Adaptive Modulation and Interleaving for BICM-OFDM

Carsten Bockelmann and Karl-Dirk Kammeyer
Department of Communications Engineering
University of Bremen, 28359 Bremen, Germany
Email:{bockelmann, kammeyer}@ant.uni-bremen.de

Abstract— **The optimisation of communication systems with respect to the applied channel code is a more recent topic in the field of adaptive communications compared to the optimisation of uncoded transmissions. Besides the information theoretical statements the applied codes are not perfect but practical codes which poses the additional problem to capture the structure and overall behaviour of a code sufficiently. Intelligent interleaver design has been proposed to enhance the performance of convolutional codes, but adaptive modulation approaches using the mutual information of Bit Interleaved Coded Modulation (BICM) systems as a figure of merit seem well suited to enhance the error rate, too. To this end, we propose a new loading algorithm and compare its performance to the interleaver approach [1] and a combination of both for several system configurations.**

I. Introduction

Adaptive communications for Orthogonal Frequency Division Multiplexing (ODFM) systems have been studied extensively, but mostly limited to the uncoded case. Practical coding typically has been neglected, but recently more attention has been given to this topic. The two objectives are the maximisation of the overall transmitted data rate or the minimisation of the error rate under a given rate constraint. Regarding the first problem, Stiglmayr et. al [2] introduced an algorithm to adapt the code rate of a duo-binary turbo code and the modulation alphabet jointly via SNR switching thresholds, which is very similar to [3]. Furthermore, Sankar et. al [4] introduced a simple scheme based on an approximation of the BICM capacity allowing for a closed form waterfilling solution of the power allocation problem.

Regarding the second aim, Stierstorfer et. al [1] proposed a new angle of view, namely to adapt the interleaver to the channel conditions, but keep the modulation and coding fixed. Opposed to uncoded bit and power loading this performs very well and offers new insight how a specific code structure can be exploited. In this context the bit level capacities have been shown to be a good optimisation criterion with regard to the bit error rate after decoding. Especially [1] shows the impact of bit level capacities on the decoding performance of convolutional codes and their contribution to the Viterbi path metrics.

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Additionally, insights from [3] and [5] show that the overall performance of a coded system can be described via the average mutual information of a code word. This particularly holds for capacity achieving codes and with some limitations can also be applied to weaker codes. We propose a loading scheme which enhances the average mutual information over one code word by adapting the modulation alphabets on the subcarriers of an OFDM symbol and by distributing the overall transmit power over all active subcarriers evenly. For a variety of data rates and codes this approach is compared to and combined with the interleaver adaptation [1].

The remainder of this paper is organised as follows. Section II describes the system model and important assumptions, which are used to discuss the concept of bit level capacities in Section III. In Section IV interleaver design and influences are discussed additionally to Section I. Section V then introduces a new adaptive modulation algorithm to enhance the overall performance of a channel coded OFDM system, whose performance is then analysed and compared to other approaches in Section VI. Finally, Section VII concludes this work.

Notation

In the following, vectors and sets are denoted by lower case bold and calligraphic letters, respectively. Furthermore, expectation and probabilities are denoted as $\mathrm{E}\{\cdot\}$ and $\mathrm{p}(\cdot)$. $\mathcal{N}_C(\mu, \sigma)$ describes a complex Gaussian distribution with mean μ and variance σ.

II. System Model

We consider an equivalent baseband model of the OFDM system with N_C subcarriers assuming perfect synchronisation, a sufficient guard interval (GI) and perfect knowledge of the channel state information at both transmitter and receiver. Thus, the system can be described in frequency domain as

$$y_k = h_k \cdot \sqrt{p_k} \cdot d_k + n_k \,, \qquad (1)$$

where h_k denotes the channel coefficient in frequency domain on subcarrier $k = 1, \cdots, N_C$ and p_k, d_k, n_k and y_k denote the transmit power, transmit symbol, Gaussian noise and receive symbol, respectively. The overall transmit power is chosen as $P = \sum_{k=1}^{N_C} p_k$ and the power

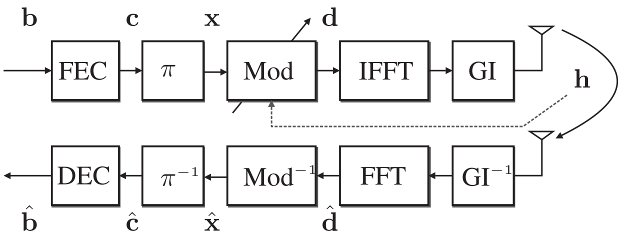

Fig. 1. System Model

of the noise $n_k \sim \mathcal{N}_C(0, \sigma_n^2)$ is fixed to $\sigma_n^2 = 1$. The N_C frequency domain channel coefficients are obtained through

$$h_k = \sum_{\ell=0}^{L_F-1} \tilde{h}(\ell) e^{-j\Omega_k \ell}, \tag{2}$$

where the L_F taps of the time domain channel are following $\tilde{h}(\ell) \sim \mathcal{N}_C(0, 1/L_F)$.

Specifically, transmit symbols stemming from M-QAM modulation alphabets $\mathcal{A}$ with binary reflected gray mapping [6] are considered throughout this paper. To each subcarrier k an individual alphabet of cardinality $|\mathcal{A}_k| = M_k$ may be assigned. The bit label of a specific symbol of this alphabet $d_k \in \mathcal{A}_k$ is denoted by $\mathbf{x}_k \in \mathbb{C}^{\log_2(M_k)}$ and the specific bit levels are $x_{k,i}\ \forall i = 1, \cdots, \log_2 M_k$. Soft-Demapping via a-posteriori-probability (APP) detection is used to supply soft information to the decoder.

Figure 1 shows the general system model including the channel code and interleaving, where on the one hand non-systematic non-recursive convolutional codes of rate $R_C = 1/2$ and constraint length $L_C = 3$ and $L_C = 7$ and on the other hand a turbo code of rate $R_C = 1/2$ are considered. The turbo code is composed of two parallel identical systematic recursive convolutional codes ($R_C = 1/2$, $L_C = 3$) with termination for the first code and no termination for the second code. A BCJR algorithm has been used for soft-decoding; the interleaver design will be detailed in section IV.

III. Bit level capacities

The capacity of a communication system (1) applying bit interleaved coded modulation is well known to be [7]

$$C_{\text{BICM},k} = \sum_{i=1}^{\log_2(M_k)} I\left(y_k; x_{k,i} | M_k, h_k\right), \tag{3}$$

where M_k indicates the applied modulation alphabet and mapping. The $I\left(y_k; x_{k,i} | M_k, h_k\right)$ are the so called bit level capacities, depending on the chosen modulation and the current channel state. Specifically, these can be calculated as

$$I\left(y_k; x_{k,i} | M_k, h_k\right) = \tag{4}$$
$$1 - \mathrm{E}_{y_k,\mu}\left\{\log_2\left(\frac{\sum\limits_{a\in\mathcal{A}_k} p\left(y_k | a, h_k\right)}{\sum\limits_{a\in\mathcal{A}_{k,i}^{\mu}} p\left(y_k | a, h_k\right)}\right)\right\},$$

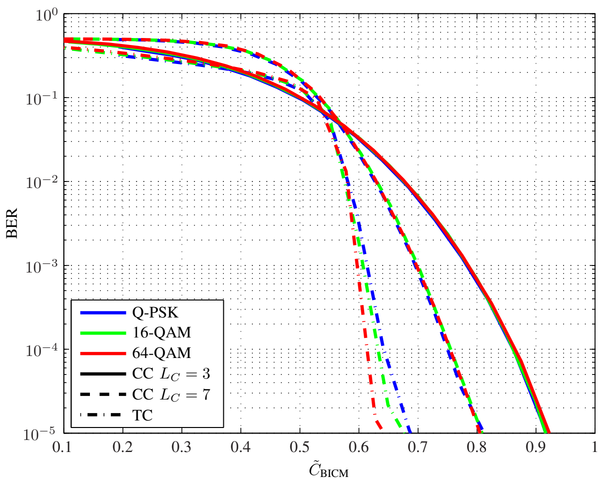

Fig. 2. Comparison of the BER vs. C_{BICM} with regard to two convolutional codes ($R_C = 0.5$ with $L_C = 3$ (solid), $L_C = 7$ (dashed)) and one turbo code (dash dotted) applying Q-PSK (blue), 16-QAM (green) and 64-QAM (red); $N_C = 1024$ subcarriers, $L_F = 10$ channel taps

with $\mathcal{A}_{k,i}^{\mu}$ denoting the set of all symbols in $\mathcal{A}_k$ with $\mu \in \{0, 1\}$ for the bit label $x_{k,i}$ at position i and subcarrier k. These bit level capacities can be calculated by Monte Carlo integration and stored as a look-up table for further use as channel variations influence the outcome simply by a shift in the signal-to-noise ratio.

An important property of the average BICM capacity over one code word (OFDM symbol)

$$\begin{aligned} C_{\text{BICM}} &= \frac{1}{N_C}\sum_{k=1}^{N_C} C_{\text{BICM},k} \\ &= \frac{1}{N_C}\sum_{k=1}^{N_C}\sum_{i=1}^{\log_2(M_k)} I\left(y_k; x_{k,i} | M_k, h_k\right) \end{aligned} \tag{5}$$

is an indication of the decoding performance of nearly capacity achieving codes independent of the applied modulation alphabet. This has been used to design loading algorithms which include the influence of channel coding, e.g. [3]. However, for shorter code word lengths and weaker codes C_{BICM} may not be a sufficient criterion of the decoding performance which is strongly connected to the bit level capacities and interleaver optimisation in the following section.

Figure 2 shows results for the three coding schemes, which will be applied throughout this paper. The normalised BICM capacity is simply obtained as

$$\tilde{C}_{\text{BICM}} = C_{\text{BICM}} / \log_2(M), \tag{6}$$

where M is the size of the modulation alphabet common for all subcarriers. In order to obtain some insight into the quality of the BICM capacity as an indicator in OFDM systems the following system has been simulated. The number of subcarriers is fixed to $N_C = 1024$ with a time domain channel of $L_F = 10$ taps, and coding applied over the bits contributing to one OFDM symbol. Despite the relatively short code word length all coding schemes show a very similar behaviour for the applied modulation alphabets. Only the turbo code performs slightly different for the shown modulation alphabets because of the increased interleaver length with increasing modulation size. Overall, we can conclude that optimising C_{BICM} seems to be a good criterion to enhance the error rate performance of such a system independent of the specific code.

IV. Influence of Interleaving

For bit interleaved coded modulation system analysis the interleaver is generally chosen to be random, which ensures, that the quality of such interleavers is sufficiently distributed (balanced between good and bad interleavers) and correlations between the transmitted bits are combated sufficiently well to avoid block errors and enhance the decoding process. However, interleavers are focus of optimisation and design if actual communication systems have to be engineered. Hence, an interleaver could also be adapted to the communication channel, which is the idea of [1]. Their analysis showed that an appropriately designed interleaver considering the trellis representation of the decoding process of a Viterbi decoder, should combine the bit level capacities of a code word to enhance the metric of a trellis segment.

If we assume $\mathbf{x} = [\mathbf{x}_1, \cdots, \mathbf{x}_{N_C}]^T$ to include all bits of a code word, where $\mathbf{x}_k$ are the bit labels of symbol d_k, then $\pi^{-1}(\mathbf{x})$ denotes the deinterleaver function. According to the BICM model the transmission can be described by the bit levels as parallel channels, each with its own bit level capacity $I(y_k; x_{k,i}|M_k, h_k)\ \forall k, i$, leading to a vector of bit capacities (omitting the conditioning for the sake of brevity)

$$\mathbf{I} = [I(y_1; x_{1,1}), \cdots, I(y_{N_C}; x_{N_C,\log_2(M_{N_C})})]^T, \quad (7)$$

which corresponds to $\mathbf{x}$. Accordingly, the deinterleaver changes the order of the individual bit level capacities $\pi^{-1}(\mathbf{I})$. Considering that the Viterbi metric of each trellis segment is influenced by $1/R_C$ bits or bit level capacities, respectively, the combined bit level capacities can be interpreted as a quality indicator of each trellis segment. Optimising the combination of bit level capacities contributing to one trellis segment seems a good heuristic to enhance the overall decoding performance. To this end a simple design rule based on sorting of the bit level capacities has been proposed [1], which leads to combined capacities in a trellis segment close to the average bitwise capacity of the code word. Additional randomisation ensures that block errors are still separated reasonably well.

The observation, which can be drawn from this, is that the structure of the code, or the decoder respectively, has an important role in the overall performance. As the interleaver design does not change the average bitwise capacity of a code word and in general a higher bitwise capacity translates to a better error performance, a combination of the interleaver design approach with a loading algorithm that enhances the bitwise capacity should lead to further gains. This will be investigated in section VI.

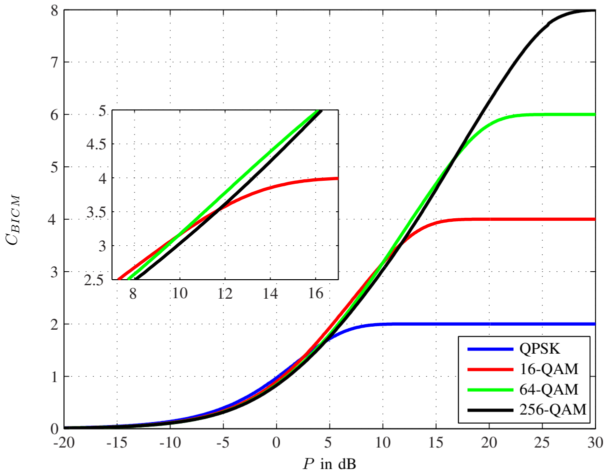

Fig. 3. Bit Interleaved Coded Modulation capacity for the modulation schemes QPSK, 16-QAM, 64-QAM and 256-QAM using binary reflected gray-mapping for an AWGN channel with noise power $\sigma_n^2 = 1$ and power P

V. Adaptive Modulation and Coding

Following the conclusions from Section IV, we propose a new method to adapt the number of bits $\log_2(M_k)$ contained in symbol d_k on a subcarrier k with respect to the sum bit level capacity $C_{\text{BICM},k}$, which depends on the modulation alphabet M_k, channel coefficient h_k and assigned transmit power p_k. In order to enhance the overall average bitwise capacity of the code word (OFDM symbol), the following heuristic approach will be used.

Reviewing the general property of the BICM capacity as shown in Figure 3, it is clear that there exists a modulation alphabet, which is superior to all other choices at a fixed signal-to-noise ratio (SNR). Therefore, it would be sufficient to use SNR thresholds to identify the modulation alphabet used on a subcarrier if transmit power p_k and channel coefficient h_k are both known. However, given a fixed target transmission rate we have to ensure, that this rate is fulfilled. To be more precise, for each subcarrier k whose rate is raised, the rate on another subcarrier n has to be lowered. Ultimately, this should lead to an overall increase in C_{BICM}, which makes calculation of both $C_{\text{BICM},k}$ and $C_{\text{BICM},n}$ necessary. In the special case that one subcarrier could be switched off entirely, it is also necessary to distribute the transmit power evenly over all remaining active subcarriers to check, if this would lead to an overall capacity gain. The algorithm *MAB* (Maximize Average BICM capacity) details the steps of such an approach assuming, that the applied modulation alphabets differ by 2 bits each (e. g. Q-PSK → 16-QAM). This can be easily extended to other ensembles of alphabets.

MAB:

1) Initialise: Calculate all subcarrier capacities $C_{\text{BICM},k,i} = f(h_k, p_k, \mathcal{A}_i)$ for all allowed

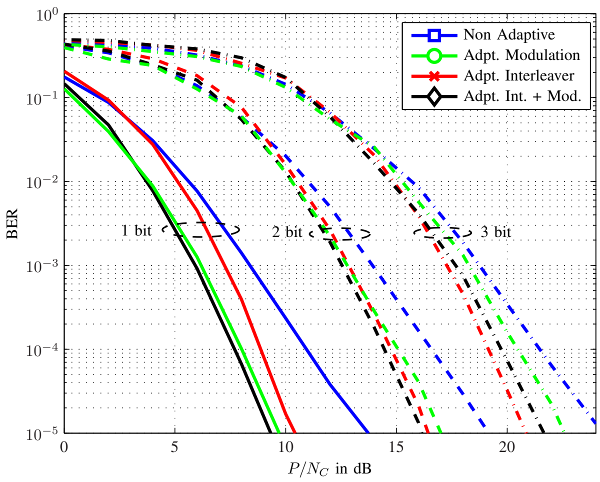

Fig. 4. BER vs. transmit power P, frequency selective channel with $L = 10$ equal power taps, $N_C = 1024$ subcarriers, $[7,5]_8$ convolutional code ($R_C = 1/2$, $L_C = 3$) over one OFDM symbol; QPSK (solid), 16-QAM (dashed), 64-QAM (dash dotted)

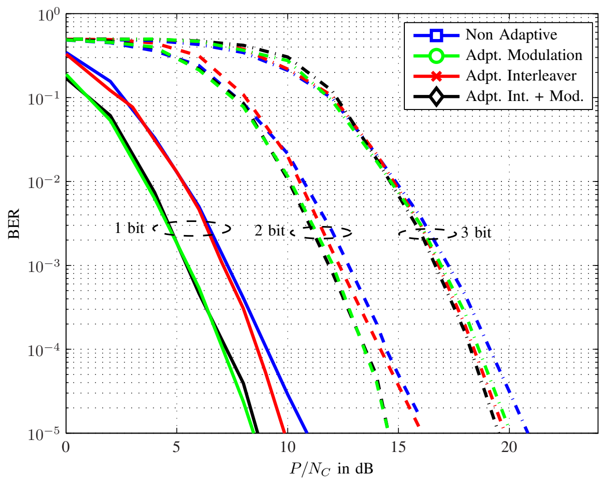

Fig. 5. BER vs. transmit power P, frequency selective channel with $L = 10$ equal power taps, $N_C = 1024$ subcarriers, $[171,133]_8$ convolutional code ($R_C = 1/2$, $L_C = 7$) over one OFDM symbol; QPSK (solid), 16-QAM (dashed), 64-QAM (dash dotted)

modulation alphabets $\mathcal{A}_i \quad \forall i = 0, \cdots, i_{max}$ with $p_k = P/N_C$ and determine the differences $\Delta C_{k,i} = C_{\text{BICM},k,i} - C_{\text{BICM},k,i+1}$ characterising the capacity loss for increasing the modulation order.

2) Find the largest $\Delta C_{k,i(k)-1} \ \forall k$, where $i(k)$ indicates the current modulation alphabet on a subcarrier k, resulting in a subcarrier index k_{low}. If $i(k_{\text{low}}) - 1$ is equal to zero, goto b), else goto a)
   a) Find the smallest $\Delta C_{k,i(k)} \quad \forall k$ resulting in a subcarrier k_{high}. If $\Delta C_{k_{\text{low}},i(k)-1} > \Delta C_{k_{\text{high}},i(k)}$ Goto 2), else stop.
   b) Due to $i(k_{\text{low}}) == 0$, the power of the subcarrier k_{low} has to be redistributed, resulting in $p_k = P/N_{C,eff} \ \forall k$ with $N_{C,eff}$ denoting the remaining active subcarriers. Recalculate $C_{\text{BICM},k,i}$ and $\Delta C_{k,i}$, then goto a).
3) Increase $i(k_{\text{high}})$ by one and decrease $i(k_{\text{low}})$ by one.
4) Repeat 2) to 3) until no further enhancement is possible or 2a) leads to a stop.

VI. Results

We employed simulations for all three codes of code rate $R_C = 0.5$ (excluding termination aspects) for the modulation alphabets Q-PSK, 16-QAM and 64-QAM, resulting in a rate of 1, 2, and 3 bits/s/Hz. Besides the non-adaptive transmission with a common modulation alphabet for all subcarriers three additional schemes have been simulated: interleaver optimisation, adaptive modulation and the combination of both approaches. Specifically, the maximum number of bits, which are assigned to a single subcarrier has been set to $i_{\text{max}} = 10$. The size of a code word is limited to the $N_C = 1024$ subcarriers, leading to differently sized code word lengths depending on the transmitted rate. The channel has been chosen to be of length $L_F = 10$, constant for one OFDM symbol, but changing independently from symbol to symbol.

Figure 4 shows the results for the $[7,5]_8$ convolutional code, where we can observe, that the proposed scheme enhances the transmission in all cases. However, for 3 bit/s/Hz the overall gain decreases, resulting in an overall worse performance of the combined scheme than the interleaver optimisation alone. This can be explained with the bit level capacities following from the higher modulations used to compensate for weak subcarriers. A modulation with two additional bits adds one additional bit level capacity (due to the structure of M-QAM there are only $\log_2(M)/2$ different bit level capacities) which is smaller than all bit level capacities of the base modulation. Even though the mean normalised BICM capacity of the code word is enhanced, it seems that the less reliable bits limit the overall performance. Another surprising outcome is that even the combination of our scheme and the interleaver optimisation results in a worse performance than the interleaver optimisation alone. This hints at a problem regarding the bit level capacities, which is not well captured by our approach as only C_{BICM} over the whole OFDM symbol is enhanced. Nonetheless, the combination of both approaches leads to further gains especially in the case of a low data rate.

A similar behaviour can be observed for the $[171,133]_8$ convolutional code as shown in Figure 5. The gains obtained by interleaver optimisation are smaller, which is a consequence of the increased encoder memory. With respect to the trellis interpretation this leads to more dependencies between trellis segments and thus an averaging effect making good bit level capacities in specific trellis segments less important. Furthermore, a higher average bit level capacity can be better utilised. Due to this, the proposed scheme performs better at 3 bit/s/Hz, even leading to the overall best performance if both schemes are combined. The effective gains, though, are smaller for the stronger code than in Figure 4, which clearly shows, that the limited degrees of freedom (namely

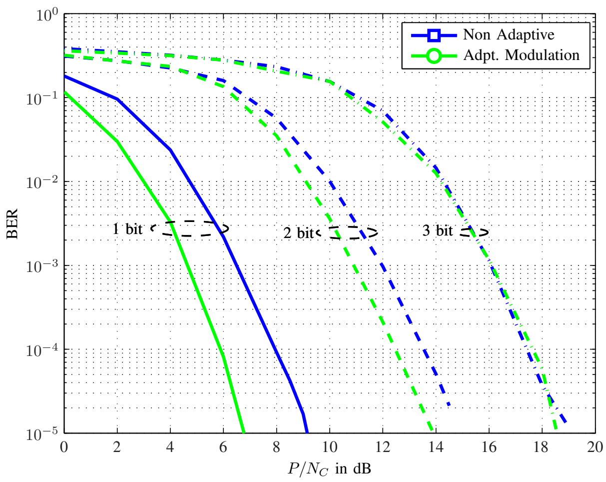

Fig. 6. BER vs. transmit power P, frequency selective channel with $L = 10$ equal power taps, $N_C = 64$ subcarriers, modulation 16-QAM, Turbo code ($R_C = 1/2$, 8 decoder iterations) over one OFDM symbol

the number of channel taps) in this case are well exploited by the code alone.

Considering the turbo code, an interleaver optimisation as previously discussed does not make much sense because of the interleaved information bits fed to the second encoder. Optimising the bit level capacity combination of the first code leads to an enhanced performance in the first half iteration, but may pose a problem decoding the second code. Here, the performance is generally more limited by correlation of the bits after some iterations. Therefore, Figure 6 only shows the results for the turbo code regarding the proposed scheme. For low rates, the gains are in the order 1-2 dB, at 3 bit/s/Hz, however, there is no further gain in adapting the modulation to the channel.

VII. Conclusion

Including information theoretical propositions, but also conclusions following from the decoding procedure, into the optimisation of a coded communication system can lead to further gains, which are not obtainable simply by application of algorithms designed to enhance uncoded systems. On the other hand it has been shown, that even though interleaver optimisation and maximisation of the average BICM capacity of a code word each lead to significant gains, there is still room for improvement. Especially, the overall results depend a lot on the actual code and the chosen data rate, where stronger codes (e.g. turbo codes) more closely follow the general guideline of information theory suggestions and weaker codes (e.g. short memory length convolutional codes) seem to perform worse if the average bitwise capacity is enhanced and interleaver design is used. The latter is quite surprising as the interleaver is optimised to exploit the seemingly higher performance (in terms of bitwise capacity). This hints at additional factors which should be considered, namely the distribution of the bit level capacities and extrinsic properties of the code, which are ultimately necessary to identify unreliable code bits. The bit level capacities are one tool to analyse the performance, but not a sufficient one for all codes.

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