LINEARISATION OF TRANSMITTER AND RECEIVER NONLINEARITIES IN OPTICAL OFDM TRANSMISSION

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Abstract In this paper, linearisation of transmitter and receiver nonlinearities of an optical Intensity Modulation/Direct Detection system using digital signal processing is presented and analysed. The system performance is estimated on basis of the signal-to-interference-and-noise ratio. Furthermore, the bit error performance of a linearised system is compared to a non-linearised one.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) was recently proposed for optical long-haul high-speed data transmission over single-mode fibers due to the simplicity of equalization of the dispersion dominated optical channel, e.g., [1]. However, the modulator and detector components of the optical transmission system exhibit severe nonlinearities [2] which pose a challenge to OFDM. The impact of these nonlinear effects was presented in a previous work [3], where also the system performance's dependence on the setup parameters was analysed.
In this paper, linearisation of transmitter and receiver nonlinearities using predistortion functions is presented and analysed with regard to the signal-to-interference-and-noise ratio (SINR) at the receiver in dependence on setup parameters. Additionally, the bit error performance of a linearised system in dependence on the setup parameters is simulated and compared with the non-linearised system in [3].

2. System model

The Zero-IF (Intermediate Frequency) Intensity Modulation/Direct Detection (IM/DD) system considered in this paper is able to transmit

real valued signals only, thus, the OFDM signal has to be crafted in a special manner to ensure that this requirement is fulfilled, i.e. by complex conjugate extension of subcarriers. Note that the requirement of a real-valued time domain signal does not restrict the choice of the modulation format on the subcarriers themselves, a complex valued modulation format such as Quaternary Phase Shift Keying (QPSK) is applicable and will be used for simulations later on.
For the theoretical considerations made here, the OFDM signal is modelled by a real valued, zero mean process $x(t)$ with gaussian probability density function and variance σ_x^2.
In the following, an IM/DD system in back-to-back operation with no optical filters and amplifiers, i.e. with no optical channel involved is regarded only, resulting in a system with a highly nonlinear characteristic, caused by the modulator and detector components, which will be discussed in the following.
An extension of our considerations to a scenario including the optical fiber and filters could be performed by introduction of complex valued processes and by means of e.g. Hammerstein or Wiener models [4, 5], which are commonly used for analysis of memory nonlinearities. This extension is beyond the scope of this work, but is simplified with the approach proposed in this paper compared to the non-linearised case considered in [3].

The overall nonlinear hardware characteristic of the analysed system can be described by the expression

$$r(t) = \beta^2 \cos^2(g_{-\pi/2}^{0}(m \cdot s(t) + u_{\text{bias}})), \tag{1}$$

that maps a modulator input signal $s(t)$ onto a detector output signal $r(t)$. In this equation, $g_{-\pi/2}^{0}(\cdot)$ represents the characteristic of a hard clipping device with fixed thresholds $\vartheta_{\text{low}} = -\pi/2$ and $\vartheta_{\text{high}} = 0$, m and u_{bias} are variable hardware setup parameters, as is the power scaling factor β. The cosine characteristic is caused by a Mach-Zehnder modulator (MZM) which converts from the electrical into the optical domain, while the square operation is performed in the photo diode (PD), which performs conversion from the optical into the electrical domain by detection of the instantaneous optical power. Fig. 1 shows this system, the noise term $n(t)$ depicted there represents additive noise, but is neglected for the moment.

In a non-linearised system, the OFDM signal is directly used as input to the above system, i.e. $s(t) = x(t)$, but in the following considerations, the nonlinearity is supposed to be linearised employing digital signal processing at both transmitter as well as receiver side. This is accomplished by means of nonlinear functions $s(t) = \varsigma(x(t))$ at the transmitter side

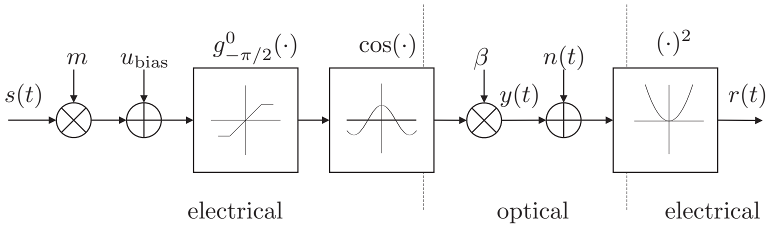

Figure 1. Block diagram of the physical components of the transmission system

and $z(t) = \nu(r(t))$ at the receiving side, resulting in an overall system with output

$$z(t) = \nu(\beta^2 \cos^2(g^{0}_{-\pi/2}(m \cdot \varsigma(x(t)) + u_{\text{bias}}))), \tag{2}$$

that is linear in a certain operating range.
When excited with a gaussian process $x(t)$, the system output signal $z(t)$ again is a stochastic process which generally neither is zero-mean, nor follows a gaussian distribution. This signal will be decomposed into a signal and an interference term in the next section.

2.1 Linearisation Approach

If predistortion is performed by use of an arc cosine expression

$$\varsigma(\xi) = \frac{1}{m}\big(- \arccos(m_{\text{pre}} \cdot \xi + b_{\text{pre}}) - u_{\text{bias}}\big), \tag{3}$$

similar to the arc sine function proposed in [6] for coherent systems, and at the receiver side the square root of the detector output signal $r(t)$ is taken, i.e.

$$z(t) = \nu(r(t)) = \sqrt{r(t)}, \tag{4}$$

the resulting overall system can be described by

$$z(t) = \beta g^{1}_{0}(m_{\text{pre}} \cdot x(t) + b_{\text{pre}}), \tag{5}$$

with $g^{1}_{0}(\cdot)$ being a hard clipping function with lower threshold 0 and upper threshold 1.
m_{pre} and b_{pre} are user-selectable setup parameters that are introduced to control the system performance by means of operation point (b_{pre}) and drive level (m_{pre}), their exact influence on the system performance will by analysed in the next section. Note that m and u_{bias} in (1) are setup parameters as well, which are controlling drive level and operation point in the non-linearised case, but in contrast to the former ones, these are representing physical quantities, i.e. voltage gain and bias voltage. In

the linearised case, these are kept fixed at an arbitrary value, but must be known to the digital preprocessing, as (3) implies.
Since both pairs of system parameters are user-controllable and determine the system performance in the linearised or non-linearised case, respectively, there exists a correspondence between them. This has to be regarded especially when these cases are supposed to be compared, as it will be the case later. Examining the MZM input signal u_{MZM} within the clipping thresholds:

$$u_{\mathrm{MZM}} = m \cdot s(t) + u_{\mathrm{bias}}, \tag{6}$$

it evaluates for the non-predistorted case, i.e. $s(t) = x(t)$, to $u_{\mathrm{MZM}} = m \cdot x(t) + u_{\mathrm{bias}}$, while in the linearised case, it equals

$$u_{\mathrm{MZM}} = -\arccos(m_{\mathrm{pre}} \cdot x(t) + b_{\mathrm{pre}}), \tag{7}$$

which implies that a predistortion bias b_{pre} at the MZM input will cause a physical bias

$$u_{\mathrm{bias}} = -\arccos\left(b_{\mathrm{pre}}\right), \tag{8}$$

which can be found by Taylor series expansion of (7) around $x(t) = 0$. The linear term of this expansion is equivalent to the driving level m and evaluates to

$$m = \frac{m_{\mathrm{pre}}}{\sqrt{1 - b_{\mathrm{pre}}^2}} \quad . \tag{9}$$

This correspondence will be used when the performance of the linearised and the non-linearised case will be compared later on.

3. Stochastic Analysis of the System Output Process

As motivated above, the system performance of the linearised system depends on the system setup parameters m_{pre} and b_{pre}, analogue to the non-linearised system, which depends on the hardware system setup parameters. To analyse this dependence, the same approach as in [3] is used: Modelling the output

$$z(t) = \beta g_0^1(m_{\mathrm{pre}} \cdot x(t) + b_{\mathrm{pre}}), \tag{10}$$

of the linearised system as a scaled version of $x(t)$ with an additional, uncorrelated distortion term $d(t)$ [7]

$$z(t) = \alpha_{\mathrm{lin}} \cdot x(t) + d(t), \tag{11}$$

the scaling factor α_{lin} can be determined by means of the crosscorrelation

$$r_{XZ}(\tau) = \mathrm{E}\left\{x^*(t) z(t+\tau)\right\} = \alpha_{\mathrm{lin}} r_{XX}(\tau) \quad , \tag{12}$$

since $x(t)$ is zero mean and $d(t)$ is uncorrelated to it. $r_{XZ}(0)$ can be calculated analytically using the definition of the expectation in (12):

$$\begin{aligned}
r_{XZ}(0) &= \mathrm{E}\{x^* z\} \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^* z p_{\mathrm{X,Z}}(x,z)dxdz \\
&= \int_{-\infty}^{\infty} x\beta g_0^1(m_{\mathrm{pre}}\cdot x + b_{\mathrm{pre}})p_{\mathrm{X}}(x)dx \\
&= \beta\cdot\frac{m_{\mathrm{pre}}\sigma_x^2}{2}\left[\mathrm{erf}\left(\frac{1-b_{\mathrm{pre}}}{\sqrt{2}m_{\mathrm{pre}}\sigma_x}\right)-\mathrm{erf}\left(\frac{-b_{\mathrm{pre}}}{\sqrt{2}m_{\mathrm{pre}}\sigma_x}\right)\right].
\end{aligned} \tag{13}$$

Rearranging the last term in (12) for $\tau = 0$ using $r_{XX}(0) = \sigma_x^2$, we get

$$\alpha_{\mathrm{lin}} = \beta\cdot\frac{m_{\mathrm{pre}}}{2}\left[\mathrm{erf}\left(\frac{1-b_{\mathrm{pre}}}{\sqrt{2}m_{\mathrm{pre}}\sigma_x}\right)-\mathrm{erf}\left(\frac{-b_{\mathrm{pre}}}{\sqrt{2}m_{\mathrm{pre}}\sigma_x}\right)\right], \tag{14}$$

where $\mathrm{erf}(\cdot)$ is the well-known error function. This factor denotes the end-to-end amplitude gain of the OFDM signal.
The system output power $\mathrm{E}\left\{|z(t)|^2\right\}$ can be calculated analytically as

$$\begin{aligned}
\mathrm{E}\left\{|z|^2\right\} &= \int_{-\infty}^{\infty}|z|^2 p_{\mathrm{Z}}(z)dz \\
&= \beta^2\int_{-\infty}^{\infty}\left(g_0^1(m_{\mathrm{pre}}\cdot x + b_{\mathrm{pre}})\right)^2 p_{\mathrm{X}}(x)dx \\
&= \beta^2\left[\frac{m_{\mathrm{pre}}\sigma_x}{\sqrt{2\pi}}\left(b_{\mathrm{pre}}\mathrm{e}^{-\frac{b_{\mathrm{pre}}^2}{\sqrt{2}m_{\mathrm{pre}}\sigma_x}}-(1+b_{\mathrm{pre}})\mathrm{e}^{-\frac{(1-b_{\mathrm{pre}})^2}{\sqrt{2}m_{\mathrm{pre}}\sigma_x}}\right)\right. \\
&\quad + \frac{m_{\mathrm{pre}}^2\sigma_x^2+b_{\mathrm{pre}}^2+1}{2}\mathrm{erf}\left(\frac{1-b_{\mathrm{pre}}}{\sqrt{2}m_{\mathrm{pre}}\sigma_x}\right) \\
&\quad \left. - \frac{m_{\mathrm{pre}}^2\sigma_x^2+b_{\mathrm{pre}}^2}{2}\mathrm{erf}\left(\frac{-b_{\mathrm{pre}}}{\sqrt{2}m_{\mathrm{pre}}\sigma_x}\right)+\frac{1}{2}\right],
\end{aligned} \tag{15}$$

the power of the interference $\mathrm{E}\left\{|d(t)|^2\right\}$ thus is known using (11):

$$\mathrm{E}\left\{|d(t)|^2\right\} = \mathrm{E}\left\{|z(t)|^2\right\} - \alpha_{\mathrm{lin}}^2\sigma_x^2 \quad . \tag{16}$$

Since for practical reasons the DC subcarrier of an OFDM system usually is not used for data transmission, only the variance $\sigma_d^2 = \mathrm{E}\left\{|d(t)|^2\right\} - \mu_d^2$ of the distortion term is of interest regarding the deterioration of the system performance. For this reason, an expression for μ_d has to be found. Since $x(t)$ is zero mean, μ_d is identical to the mean μ_z of $z(t)$ and can be calculated by

$$\begin{aligned}
\mu_d = \mu_z = \mathrm{E}\{z\} &= \int_{-\infty}^{\infty} z p_{\mathrm{Z}}(z) dz \\
&= \beta \int_{-\infty}^{\infty} g_0^1(m_{\mathrm{pre}} \cdot x + b_{\mathrm{pre}}) p_{\mathrm{X}}(x) dx \\
&= \beta \left[\frac{m_{\mathrm{pre}}\sigma_x}{\sqrt{2\pi}} \left(\mathrm{e}^{-\frac{b_{\mathrm{pre}}^2}{\sqrt{2}m_{\mathrm{pre}}\sigma_x}} - \mathrm{e}^{-\frac{(1-b_{\mathrm{pre}})^2}{\sqrt{2}m_{\mathrm{pre}}\sigma_x}} \right) + \frac{1}{2} \right. \\
&\quad \left. + \frac{b_{\mathrm{pre}}-1}{2} \mathrm{erf}\left(\frac{1-b_{\mathrm{pre}}}{\sqrt{2}m_{\mathrm{pre}}\sigma_x} \right) - \frac{b_{\mathrm{pre}}}{2} \mathrm{erf}\left(\frac{-b_{\mathrm{pre}}}{\sqrt{2}m_{\mathrm{pre}}\sigma_x} \right) \right] .
\end{aligned} \tag{17}$$

Using these results, the signal-to-interference ratio (SIR) at the receiver is given by

$$\mathrm{SIR} = \frac{\alpha_{\mathrm{lin}}^2 \sigma_x^2}{\sigma_d^2} = \frac{\alpha_{\mathrm{lin}}^2 \sigma_x^2}{\mathrm{E}\{|z|^2\} - \mu_z^2 - \alpha_{\mathrm{lin}}^2 \sigma_x^2} \quad . \tag{18}$$

Note that this expression is independent of β since the occurrences of this variable cancel in this fraction, it is a function in b_{pre} and $m_{\mathrm{pre}} \cdot \sigma_x$ only.
As soon as additive noise is introduced, i.e. $n(t) \neq 0$ in Fig. 1, the signal-to-interference-and-noise ratio (SINR) can be formulated:

$$\mathrm{SINR} = \frac{\alpha_{\mathrm{lin}}^2 \sigma_x^2}{\mathrm{E}\{|z|^2\} - \mu_z^2 - \alpha_{\mathrm{lin}}^2 \sigma_x^2 + P_{\mathrm{n}}} \quad . \tag{19}$$

This measure not only depends on the noise power P_{n} and the parameters b_{pre} and $m_{\mathrm{pre}} \cdot \sigma_x$, but also on the parameter β. In practical systems, a constraint has to be applied, which will be performed in the following.

4. Power Constraint

In optical transmission, the transmitting power is constrained to a value of approximately $0\,\mathrm{dBm} = 1\,\mathrm{mW}$ to avoid nonlinear effects in the

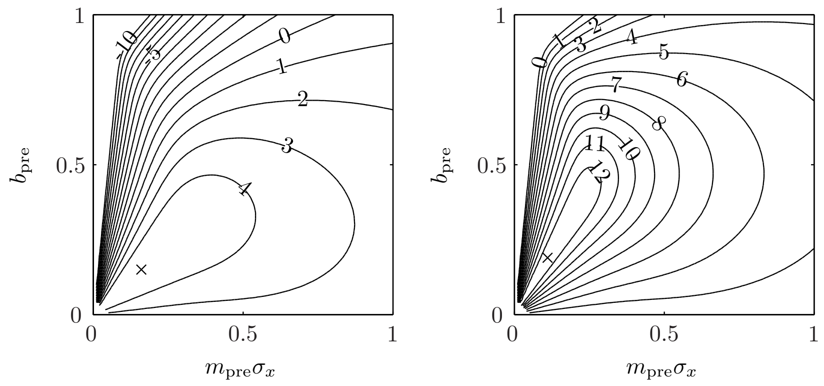

Figure 2. Signal-to-interference-and-noise ratio in dB for noise powers $P_{\mathrm{opt}}/P_{\mathrm{n}} =$ 10 dB (left), 20 dB (right) and varying parameters $b_{\mathrm{pre}}, m_{\mathrm{pre}}\sigma_x$

optical fiber. The instantaneous electric field on the optical fiber is denoted by $y(t)$ in Fig. 1. Its second order moment $\mathrm{E}\left\{|y(t)|^2\right\}$, i.e. the optical power, is identical to $\mathrm{E}\left\{|z(t)|^2\right\}$, since $z(t) = \sqrt{r(t)} = \sqrt{|y(t)|^2} = |y(t)|$. As shown in (15), it contains a factor β^2 whose purpose is to allow adjustment of the optical power. In practical systems, this adjustment is accomplished by variation of the laser power.

If the optical power is supposed to be fixed to a level P_{opt}, β has to be chosen such that

$$\begin{aligned}\mathrm{E}\left\{z^2\right\} = P_{\mathrm{opt}} = \beta^2 \Bigg[& \frac{m_{\mathrm{pre}}\sigma_x}{\sqrt{2\pi}} \left(b_{\mathrm{pre}} \mathrm{e}^{-\frac{b_{\mathrm{pre}}^2}{\sqrt{2} m_{\mathrm{pre}}\sigma_x}} - (1+b_{\mathrm{pre}}) \mathrm{e}^{-\frac{(1-b_{\mathrm{pre}})^2}{\sqrt{2} m_{\mathrm{pre}}\sigma_x}} \right) \\ & + \frac{m_{\mathrm{pre}}^2\sigma_x^2 + b_{\mathrm{pre}}^2 + 1}{2} \mathrm{erf}\left(\frac{1-b_{\mathrm{pre}}}{\sqrt{2} m_{\mathrm{pre}}\sigma_x} \right) \\ & - \frac{m_{\mathrm{pre}}^2\sigma_x^2 + b_{\mathrm{pre}}^2}{2} \mathrm{erf}\left(\frac{-b_{\mathrm{pre}}}{\sqrt{2} m_{\mathrm{pre}}\sigma_x} \right) + \frac{1}{2} \Bigg]\end{aligned} \tag{20}$$

is fulfilled. Using this constraint, the SINR (19) becomes a function only depending on b_{pre}, $m_{\mathrm{pre}}\sigma_x$ and $P_{\mathrm{opt}}/P_{\mathrm{n}}$. As an example, Fig. 2 shows the signal-to-interference-and-noise ratio for noise powers $P_{\mathrm{opt}}/P_{\mathrm{n}} = 10\,\mathrm{dB}$ (left) and 20 dB (right) in dB. The maxima have been found numerically and are denoted by "x" and correspond to parameter values $b_{\mathrm{pre}} = 0.15, m_{\mathrm{pre}}\sigma_x = 0.16$ in the first and $b_{\mathrm{pre}} = 0.19, m_{\mathrm{pre}}\sigma_x = 0.11$ in the second case. The resulting system performance for these parameter pairs will be evaluated in the following by means of bit error simulations.

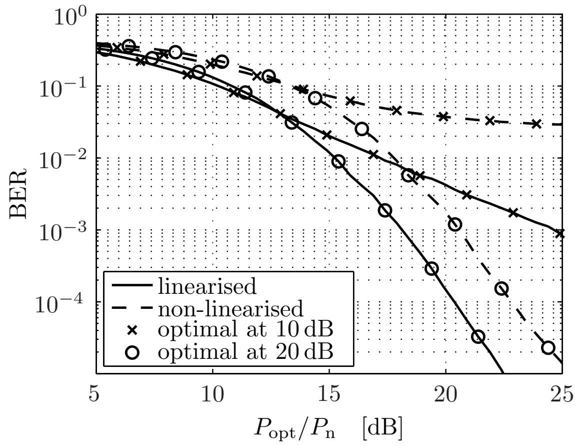

Figure 3. Bit error performance of a linearised IM/DD OFDM system for the optimal parameters $b_{\rm pre}, m_{\rm pre}\sigma_x$ in two operating points in dependence on the power ratio $P_{\rm opt}/P_{\rm n}$ and bit error performance of the non-linearised system as a reference

5. Simulation Results

The bit error performance of a linearised IM/DD optical OFDM system with 1024 subcarriers employing QPSK modulation has been evaluated by means of Monte Carlo simulations. Due to the requirement of a real valued time domain OFDM signal, DC and Nyquist frequency subcarrier are able to convey real valued symbols only, they have been set to zero in this case. For the same reason only 511 out of the remaining 1022 subcarriers are carrying independent information, while the remaining have been extended in conjugate complex fashion. Since here only the back-to-back case is considered, the length of the cyclic prefix has been set to zero. The solid lines in Fig. 3 show the bit error performance of this system for the optimal parameters $b_{\rm pre}, m_{\rm pre}\sigma_x$ for $P_{\rm opt}/P_{\rm n} = 10\,{\rm dB}$ and $20\,{\rm dB}$ as determined in the previous section. It can be seen that each one shows a lower bit error rate (BER) than the other at the corresponding power ratio $P_{\rm opt}/P_{\rm n}$ it was optimized for. For comparison, the bit error rates of the non-linearised system in [3] with separately optimized parameters have been plotted in dashed line style. A gain of approximately $2.5\,{\rm dB}$ can be observed for $P_{\rm opt}/P_{\rm n} = 20\,{\rm dB}$, for the case of $10\,{\rm dB}$, the gain is even higher, since in the non-linearised case, an error floor is reached.

Finally, the influence of the size of the resulting linear range after linearisation is analysed by means of bit error simulations. For this purpose, $b_{\rm pre}$ was fixed to a value of 0.5, i.e. the operation point was established in the middle of the linear range, while for $m_{\rm pre}\sigma_x$, values

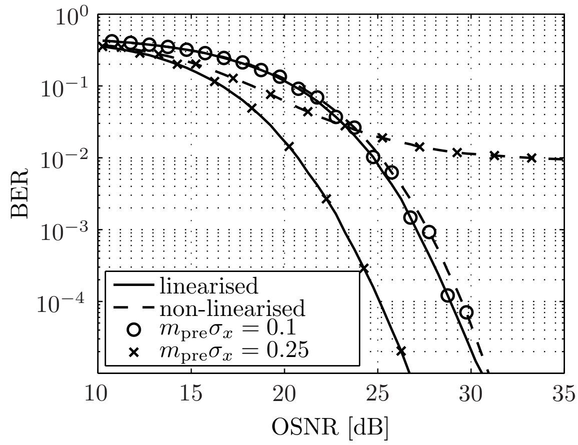

Figure 4. Bit error performance of a linearised IM/DD OFDM system in comparison to a non-linearised system

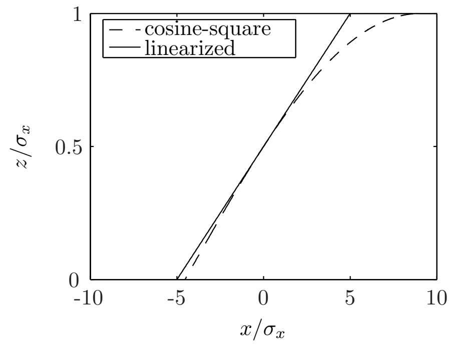

Figure 5. Comparison of characteristics of linearised and non-linearised system for $b_{\mathrm{pre}} = 0.5$ and $m_{\mathrm{pre}}\sigma_x = 0.1$

of 0.1 and 0.25 were chosen. For comparison, a non-linearised system with hardware system setup parameters u_{bias} and m chosen according to (8) and (9) was simulated. Fig. 4 shows the simulated bit error rates for the linearised system in solid line style and for the non-linearised system in dashed line style. For the abscissa labelling, the optical signal-to-noise ratio (OSNR) was chosen, a measure commonly used in the optical literature, relating the overall received optical signal power to the noise power in a defined bandwidth of 0.1 nm. For calculation of the OSNR, a transmission with 42.8 Gb/s at 1550 nm was assumed. It can be seen that the linearised and non-linearised system perform similarly for driving level $m_{\mathrm{pre}}\sigma_x = 0.1$, with a slight advantage for the linarised system. This can be explained by the fact that in a small region around the operation point the characteristics do not deviate significantly from each other, while the linear range of the linearised system is given by $-5\sigma_x \leq x \leq 5\sigma_x$, as can be seen in Fig. 5.

If $m_{\text{pre}}\sigma_x$ is increased to 0.25, α_{lin} is increased as well as can be verified by evaluation of (14), resulting in an improvement in the linearised case, even though the linear range is reduced to $-2\sigma_x \leq x \leq 2\sigma_x$. The non-linearised system is limited by interference introduced by its cosine-square characteristic, resulting in an error floor at a bit error rate of approximately 10^{-2} [3].

6. Conclusion

In our paper, an approach for linearisation of the cosine-square overall characteristic of an IM/DD optical OFDM system is presented. Its performance is estimated by means of the signal-to-interference-and-noise ratio, calculated by means of statistical measures of the system's output process. The required derivations were presented and also an analysis of a power constrained transmission system was introduced. Bit error simulations were performed and their results were compared to those for the non-linearised system. It was shown that linearisation by digital pre- and postprocessing is improving the system performance significantly.

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