pre-filter represent directly the impulse response of the intended time-truncation filter whereas the decision feedback coefficients combined with an additional unit impulse describe the residual impulse response at the output of the pre-filter which is to be fed to the Viterbi-detection unit.

Both solutions are derived in Section 3; in Section 4 a comparison between these methods is carried out under the aspect of the Maximum-Likelihood Sequence Estimation (MLSE) error performance. For convenience, in the next section a brief analysis of the MLSE performance will be given which is necessary to understand the different properties of the eigenvector and the MMSE time truncation methods.

2. Analysis of Maximum-Likelihood Sequence Estimation

Fig. 1 shows the equivalent symbol-rate model of a data transmission system. The system part drawn in dashed lines should be neglected for the moment. The impulse response $f(i)$ – i means the symbol-rate time index – is calculated by convolution of the transmitter filter, channel, and receiver matched filter impulse responses, symbol-rate sampling and convolution with the symbol-rate decorrelation filter impulse response. The additive noise is white and gaussian [2]. The Viterbi-detector must be supplied with the impulse resonse $\hat{f}(i)$ which has to be estimated by means of certain training sequences or by the decided data, respectively. Due to the v-symbol delay of the Viterbi-detector the received signal $y(i)$ has to be delayed by

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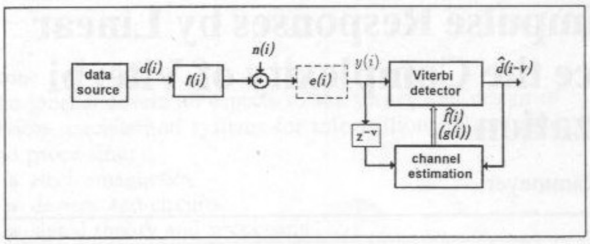

Fig. 1. Symbol-rate model of a data transmission system.

v symbols as well in order to get a correct channel estimate.

The receiver shown in Fig. 1 is optimum, i.e. the symbol error rate is minimum for any transmission channel actually given. On the other hand, the value of the symbol error rate depends on the specific channel impulse response. Thus an analysis of *worst-case channel configurations* is of particular interest. This problem has been investigated by several authors. In the present paper we follow the analysis given in [4].

During the Viterbi-detection an error event is characterized by a specific divergence between the estimated and the true path in the trellis diagram. The length of an error event is denoted as L. Then we get specific sequences of symbol errors which are decribed by corresponding *error vectors*

$$\mathbf{e} = [e_0, e_1, \cdots, e_{L-1}]^{\mathrm{T}} \quad (1)$$

where

$$e_\nu = \frac{1}{d_{\min}} \left(d(i_e + \nu) - \hat{d}(i_e + \nu) \right).$$

Here i_e describes the beginning of an error event; $d(i)$ and $\hat{d}(i)$ are the true and the decided data. Note that the minimum value of any error vector element is one due to the normalization by the minimum distance between any pair of data symbols

$$d_{\min} = \min\{|d_\nu - d_\mu|\}, \quad \nu \neq \mu.$$

The first and the last elements of any error vector are non-zero.

The error analysis given in [4] shows that the symbol error rate is characterized by the well-known errorfunction complement erfc$(\cdot)$; in case of M-ary PSK (phase shift keying) we get

$$P_s \approx K_{\gamma_{\min}} \operatorname{erfc}\left(\sqrt{\mathrm{ld}(M) \gamma_{\min}^2 \frac{E_b}{N_0}} \sin \frac{\pi}{M} \right), \quad (2)$$

where $K_{\gamma_{\min}}$ is a positive factor which is of minor interest, in contrast to the value $\gamma_{\min}$ in the argument of the erfc-function by which the E_b/N_0-ratio (E_b = symbol energy per bit, $N_0/2$ = noise power density) is reduced. The value of $\gamma_{\min}^2$ is called *SNR-loss*: if $\gamma_{\min}^2 < 1$ a large degradation of the Viterbi-detection may occur due to the extremely steep descent of the erfc-function. Thus the SNR-loss is a very important parameter to describe the symbol error performance of the Viterbi-detector; the SNR-loss is determined by

$$\gamma_{\min}^2 = \min_{\mathbf{e}}\{\mathbf{e}^*\mathbf{F}^*\mathbf{F}\mathbf{e}\} = \min_{\mathbf{e}}\{\mathbf{f}^*\mathbf{R}_{ee}\mathbf{f}\} \quad (3)$$

with the definitions

- $\mathbf{f} = [f_0, f_1, \ldots, f_m]^{\mathrm{T}}$, vector of equivalent symbol-rate impulse response,
- $\mathbf{F}$, convolution matrix corresponding to $\mathbf{f}$,
- $\mathbf{R}_{ee}$, energy autocorrelation matrix of error vector $\mathbf{e}$.

The asterisks denote the *transjugated* forms of vectors or matrices, respectively (i.e. transposed with conjugate complex elements).

Eq. (3) can be exploited to calculate the SNR-loss for a given impulse response vector $\mathbf{f}$: From a theoretical point of view all error vectors $\mathbf{e}$ possible have to be examined in order to find the minimum value of $\gamma_{\min}^2$. On the order hand under practical conditions the lengths of the worst case error vectors usually lie below a certain maximum value, say 4 or 5. Thus a pragmatic solution to determine the SNR-loss under a fixed given vector $\mathbf{f}$ is to restrict the maximum length of $\mathbf{e}$ to a certain limit and then examine the finite number of error vectors in order to find the minimum value of $\mathbf{e}^*\mathbf{F}^*\mathbf{F}^*\mathbf{e}$. In most cases the true SNR-loss will be found in that simple way. It should be mentioned, however, that there exists a straight forward method to find the global minimum even in presence of limit cycles, i.e. vectors of infinite length [5]. This method will not be further discussed in this paper.

A very interesting problem is a unique formulation of those *worst case channels* which lead to the gobal minimum value of $\gamma_{\min}^2$. The solution of this problem is explained e.g. in [4]:

- The global minimum of $\gamma_{\min}^2$ is identical to the minimum eigenvalue of the energy autocorrelation matrix $\mathbf{R}_{ee}$ corresponding to the worst case error vector.
- The corresponding worst case channel impulse response $\mathbf{f}_{\min}$ is identical to the corresponding eigenvector.

The fundamental problem is to find the worst case error vectors that result in the minimum eigenvalue of $\mathbf{R}_{ee}$. Some results of worst case channels with special channel order constraints published e.g. in [2], [6] are summarized below.

- real- and complex-valued 1st-order-channels [2], [6]

$$\gamma_{\min}^2 = 1 \rightarrow \text{SNR-loss 0 dB}$$

$$\mathbf{f} = \frac{1}{\sqrt{2}}[1, -\mathrm{e}^{-\mathrm{j}\theta}]^{\mathrm{T}}, \quad (4)$$

- real-valued 2nd-order-channels, real data [2]

$$\gamma_{\min}^2 = 2 - \sqrt{2} \rightarrow \text{SNR-loss 2.3 dB}$$

$$\mathbf{f} = \frac{1}{2}[1, \sqrt{2}, 1]^{\mathrm{T}}, \quad (5)$$

- complex-valued 2nd-order-channels, complex data [6]

$$\gamma_{\min}^2 = 2 - \sqrt{2} \rightarrow \text{SNR-loss 2.3 dB}$$

$$\mathbf{f} = \frac{1}{2}[\mathrm{e}^{-\mathrm{j}\theta}, \sqrt{2}, \mathrm{e}^{\mathrm{j}\theta}]^{\mathrm{T}}. \quad (6)$$

For the phases θ the following values have to be picked

$$\text{QPSK}: \theta \in \{0, \frac{\pi}{2}, \pi, \frac{3}{2}\pi\} \qquad (7a)$$

$$\text{8PSK}: \theta \in \{0, \frac{\pi}{4}, \frac{\pi}{2}, \ldots, \frac{7}{4}\pi\}. \qquad (7b)$$

The case of complex-valued 2nd-order channels with complex data was discussed in [6] under the assumption of error vectors with a maximum length of 2. This is true for real-valued channels in presence of real data. However, further investigations show that the worst case is obtained for increased length. For complex-valued channels and QPSK transmission we get [7]

$$\gamma^2_{\min} = 0.4689 \rightarrow \text{SNR-loss: } 3.3 \text{ dB}$$

error vectors:

$$\mathbf{e} = \begin{cases} \mathrm{e}^{\mathrm{j}\theta}[1, \pm(1+j), j]^{\mathrm{T}} \\ \mathrm{e}^{\mathrm{j}\theta}[1, \pm(1-j), -j]^{\mathrm{T}} \end{cases} \qquad (8a)$$

equivalent impulse response:

$$\mathbf{f} = \begin{cases} f_0[1, \mp a(1+j), j]^{\mathrm{T}} \\ f_0[1, \mp a(1-j), -j]^{\mathrm{T}} \end{cases} \qquad (8b)$$

with

$$a = 1.132782,$$

symbolic error rate:

$$P_s \approx \frac{3}{8}\mathrm{erfc}(\sqrt{0.4689 \cdot E_b/N_0}). \qquad (8c)$$

3. Time Truncation of the Channel Impulse Response by Linear Filtering

In the previous section some fundamental relations between the equivalent symbol-rate channel impulse response and the symbol error rate performance of the Viterbi-detector were discussed. These considerations were restricted to 1st- and 2nd-order channels. In realistic data transmission systems channel impulse responses my occur that are far longer; in these cases the complexity of the Viterbi-dector is extremely high. Thus an appropriate (suboptimum) solution is the introduction of a pre-filter by which the channel impulse response is time truncated. One of the very first approaches by Qureshi and Newhall [8] was improved by Falconer and Magee [3] by the introduction of a constant energy constraint (at the Viterbi input). An alternative method was presented in [6] which is based on the analysis of the channel: zeros near the unit circle are taken into account by the Viterbi-detector whereas the non-critical zeros with sufficient distance from the unit circle are canceled by linear equalization. In the present paper an alternative very simple method will be presented which is based on the closed-form MMSE solution to the decision feedback equalization combined with a linear pre-filter.

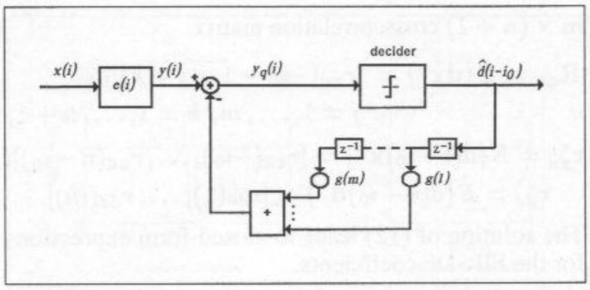

Fig. 2. Decision-feedback equalizer with non-recursive pre-filter (FIR-DF equalizer).

3.1 MMSE-Solution

Consider Fig. 2 which demonstrates the concept of an FIR-DF-equalizer. The linear pre-filter $e(i)$ is introduced to restrict the number of the feedback coefficients $g(i)$ to a certain prescribed number m. A unique solution to the problem can be given by the minimization of the power of the decider-input noise (which is composed of gaussian channel noise and residual intersymbol interference). For a compact mathematical formulation some vectors are defined.

- state variables of the FIR pre-filter:

$$\mathbf{x} = [x(i), x(i-1), \ldots, x(i-n)]^{\mathrm{T}} \qquad (9a)$$

- decided data (assume correct decisions):

$$\mathbf{d} = [d(i-i_0-1), d(i-i_0-2), \ldots, d(i-i_0-m)]^{\mathrm{T}} \qquad (9b)$$

- transjugated coefficient vectors:

$$\mathbf{e}^* = [e(0), e(1), \ldots, e(n)], \qquad (9c)$$

$$\mathbf{g}^* = [g(1), g(2), \ldots, g(m)]. \qquad (9d)$$

In (9b) i_0 describes the time delay introduced by the FIR pre-filter. By means of these definitions an appropriate MMSE cost-function is given as

$$F_{\mathrm{MSE}} = E\{|\, y_q(i) - d(i-i_0)\,|^2\} = E\{|\,\mathbf{e}^*\mathbf{x} - \mathbf{g}^*\mathbf{d} - d(i-i_0)\,|^2\}. \qquad (10)$$

This cost-function is minimized on condition that

$$\frac{\partial F_{\mathrm{MSE}}}{\partial \mathbf{e}} = \mathbf{0}, \quad \frac{\partial F_{\mathrm{MSE}}}{\partial \mathbf{g}} = \mathbf{0}. \qquad (11)$$

After some fundamental calculations this condition results in a set of linear equations

$$\mathbf{e}^*\mathbf{R}_{xx} - \mathbf{g}^*\mathbf{R}_{xd} = \mathbf{r}^*_{xd}, \qquad (12a)$$

$$\mathbf{e}^*\mathbf{R}^*_{xd} - \mathbf{g}^*\mathbf{R}_{dd} = \mathbf{r}^*_{dd} \qquad (12b)$$

where the following definitions are used:

$(n+1)\times(n+1)$ autocorrelation matrix of the received signal:

$$\mathbf{R}_{xx} = E\{\mathbf{x}\mathbf{x}^*\},$$

$m \times m$ autocorrelation matrix of the data:

$$\mathbf{R}_{dd} = E\{\mathbf{d}\mathbf{d}^*\},$$

$m \times (n+1)$ crosscorrelation matrix:

$$\mathbf{R}_{xd} = E\{\mathbf{dx}^*\} = [r_{xd}(-i_0 - 1 - j + k)]_{j,k}, \quad j = 1, \ldots, m,\ k = 1, \ldots, n+1,$$

$$\mathbf{r}_{xd}^* = E\{d(i-i_0)\mathbf{x}^*\} = [r_{xd}(-i_0), \ldots, r_{xd}(n-i_0)],$$

$$\mathbf{r}_{dd}^* = E\{d(i-i_0)\mathbf{d}^*\} = [r_{dd}(1), \ldots, r_{dd}(m)].$$

The solution of (12) leads to closed-form expressions for the FIR-DF coeffcients.

$$\mathbf{e}^* = (\mathbf{r}_{xd}^* - \mathbf{r}_{dd}^*\mathbf{R}_{dd}^{-1}\mathbf{R}_{xd})(\mathbf{R}_{xx} - \mathbf{R}_{xd}^*\mathbf{R}_{dd}^{-1}\mathbf{R}_{xd})^{-1}, \tag{13a}$$

$$\mathbf{g}^* = (\mathbf{e}^*\mathbf{R}_{xd}^* - \mathbf{r}_{dd}^*)\mathbf{R}_{dd}^{-1}. \tag{13b}$$

For the important case of *uncorrelated data* the equation can be simplified. Since

$$\mathbf{R}_{dd} = \sigma_d^2 \cdot \mathbf{I}, \quad \mathbf{r}_{dd} = 0$$

(where $\mathbf{I}$ denotes the unit matrix), we get

$$\mathbf{e}^* = \mathbf{r}_{xd}^*(\mathbf{R}_{xx} - \frac{1}{\sigma_d^2}\mathbf{R}_{xd}^*\mathbf{R}_{xd})^{-1}, \tag{14a}$$

$$\mathbf{g}^* = \frac{1}{\sigma_d^2}\mathbf{e}^*\mathbf{R}_{xd}^*. \tag{14b}$$

Eq. (14b) shows that the decision-feedback coeffficients $g(1), \ldots, g(m)$ are identical with the convolution result

$$f(i) * e(i)\ |_{i=i_0+\nu} = g(\nu), \quad \nu = 1, \ldots, m, \tag{15}$$

i.e. the impulse response samples at the output of the pre-filter. This is true even under additive noise influence as long as the noise is uncorrelated with the data.

The sample $g(0) = f(i) * e(i)\ |_{i=i_0}$ should be one under ideal conditions – in case of additive noise it is slightly less than unity. Its value is uniquely determined by correlation: In case of uncorrelated data we get

$$g(0) = f(i) * e(i)\ |_{i=i_0} = \frac{1}{\sigma_d^2}E\{y(i)d^*(i-i_0)\} = \frac{1}{\sigma_d^2}\mathbf{e}^*\mathbf{r}_{xd}. \tag{16}$$

The relation between the FIR-DF solution derived above and the time truncation problem is obvious: If we disconnect the feedback path of the equalizer we get the time truncated impulse $g(0), g(1), \ldots, g(m)$ at the pre-filter output. The samples outside the time intervall $i_0 \leq i \leq i_0 + m$ are suppressed in the minimum mean-squared error sense. In connection with the MLSE structure the nonlinear part of the equalizer has to be replaced by the Viterbi-detector (see Fig. 1) which has to be supplied with the coefficients $g(0), \ldots, g(m)$ determined by (13a,b) or (14a,b), respectively, and (16). It should be mentioned that these closed-form solutions may be replaced by iterative adaptive algorithms, e.g. the stochastic gradient search method.

3.2 Falconer's and Magee's Eigenvector Solution

The solution published in [3] will be briefly reviewed for convenience since in the original paper only real-valued channels are taken into account instead of the general complex case. The approach is based on the cost-function

$$\tilde{F}_{\text{MSE}} = E\{|\ \tilde{y}(i) - \sum_{\nu=0}^{m}\tilde{g}(\nu)d(i-i_0)\ |^2\} \tag{17}$$

which can be rewritten in vector representation

$$\tilde{F}_{\text{MSE}} = \tilde{\mathbf{e}}^*\mathbf{R}_{xx}\tilde{\mathbf{e}} - \tilde{\mathbf{e}}^*\tilde{\mathbf{R}}_{xd}^*\tilde{\mathbf{g}} - \tilde{\mathbf{g}}^*\tilde{\mathbf{R}}_{xd}\tilde{\mathbf{e}} + \tilde{\mathbf{g}}^*\tilde{\mathbf{g}}. \tag{18}$$

The symbols "~" introduced here indicate that we use slightly different definitions due to the fact that the data vector $\tilde{\mathbf{d}}$ now contains the additional element $d(i-i_0)$ at the first position:

$$\tilde{\mathbf{d}}^* = [d^*(i-i_0), d^*(i-i_0-1), \ldots, d^*(i-i_0-m)], \tag{19a}$$

$$\tilde{\mathbf{R}}_{xd} = E\{\tilde{\mathbf{d}}\mathbf{x}^*\}, \tag{19b}$$

$$\tilde{\mathbf{g}}^* = [\tilde{g}(0), \tilde{g}(1), \ldots, \tilde{g}(m)]. \tag{19c}$$

At first, we formulate the MMSE solution for the pre-filter $\mathbf{e}$ under a *fixed vector* $\tilde{\mathbf{g}}$. The condition $\partial\tilde{F}_{\text{MSE}}/\partial\mathbf{e} = \mathbf{0}$ yields

$$\tilde{\mathbf{e}}^* = \tilde{\mathbf{g}}^*\tilde{\mathbf{R}}_{xd}\mathbf{R}_{xx}^{-1}. \tag{20}$$

This solution is put in eq. (18)

$$\tilde{F}_{\text{MSE}} = \tilde{\mathbf{g}}^*[\mathbf{I} - \tilde{\mathbf{R}}_{xd}\mathbf{R}_{xx}^{-1}\tilde{\mathbf{R}}_{xd}^*]\tilde{\mathbf{g}}. \tag{21}$$

This expression should be minimum under the supplementary condition of *constant energy* of $\tilde{\mathbf{g}}$

$$\tilde{\mathbf{g}}^*\tilde{\mathbf{g}} = 1. \tag{22}$$

It is well known that this problem leads to an eigenvalue problem: The optimum solution for $\tilde{\mathbf{g}}$ is the *eigenvector of* the matrix

$$[\mathbf{I} - \tilde{\mathbf{R}}_{xd}\mathbf{R}_{xx}^{-1}\tilde{\mathbf{R}}_{xd}^*]$$

corresponding to the minimum eigenvalue. For the determination of the pre-filter impulse response $\tilde{\mathbf{e}}$ the optimum vector $\tilde{\mathbf{g}}$ is put in eq. (20).

It is an important fact that the Falconer-Magee solution is *biased* in the sense that the vector $\tilde{\mathbf{g}}$ derived from the eigenvalue problem is not identical with the impulse response at the pre-filter output in presence of additive noise, i.e.

$$f(i) * e(i)\ |_{i=i_0+\nu} = \tilde{g}(\nu) + \varepsilon(\nu),\ \nu = 0, \ldots, m. \tag{23}$$

So if the vector $\tilde{\mathbf{g}}$ is fed to the Viterbi-detector as an estimate of the equivalent channel impulse response (as suggested by Falconer and Magee) the error $\varepsilon(\nu)$ will cause a degradation. To avoid this disadvange a seperate channel estimation procedure would be necessary.

4. Comparison of MMSE and Eigenvector Solution

To illustrate the different properties of both time truncation methods discussed above we apply a 16th-order

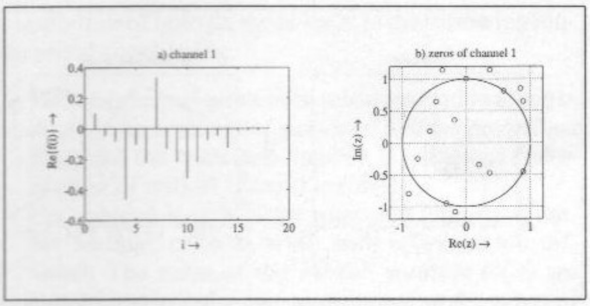

Fig. 3. Channel 1 in equivalent baseband representation: (a) real part of the impulse response, (b) zero configuration.

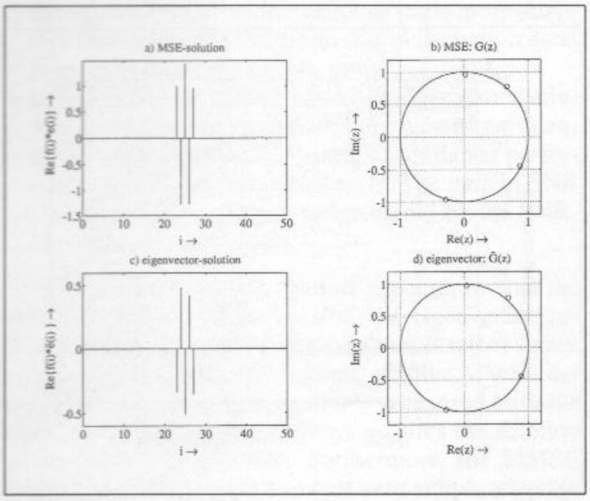

Fig. 4. Impulse truncation results ($m + 1 = 5$, no channel noise): (a) MMSE: real part of the truncated impulse $g(i)$, (b) zero configuration of $G(z)$, (c) Eigenvector-solution: real part of the truncated impulse $\tilde{g}(i)$, (d) zero configuration of $\tilde{G}(z)$.

channel model with randomly picked coefficients. This can be regarded as an instantaneous configuration of a frequency selective Rayleigh channel. The real part $f'(i)$ of the impulse response $f(i)$ ist plotted in Fig. 3a. Fig. 3b shows the corresponding zero configuration in the z-plane; note that in this example 4 zeros lie approximately on the unit circle ("critical zeros") whereas 12 zeros can be regarded as "non critical". MMSE and eigenvector solution can be compared by means of Figs. 4a–d; in this case no channel noise was present. The lengths of the truncated impulses $g(i)$ and $\tilde{g}(i)$ were prescribed as $m + 1 = 5$, both. The order of the pre-filter is $n = 32$ ($i_0 = 22$). Both results, $g(i)$ and $\tilde{g}(i)$, are approximately equal (apart from a constant factor). It is rather instructive to consider the zeros of the z-transforms of $g(i)$ and $\tilde{g}(i)$. Obviously, the are approximately the same as the critical zeros of the original channel: Only the non critical channel zeros are compensated by the pre-filter whereas the critical zeros remain nearly unchanged – provided that the value of m is sufficiently large ($m \geq$ number of critical channel zeros). If this condition is not met it is not possible to collect the m critical zeros in the truncated impulses $g(i)$ and $\tilde{g}(i)$. This fact is demonstrated

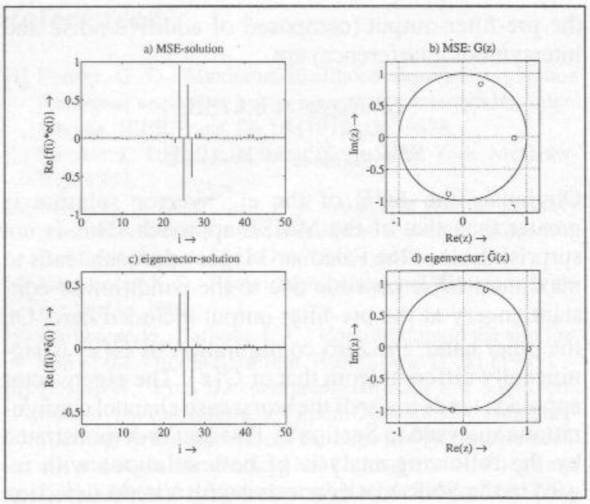

Fig. 5. Impulse truncation with reduced length ($m + 1 = 4$, no channel noise): (a) MMSE: real part of the truncated impulse $g(i)$, (b) zero configuration of $G(z)$, (c) Eigenvector-solution: real part of the truncated impulse $\tilde{g}(i)$, (d) zero configuration of $\tilde{G}(z)$.

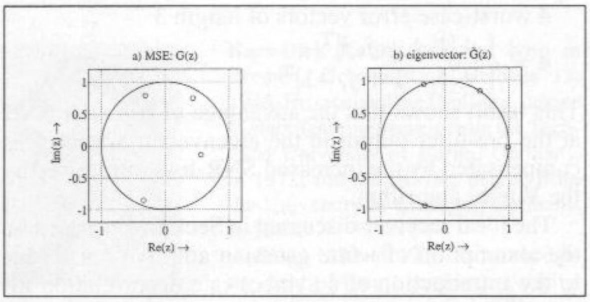

Fig. 6. Impulse truncation under additive noise ($E_b/N_0 = 10dB, m+1 = 5$): (a) MMSE, zeros of $G(z)$, (b) eigenvector-solution, zeros of $\tilde{G}(z)$.

by Figs. 5a–d for $m = 4$: The zeros of the MMSE solution are shifted inwards the unit circle whereas the zeros of the eigenvector solution remain located very near to the unit circle (of course, their positions differ from the positions of the original critical zeros). The time truncation of the impulse response is satisfactorily solved in both cases.

In the next example additive white noise is introduced. The ratio E_s/N_0 (symbol energy/ noise spectral density) is 13 dB. In case of QPSK transmission this corresponds to $E_b/N_0 = 10$ dB. The zero configurations of $G(z)$ and $\tilde{G}(z)$ are depicted in Figs. 6a, b. Although the length of the truncated impulses is chosen sufficiently large ($m + 1 = 5$) the zeros are removed from the ideal positions of the 4 critical channel zeros: In case of the MMSE solution they are again shifted inwards the unit circle whereas in the eigenvector approach they remain on the unit circle – the zero angles, however, are changed in comparison with the critical channel zeros.

It is instructive to compare the signal-to-noise properties of both solutions. The signal to noise ratios at

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the pre-filter output (composed of additive noise and intersymbol interference) are

$$SNR_{\text{MSE}} = 8.61\text{dB},$$

$$SNR_{\text{Eigenvec}} = 10.12\text{dB}.$$

Obviously the *SNR* of the eigenvector solution is greater than that of the MMSE approach. This is not surprising since the Falconer-Magee approach leads to maximum *SNR* possible due to the condition of constant engery at the pre-filter output included here. On the other hand, the zero configuration of $\tilde{G}(z)$ is significantly different from that of $G(z)$. The eigenvector approach tends towards the worst case channel configurations analysed in Section 2. This fact is demonstrated by the following analysis of both solutions with regard to the SNR-loss connected with Viterbi detection (according to eq. (3))

MSE-result:
SNR-loss = 0 dB,
error vectors reduce to length 1.

Eigenvector-result:
SNR-loss = 2.58 dB,
4 worst-case error vectors of length 3

$$\mathbf{e} = \begin{cases} \pm[1, 1+j, j]^{\text{T}} \\ \pm[j, -1+j, -1]^{\text{T}} \end{cases}$$

This result shows that the advantage of a greater *SNR* at the pre-filter output in the eigenvector approach is compensated by the increased SNR-loss introduced by the Viterbi algorithm.

The ideal receiver discussed in Section 2 is based on the assumption of *white* gaussian additive noise (due to the introduction of a symbol-rate decorrelation filter). In the suboptimum method with impulse truncation noise colouring is introduced. Under the application of Euclidean metric in the Viterbi-detector this results in a more of less severe degradation in symbol error performance. In general the noise colouring influence in the eigenvector solution is significantly more intensive than in the MMSE result. This can be explained as follows. In the eigenvector solution the remaining m zeros are located on (or near) the unit circle. This is true even under additive noise – in this case, however, their positions are different from the positions of the critical channel zeros. Consequently, these "new" critical zeros must have been introduced by the pre-filter which results in a more intensive noise colouring than in the MMSE solution where the new zeros are non critical. The different noise colouring of both solutions is illustrated in Fig. 7b which shows the spectral density of the pre-filter output noise. In this case a real-valued channel impulse response was applied (see Fig. 7a) which has been used in [3] as a worst-case channel example.

The present section is concluded with some investigations on the QPSK symbol error rate under white gaussian noise and intersymbol interference introduced by the channel impulse response shown in Fig. 7a.

At first consider the simulation results for both time truncation methods given in Fig. 8. Obviously the MMSE solution is slightly superior over the eigenvector apprach. For comparison the theoretical symbol-

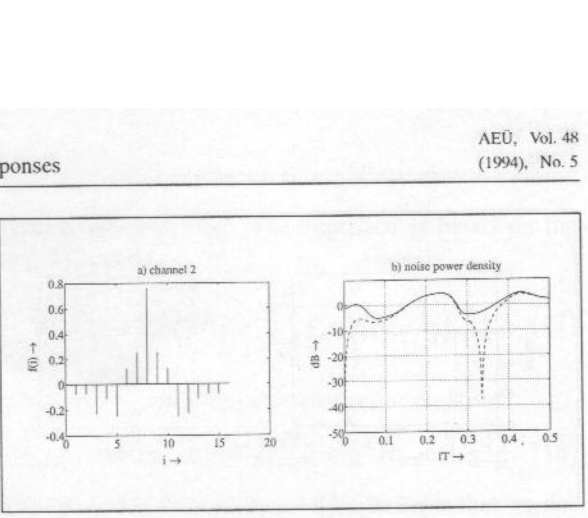

Fig. 7. Noise colouring by pre-filtering: (a) example channel 2, impulse response, (b) noise power density after pre-filtering (—MMSE; – – – Eigenvector solution).

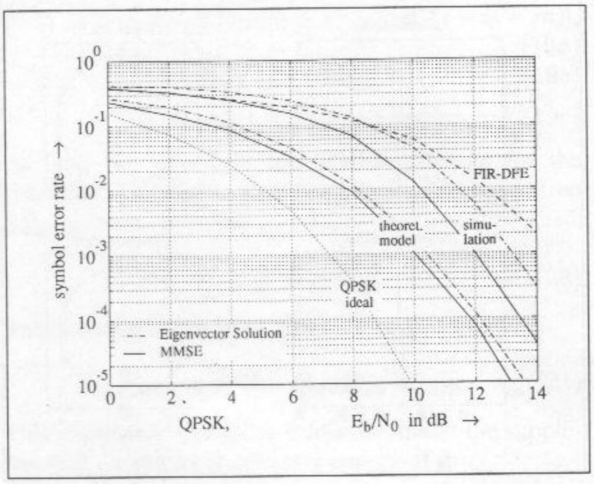

Fig. 8. Symbol error rates in QPSK transmission (channel 2, see Fig. 7a).

error rates are shown where the SNR-loss of the Viterbi-detection is taken into account. As explained above the SNR-loss in the eigenvector solution is greater than the gain in SNR at the pre-filter output. The difference between this theoretical model and the simulation can be explained by the influence of the noise colouring which is not included in the computational solution. Finally the symbol error rate of the classical decision feedback equalizer (based on the MMSE solution) is given in Fig. 8. Compared with the Falconer-Magee solution it shows only a small degradation in symbol error performance whereas the MMSE time truncation method combined with Viterbi-detection leads to an improvement of about 2.5 dB (at a symbol error rate of 10^{-3}).

5. Conclusion

In the present paper suboptimum receivers with Viterbi-detection were discussed. A non-recursive pre-filter was introduced for channel impulse response time truncation in order to reduce the complexity of the Viterbi algorithm. Two different time truncation methods were compared: the well-known eigenvector solution by Falconer and Magee and a simple MMSE approach. The comparison of both methods leads to the following fundamental conclusions:

- Without channel noise both solutions lead to approximately the same result, provided that the prescribed length of the truncated impulse is sufficient ($m \geq$ number of critical channel zeros).
- For reduced length of the truncated impulse or under additive channel noise both solutions are different. The zeros of the MMSE solution $G(z)$ are shifted inwards the unit circle whereas the zeros of the eigenvector solution $\tilde{G}(z)$ are located near the unit circle.
- Noise colouring is intruduced by the pre-filters. In the eigenvector solution this colouring is more intensive than in the MMSE result due to the new critical zeros introduced by the pre-filter.
- In contrast to the MMSE result the eigenvector solution tends towards the well-known worst-case channel configuration which causes a significant performance loss in the Viterbi-detector. The gain in SNR at the pre-filter output is compensated by the SNR-loss of the Viterbi-detector.

The examples regarded in this paper show that the MMSE solution is superior over the eigenvector approach. Indeed, the gain in performance is rather small, but is has to be taken into account that the MMSE design of the pre-filter is very simple compared with the Falconer-Magee approach which requires the solution of an eigenvalue problem. Furthermore, the MMSE method allows the formulation of very simple adaptive algorithms, e.g. on the basis of the stochastic gradient search.

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