Evaluation of Outage Restricted Distributed MIMO Multi-Hop Networks by the Improved Approximative Power Allocation

Dirk Wübben
Department of Communications Engineering, University of Bremen, Otto-Hahn-Allee, D-28359 Bremen, Germany
Email: wuebben@ant.uni-bremen.de, Phone: +49 421 218 2407

***Abstract*—The concept of Virtual Antenna Array (VAA) is a promising approach to apply MIMO concepts in relaying systems. In order to fulfill a given Quality-of-Service (QoS) requirement while reducing the power consumption of the entire system, efficient resource allocation strategies have to be developed. The optimum power allocation solution corresponds to a convex optimization problem. To achieve an analytical solution, the approximative approach IAPA (Improved Approximative Power Allocation) has been proposed by the author. Within this paper this closed-form solution is further analyzed and analytical expressions for the power allocation in case of symmetric relaying networks are derived. Thereby, an approach for analyzing and optimizing different multi-hop scenarios is developed.**

I. Introduction

Recently, the application of MIMO techniques to spatially separated relaying nodes has gained considerable interest in the research community [1]–[4]. In case of distributed MIMO multi-hop networks adjacent nodes are combined into so called VAAs, whereby the source-destination link is separated into several hops. In each hop the nodes of the transmit VAA serve as virtual transmit antennas of a "distributed" space-time code and the nodes of the receive VAA perform independent data detections. It has been shown, that such a distributed MIMO multi-hop network achieves significant capacity improvements in comparison to single-hop communications.

One of the most important issues in the design of such a wireless multi-hop network is the optimized allocation of transmit power to the distinct VAAs, as the power consumption of the terminals may be one of the most limiting factors for future communication systems. On the other hand the power allocated to the nodes has to be large enough to support the required QoS. To this end, several resource allocation strategies have been presented to meet an end-to-end (e2e) ergodic capacity or error-rate, e.g., [2]–[5]. However, as the majority of today's wireless communications happen over slow-fading channels, i.e., non-ergodic in the capacity sense, the consideration of the e2e outage probability is of higher practical relevance. For multi-hop networks applying the Decode-and-Forward (D&F) [1] protocol several approaches for minimizing the total transmit power while meeting the e2e outage constraint have been proposed by the author in [6]–[8]. In this paper the near-optimum closed-form solution IAPA [7] is further analyzed and used to derive analytical expressions for the power assignment in symmetric relaying systems.

The remainder of this paper is organized as follows. The system model and the optimization task are introduced in Sections II and III, respectively. The approximated power allocation problem as well as algebraic solutions are presented in Section IV and analytical investigations with respect to the IAPA approach are given in Section V. The performance is evaluated in Section VI and the paper is concluded by the summary in Section VII.

II. System description

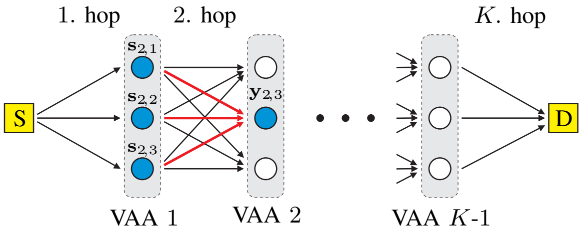

Fig. 1. System model for a distributed MIMO multi-hop network with highlighted MISO link in the second hop.

We consider a multi-hop system where the source communicates with the destination via K–1 VAAs as shown in Fig. 1. The nodes within one VAA decode the information separately but re-encode the decoded information by using a spatial fraction of the choosen space-time code word. Therefore, the transmission within one hop can be modeled as several multiple-input single-output (MISO) systems. It is assumed that each VAA transmits signals with the same rate R and all hops use the total bandwidth W that is available to the network. Let k index the hop and t_k, r_k denote the number of transmit nodes and receive nodes within the kth hop, respectively. With $\mathbf{S}_k \in \mathbb{C}^{t_k \times T}$ defining the space-time encoded signal of length T transmitted from the t_k nodes at hop k, the corresponding received signal $\mathbf{y}_{k,j} \in \mathbb{C}^{1\times T}$ at node $1 \le j \le r_k$ of VAA $k+1$ is given by

$$\mathbf{y}_{k,j} = \sqrt{\frac{\theta_k \mathcal{P}_k}{t_k}} \mathbf{h}_{k,j} \mathbf{S}_k + \mathbf{n}_{k,j} \ , \qquad (1)$$

where $\mathbf{n}_{k,j}$ denotes the Gaussian noise vector with power spectral density N_0 and $\mathcal{P}_k$ is the total power of the kth VAA. The channel from the t_k transmit nodes to the jth receive node within hop k is expressed as $\mathbf{h}_{k,j} \in \mathbb{C}^{1\times t_k}$ containing complex zero-mean circular symmetric Gaussian distributed elements with variance 1. We assume that the relaying nodes belonging to the same VAA are spatially sufficiently close as to justify a common path loss $\theta_k = d_k^{-\epsilon}$ between two adjacent VAAs, where d_k is the distance between the transmit and receive nodes at hop k and ϵ is the path loss exponent within range of 2 to 5.

III. Optimum Power Allocation

The outage probability $P_{\mathrm{out},k,j} = \Pr(R > C_{k,j})$ of receiving node j in hop k defines the probability that the transmit rate R is greater than the rate

$$C_{k,j} = W \log_2 \left(1 + \frac{\mathcal{P}_k}{d_k^{\epsilon} t_k W N_0} \|\mathbf{h}_{k,j}\|^2\right) \tag{2}$$

supported by the instantaneous channel realization [9]. It is expressed as a cumulative distribution function (CDF) and depends on the fixed transmission parameters and the channel condition within the hop and equals

$$P_{\mathrm{out},k,j} = \Pr\left(\|\mathbf{h}_{k,j}\|^2 < x_k\right) = \frac{\gamma(t_k, x_k)}{\Gamma(t_k)} \ . \tag{3}$$

To ease notation, the system parameters of the kth hop have been collected in the variable

$$x_k = (2^{\frac{R}{W}} - 1) d_k^{\epsilon} t_k W N_0 / \mathcal{P}_k = Q d_k^{\epsilon} t_k / \mathcal{P}_k \tag{4}$$

being inversely proportional to the signal-to-noise-ratio of hop k and the variable $Q = (2^{\frac{R}{W}} - 1) W N_0$ contains the system-wide parameters bandwidth, noise power and data rate. Furthermore, as $\|\mathbf{h}_{k,j}\|^2$ obeys a Gamma distribution, the CDF is given by the incomplete Gamma function $\gamma(t_k, x_k) = \int_0^{x_k} e^{-u} u^{t_k - 1}\, \mathrm{d}u$ normalized by the Gamma function $\Gamma(t_k)$ [9].

As the same path loss is assumed within a hop, each MISO system of the hop has the same outage probability and hence the outage probability of the kth hop is given by

$$P_{\mathrm{out},k} = 1 - \prod_{j=1}^{r_k} (1 - P_{\mathrm{out},k,j}) = 1 - (1 - P_{\mathrm{out},k,j'})^{r_k} \ , \tag{5}$$

where j' indexes an arbitrary $j \in \{1, \dots, r_k\}$. Since D&F is applied, the signals are completely decoded at each VAA, so that the outage probabilities per hop $P_{\mathrm{out},k}$ are mutually independent. For simplicity, it is assumed that the e2e communication is in outage if any of the MISO systems can not correctly decode the information [2]. With this pessimistic assumption the e2e outage probability corresponds to

$$P_{\mathrm{e2e}} = 1 - \prod_{k=1}^{K} (1 - P_{\mathrm{out},k}) = 1 - \prod_{k=1}^{K} (1 - P_{\mathrm{out},k,j'})^{r_k} \tag{6}$$

and serves as the measurement for the required QoS in the subsequent investigation. The optimal power allocation minimizes the transmit power $\mathcal{P}_{\mathrm{tot}}$ of the whole network while meeting the e2e outage probability requirement e, i.e., $P_{\mathrm{e2e}} \le e$. It can be shown that the optimization problem

$$\text{minimize} \quad \mathcal{P}_{\mathrm{tot}} = \sum_{k=1}^{K} \mathcal{P}_k \tag{7a}$$

$$\text{subject to} \quad P_{\mathrm{e2e}} = 1 - \prod_{k=1}^{K} (1 - P_{\mathrm{out},k,j'})^{r_k} \le e \ . \tag{7b}$$

is convex for low outage probability constraints e [10]. Unfortunately, no closed-form solution in terms of the power $\mathcal{P}_k$ per hop is available for this convex optimization problem, so that standard optimization tools, as presented in [11], have to be used. In order to ease calculation and to achieve a closed-form solution, the Improved Approximative Power Allocation (IAPA) has been proposed in [7]. In the sequel the applied approximations as well as the achieved closed-form solution for the power allocation will be summarized.

IV. Approximated Power Allocation

A. Near-Optimum Optimization Problem

For practical systems the low outage probability region (e.g., $e = 1\%$) is of interest requiring sufficiently large SNR, i.e., $x_k \to 0$. Based on this assumption the series expansion of the incomplete gamma function can be truncated after the leading term resulting in the approximation $\gamma(t_k, x_k) \approx t_k^{-1} x_k^{t_k}$ for $x_k \to 0$. Thus, a simple approximation for (3) is achieved by [6], [12]

$$\tilde{P}_{\mathrm{out},k,j} = \frac{t_k^{-1} x_k^{t_k}}{\Gamma(t_k)} = \frac{x_k^{t_k}}{\Gamma(t_k + 1)} \ . \tag{8}$$

In the sequel symbols labeled by tilde indicate approximated terms, i.e., $\tilde{P}_{\mathrm{out},k,j}$ denotes the approximation of $P_{\mathrm{out},k,j}$. By further approximating the product representation of the outage probability by a sum expression, the outage probability per hop (5) is expressed as $\tilde{P}_{\mathrm{out},k} = r_k \tilde{P}_{\mathrm{out},k,j'} = r_k x_k^{t_k} / \Gamma(t_k + 1)$ and the e2e outage probability (6) can be approximated by [6], [7]

$$\tilde{P}_{\mathrm{e2e}} = \sum_{k=1}^{K} \tilde{P}_{\mathrm{out},k} = \sum_{k=1}^{K} \frac{r_k x_k^{t_k}}{\Gamma(t_k + 1)} \ . \tag{9}$$

Considering this approximated e2e outage probability, the original optimization problem (7) can be simplified to

$$\text{minimize} \quad \mathcal{P}_{\mathrm{tot}} = \sum_{k=1}^{K} \mathcal{P}_k \tag{10a}$$

$$\text{subject to} \quad \tilde{P}_{\mathrm{e2e}} = \sum_{k=1}^{K} \tilde{P}_{\mathrm{out},k} = \sum_{k=1}^{K} \frac{r_k x_k^{t_k}}{\Gamma(t_k + 1)} \le e \ . \tag{10b}$$

As the union bound approximation (9) serves as an upper bound for the exact outage probability (6), i.e., $P_{\mathrm{e2e}} \le \tilde{P}_{\mathrm{e2e}}$, the solution of problem (10) automatically fulfills the original constraint P_{e2e} due to the more stringent e2e outage requirement [6]. The solution of (10) leads to a near-optimal power allocation with increased power consumption compared to the exact solution formulated in (7). However, this form permits the construction of analytical or even closed-form approaches.

For this near-optimum power allocation the derivation of the Lagrangian $L(\mathcal{P}_k, \lambda) = \sum_{k=1}^{K} \mathcal{P}_k + \lambda(\tilde{P}_{\mathrm{e2e}} - e)$ with respect to $\mathcal{P}_k$ has to be equal for all k and the outage constraint (10b) has to be met with equality, i.e., $\tilde{P}_{\mathrm{e2e}} = e$ [6]. By setting the derivations equal to a constant value, the relation

$$A = -Q \frac{\partial \tilde{P}_{\mathrm{e2e}}}{\partial \mathcal{P}_k} = \frac{r_k \tilde{P}_{\mathrm{out},k,j'}^{\frac{t_k + 1}{t_k}} \Gamma(t_k + 1)^{\frac{1}{t_k}}}{d_k^{\epsilon}} \ , \quad \forall k \ , \tag{11}$$

is derived and has to be fulfilled for all k [7]. The value of A will be $0 < A \ll 1$ as the lower outage probability region as well as distances $d_k \geq 1000\,\mathrm{m}$ between the VAAs are considered here. Several approaches for determining A have been presented in [6], [7] and are outlined in the sequel.

B. Near-Optimal Power Allocation (NOPA)

By rewriting (11) with respect to $\tilde{P}_{\mathrm{out},k,j'}$, the outage probability per hop $\tilde{P}_{\mathrm{out},k} = r_k \tilde{P}_{\mathrm{out},k,j'}$ is given by

$$\tilde{P}_{\mathrm{out},k} = r_k \left(\frac{A\, d_k^{\epsilon}}{r_k \Gamma(t_k+1)^{\frac{1}{t_k}}} \right)^{\frac{t_k}{t_k+1}} = a_k \cdot A^{\frac{t_k}{t_k+1}} \ , \qquad (12)$$

where the coefficients

$$a_k = r_k \left(\frac{d_k^{\epsilon}}{r_k \Gamma(t_k+1)^{\frac{1}{t_k}}} \right)^{\frac{t_k}{t_k+1}} = \left(\frac{r_k\, d_k^{\epsilon \cdot t_k}}{\Gamma(t_k+1)} \right)^{\frac{1}{t_k+1}} \qquad (13)$$

contain the parameters of hop k. Using this result in (10b), the relation $e = \sum_{k=1}^{K} \tilde{P}_{\mathrm{out},k} = \sum_{k=1}^{K} a_k A^{\frac{t_k}{t_k+1}}$ is achieved to determine A. Thus, solving the optimization problem (10) is equivalent to calculating the constant A that fulfills this equation. Note that this result for A depends only on the system configuration, but is independent of the system-wide parameters collected in Q. A near-optimal power allocation is achieved by rewriting the outage relation as a polynomial in A

$$f_a(A) = \sum_{k=1}^{K} a_k \cdot A^{\frac{t_k}{t_k+1}} - e \qquad (14)$$

and searching for its real-valued and positive root A_a, which can be determined by applying standard root-finding algorithms. The outage probabilities per hop $\tilde{P}_{\mathrm{out},k}$ can then be achieved by using the root A_a of $f_a(A)$ in (12). Applying this result to (8) for determining x_k, the near-optimal power allocation $\mathcal{P}_k^{\star}$ per hop follows from (4)

$$\begin{aligned} \mathcal{P}_k^{\star} &= \frac{Q d_k^{\epsilon} t_k}{x_k} = Q d_k^{\epsilon} t_k \left(\frac{r_k}{\tilde{P}_{\mathrm{out},k} \Gamma(t_k+1)} \right)^{1/t_k} \\ &= Q t_k \left(\frac{r_k d_k^{\epsilon \cdot t_k}}{\Gamma(t_k+1)} \right)^{\frac{1}{t_k+1}} A_a^{-\frac{1}{t_k+1}} = Q t_k a_k A_a^{-\frac{1}{t_k+1}} \end{aligned} \qquad (15)$$

and the total transmit power $\mathcal{P}_{\mathrm{tot}}^{\star} = \sum_{k=1}^{K} \mathcal{P}_k^{\star}$ corresponds to

$$\mathcal{P}_{\mathrm{tot}}^{\star} = Q \sum_{k=1}^{K} t_k \left(\frac{r_k d_k^{\epsilon \cdot t_k}}{\Gamma(t_k+1) A_a} \right)^{\frac{1}{t_k+1}} = Q \sum_{k=1}^{K} t_k a_k A_a^{-\frac{1}{t_k+1}} \ . \qquad (16)$$

Unfortunately, no closed-form solution is available for determining the roots of polynomials of arbitrary degrees and, consequently, no closed-form solution is achieved for the power allocation problem, yet.

C. Approximative Power Allocation (APA)

A simple closed-form solution can be derived under the assumption, that the number of transmit nodes per hop t_k for $k \geq 2$ is so large that the approximation $\frac{t_k}{t_k+1} \approx 1$ is valid. Using this assumption and the fact that the source contains only one antenna, the polynomial (14) is replaced by

$$f_b(A) = b_1 \cdot A^{\frac{1}{2}} + A \cdot \sum_{k=2}^{K} b_k - e \qquad (17)$$

with coefficients achieved from (13) using $t_1 = 1$ and $\frac{t_k}{t_k+1} \approx 1$

$$b_1 = (r_1 d_1^{\epsilon})^{\frac{1}{2}} \ \text{ and } \ b_k = d_k^{\epsilon} \Gamma(t_k+1)^{-\frac{1}{t_k}} \ \text{ for } k \geq 2\ . \qquad (18)$$

Equation (17) is a polynomial of degree two with positive root

$$A_b^{\frac{1}{2}} = \frac{\sqrt{b_1^2 + 4e \sum_{k=2}^{K} b_k} - b_1}{2 \sum_{k=2}^{K} b_k} \ . \qquad (19)$$

With this solution A_b the outage probability $\tilde{P}_{\mathrm{out},k}$ and the power allocation $\mathcal{P}_k^{\star}$ per hop can be determined [6], [7]. Due to the applied assumption for the number of transmit nodes per hop, this approach leads to rather inaccurate solutions for small t_k resulting in an increased total transmit power. To overcome this draw-back but still achieving a closed-form expression for the power allocation, the subsequent improved APA has been developed in [7].

D. Improved Approximative Power Allocation (IAPA)

To achieve an accurate but also closed-form solution for the approximated optimization problem (10), the IAPA approach incorporates the solution A_b (19) into the calculation of the root A_a of $f_a(A)$ (14). This leads to an approximated solution for the optimization parameter A_a given by [7]

$$\tilde{A}_a = e^2 \left(a_1 + \sum_{k=2}^{K} a_k \cdot A_b^{\frac{t_k-1}{2t_k+2}} \right)^{-2} \ . \qquad (20)$$

Note that this approximation for the exact root A_a of $f_a(A)$ leads to a power allocation which fulfills the e2e outage requirement e in general. Thereby, an approximation for the root of the higher-order equation (14) has been found, which can then be used for the power allocation.

Theorem 1 (IAPA): For an arbitrary number of nodes t_k per VAA and a given e2e outage probability requirement e, the near-optimal power allocation $\mathcal{P}_k^{\star}$ corresponds to

$$\mathcal{P}_k^{*} = Q \cdot t_k \left(\frac{r_k d_k^{\epsilon \cdot t_k}}{\Gamma(t_k+1)} \right)^{\frac{1}{t_k+1}} \cdot \tilde{A}_a^{-\frac{1}{t_k+1}} \qquad (21)$$

where the optimized system parameter

$$\tilde{A}_a = e^2 \left(\sqrt{r_1 d_1^{\epsilon}} + \sum_{k=2}^{K} \left(\frac{r_k d_k^{\epsilon \cdot t_k}}{\Gamma(t_k+1)} \right)^{\frac{1}{t_k+1}} \cdot A_b^{\frac{t_k-1}{2t_k+2}} \right)^{-2} \qquad (22)$$

is determined by

$$A_b^{\frac{1}{2}} = \frac{\sqrt{r_1 d_1^{\epsilon} + 4e \sum_{k=2}^{K} d_k^{\epsilon} \cdot \Gamma(t_k+1)^{-\frac{1}{t_k}}} - \sqrt{r_1 d_1^{\epsilon}}}{2 \sum_{k=2}^{K} d_k^{\epsilon} \cdot \Gamma(t_k+1)^{-\frac{1}{t_k}}} \ . \qquad (23)$$

In this theorem the given equations for A_b and $\tilde{A}_a$ have been achieved from (19) and (20) by replacing the coefficients a_k and b_k using (13) and (18), respectively. Thus, the dependency of the power allocation solution from the system parameters is illustrated.

V. Investigations

In the sequel the closed-form solution IAPA is applied to investigate properties of distributed MIMO multi-hop networks.

A. Relative Power Ratio Per Hop

Considering (16) and (21), the fraction of the total transmit power assigned to hop k can be expressed by

$$\vartheta_k = \frac{\mathcal{P}_k^\star}{\mathcal{P}_{\text{tot}}^\star} = \frac{t_k \cdot a_k \cdot \tilde{A}_a^{-\frac{1}{t_k+1}}}{\sum_{\kappa=1}^{K} t_\kappa \cdot a_\kappa \cdot \tilde{A}_a^{-\frac{1}{t_\kappa+1}}} . \quad (24)$$

As will be demonstrated in Section VI, a large fraction of the overall power is allocated to the first hop. Thus, it is evident to define also the fraction $\xi_k = \mathcal{P}_k^\star/\mathcal{P}_1^\star$ of power assigned to hop k with respect to the power $\mathcal{P}_1^\star$ transmitted by the source (i.e., $\xi_1 = 1$ holds by definition)

$$\xi_k = \frac{\mathcal{P}_k^\star}{\mathcal{P}_1^\star} = \frac{t_k a_k \tilde{A}_a^{-\frac{1}{t_k+1}}}{a_1 \tilde{A}_a^{-\frac{1}{2}}} = \frac{t_k}{\sqrt{r_1 d_1^\epsilon}} \left(\frac{r_k d_k^{\epsilon \cdot t_k}}{\Gamma(t_k+1)} \tilde{A}_a^{\frac{t_k-1}{2}} \right)^{\frac{1}{t_k+1}}, \quad (25)$$

where (13) was used to simplify the expression. With this variable ξ_k also the power fraction (24) is given due to the relation $\vartheta_k = \xi_k / \sum_{\kappa=1}^{K} \xi_\kappa$. Thus, closed-form expressions for the relative power assignment per hop have been achieved for arbitrary system configurations.

From (24) it is obvious, that the portion of power occupied by hop k is independent from the system-wide parameters bandwidth, noise power and data rate combined in the variable Q. If for example $\mathcal{P}_{\text{tot},1}^\star$ is known for a data rate R_1, respectively a configuration Q_1, the total power for another configuration Q_2 is simply given by

$$\mathcal{P}_{\text{tot},2}^\star = \frac{Q_2}{Q_1} \cdot \mathcal{P}_{\text{tot},1}^\star = \frac{2^{\frac{R_2}{W}} - 1}{2^{\frac{R_1}{W}} - 1} \cdot \mathcal{P}_{\text{tot},1}^\star \quad (26)$$

and the transmit power for node k corresponds to

$$\mathcal{P}_{k,2}^\star = \vartheta_k \cdot \mathcal{P}_{\text{tot},2}^\star = \frac{Q_2}{Q_1} \cdot \vartheta_k \cdot \mathcal{P}_{\text{tot},1}^\star = \frac{2^{\frac{R_2}{W}} - 1}{2^{\frac{R_1}{W}} - 1} \cdot \mathcal{P}_{k,1}^\star . \quad (27)$$

Thus, the optimization problem for one system configuration (i.e., number of hops k, distances d_k, and number of receive and transmit nodes per VAA) has to be solved only once for an arbitrary Q. The power assignment for the data rate of interest is then easily achieved by (26) and (27), respectively.

B. Same Number of Nodes per VAA and Equal VAA Distances

Subsequently the special case of an equal number of nodes per VAA with an constant distance $d = d_k$ for $k = 1, \ldots, K$ between the VAAs is investigated. A practical example for such a symmetric architecture is given by a wireless backhaul network with fixed relays. Let t denote the number of nodes per VAA, then $t_k = t$ for $2 \le k \le K$ and $r_k = t$ for $1 \le k \le K-1$ follows. As one antenna is assumed at the source and at the destination we have $t_1 = r_K = 1$. With these assumptions the coefficients (13) simplify to

$$a_1 = \sqrt{td^\epsilon} , \; a_k = \left(\frac{td^{\epsilon \cdot t}}{\Gamma(t+1)} \right)^{\frac{1}{t+1}} \text{ and } a_K = a_k t^{-\frac{1}{t+1}} \quad (28)$$

and the coefficients (18) become $b_1 = a_1$ and $b_k = d^\epsilon \Gamma(t+1)^{-\frac{1}{t}}$. Using these values, the root of the second order polynomial is

$$A_b^{\frac{1}{2}} = \frac{\sqrt{td^\epsilon + 4ed^\epsilon \Gamma(t+1)^{-\frac{1}{t}}} - \sqrt{td^\epsilon}}{2(K-1)d^\epsilon \Gamma(t+1)^{-\frac{1}{t}}} . \quad (29)$$

and the IAPA approximation for the root of polynomial (14) is given by

$$\begin{aligned} \tilde{A}_a &= e^2 \left[a_1 + \left(K - 2 + t^{-\frac{1}{t+1}} \right) a_k A_b^{\frac{t_k-1}{2t_k+2}} \right]^{-2} \\ &= e^2 \left[\sqrt{td^\epsilon} + \left(K - 2 + t^{-\frac{1}{t+1}} \right) \left(\frac{td^{\epsilon \cdot t}}{\Gamma(t+1)} A_b^{\frac{t_k-1}{2}} \right)^{\frac{1}{t+1}} \right]^{-2} . \end{aligned} \quad (30)$$

Furthermore, the closed-form expression for $\mathcal{P}_{\text{tot}}^\star$ (16)

$$\mathcal{P}_{\text{tot}}^\star = Q \left(a_1 \tilde{A}_a^{\frac{1}{2}} + \left(K - 2 + t^{-\frac{1}{t+1}} \right) t a_k \tilde{A}_a^{-\frac{1}{t+1}} \right) \quad (31)$$

is achieved and the power ratio (25) reduces to

$$\xi_k = \left(t^{t+3} \cdot d^{\epsilon \cdot (t-1)} \cdot \Gamma(t+1)^{-2} \cdot \tilde{A}_a^{(t-1)} \right)^{\frac{1}{2t+2}} \quad (32)$$

for $2 \le k \le K-1$ and $\xi_K = \xi_k \, t^{-\frac{1}{t+1}}$. Finally, the ratio of $\mathcal{P}_{\text{tot}}^\star$ assigned to hop $2 \le k \le K-1$ is given by

$$\vartheta_k = \left(\xi_k^{-1} + K - 2 + t^{-\frac{1}{t+1}} \right)^{-1} . \quad (33)$$

For the first and the last hop this ratio specifies to $\vartheta_1 = \vartheta_k/\xi_k$ and $\vartheta_K = \vartheta_k t^{-\frac{1}{t+1}}$, respectively. With these expressions relatively simple closed-form relations for the power assignment in symmetric relay networks have been achieved. A similar derivation can also be done for more general system configuration, e.g., same of number of nodes per VAA but varying distances. However, this leads to more complex relations.

VI. Performance Analysis

In the sequel the performance of the optimum power allocation (OPT) achieved by numerical optimization w.r.t (7) and IAPA are investigated for distributed MIMO multi-hop networks with K hops, the same number of relaying nodes $t = t_k$, $2 \le k \le K$, per hop, equal distance d between hops, and path-loss exponent $\epsilon = 3$. Furthermore, it is assumed that the e2e communication over $W = 5$ MHz should meet an e2e outage probability constraint $e = 1\%$ for noise power $N_0 = -174\,\text{dBm}$ according to the UMTS standard.

Fig. 2 shows the total power consumption for $t = 3$ nodes per hop, varying number of hops but a common distance of $d = 1\,\text{km}$ between the hops. Thus, the distance of source and destination varies with the number of hops and equals $K \cdot d$. From the figure it is obvious, that our suboptimum but efficient IAPA approach results in almost the same total power $\mathcal{P}_{\text{tot}}$ as the optimum allocation method. However, this result is achieved with significantly lower complexity. To even see the difference, a zoom into the graph for $K = 7$ has been added.

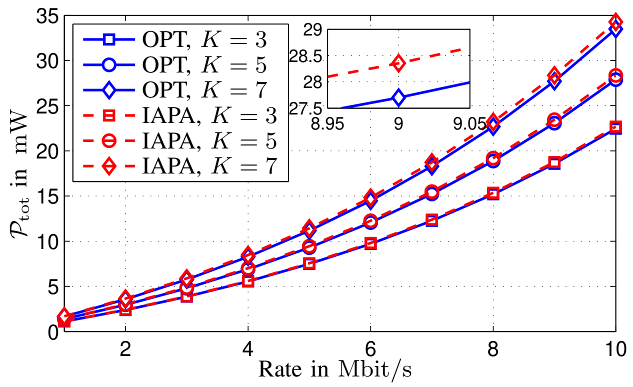

Fig. 2. Total transmit power for OPT and IAPA for $t = 3$ nodes per hop, common distance of $d = 1$ km between hops and $K = \{3, 5, 7\}$ hops.

The dependency of the required transmit power from the number of nodes t per hop can be observed from Fig. 3 for a fixed data rate of $R = 5$ Mbit/s. Based on this figure, or analytical derivations with respect to (31), it is possible to determine the optimal number of nodes per hop. It is for example favorable to use $t = 2$ nodes in case of $K = 3$ hops, but $t = 3$ nodes for a system with $K = 5$ hops. This figure highlights also the strong reduction of total transmit power required for a distributed MIMO architecture in comparison to a SISO relaying system, i.e., for $t = 1$.

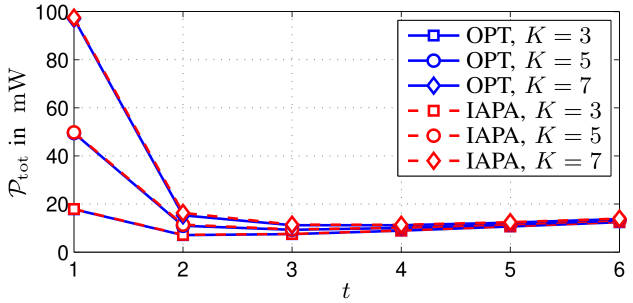

Fig. 3. Total transmit power for OPT and IAPA versus number of nodes t per hop, common distance of $d = 1$ km, varying number of hops K and $R = 5$ Mbit/s.

Fig. 4 a) depicts the relative power increment (RPI) of IAPA defined by $\mathrm{RPI} = \mathcal{P}_{\mathrm{tot}}^{\mathrm{IAPA}} / \mathcal{P}_{\mathrm{tot}}^{\mathrm{OPT}}$. It can be observed that the power increment of our sub-optimum solution vanishes almost for larger number of nodes t and is rather small also for small t in case of small number of nodes. This demonstrates, that IAPA achieves a near-optimal power allocation for a wide range of system configurations. Consequently, a very powerful analytical approach has been achieved for optimizing and analyzing even complex multi-hop networks. Finally, Fig. 4 b) shows the fraction of power assigned to the hops by the optimum power allocation and by the expression (33) for a varying number of nodes t and $K = 5$ hops. Obviously, the developed closed-form relation leads to practically the same relative power assignment. Furthermore, it is evident that most of the power is transmitted by the source, as no diversity can be exploited in the first hop. Each hop $2 \leq k \leq K-1$ has to support t MISO system and, consequently, the same amount of power is assigned to these hops. Less power is spent for the last hop, as only one MISO system has to be supported.

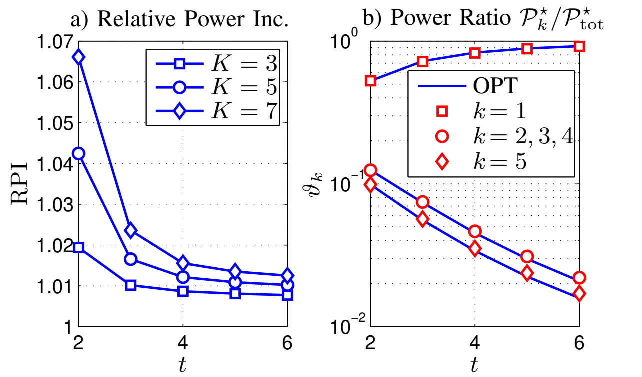

Fig. 4. a) Relative power increment of IAPA and b) fraction of $\mathcal{P}_{\mathrm{tot}}^\star$ assigned to hop k for varying number of nodes t, $d_k = 1$ km, $K = 5$ hops.

VII. Summary and Conclusions

In this paper the Improved Approximative Power Allocation (IAPA) approach was considered for the optimization of distributed MIMO multi-hop networks. This closed-form solution leads to an almost optimal power assignment also for complex relaying systems. Based on this approach the fraction of total power assigned to the hops was investigated and analytical relations for symmetric relaying systems have been presented. The shown performance results verify the applicability of our IAPA approach for optimizing and analyzing multi-hop networks. In future investigations the assumption, that the system is in outage if any MISO link is in outage, will be relaxed and the impact of MAC protocols will be considered.

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