Delay-Diversity in Multi-User Relay Systems with Interleave Division Multiple Access

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***Abstract*—In this paper, a new combination of a multiple access scheme and spatial diversity offered by relays is proposed. In order to support several users in a system, Interleave Division Multiple Access (IDMA) is suggested here due to the capability to deal with asynchronous transmission by the users. The relays apply Amplify-and-Forward combined with delay-diversity in order to exploit the spatial diversity without the need of synchronization of the relays. Furthermore, the IDMA detector can easily be extended to utilize the diversity very efficiently. So this combination of basic approaches can deal with asynchronous transmission of the users and the relays, respectively, resulting in a very flexible system with small signaling overhead.**

I. Introduction

IDMA is a non-orthogonal multiple access scheme proposed for future wireless communication systems because of several benefits. It offers a good multiple-access capability at moderate computational cost and is quite robust against multipath fading and asynchronism [1]. In contrast to CDMA, the interleavers are located after spreading and/or coding and the only means of user separation is the user-specific interleaver while the spreading and coding is the same for all users. At the receiver an iterative multi-user detector is applied which is an efficient approximation of the ML detector.

Relay assisted networks have been proposed to improve the reliability of communication between users that are effected by huge path loss and fading. In this scenario relays can significantly improve the system performance [2]. Usually, the channel access scheme in relay networks is assumed to be orthogonal which results in a rate loss that increases with the number of users and the number of relays. Non-orthogonal approaches for cooperative communication with relays like beamforming or distributed space time block codes [3] require either perfect global channel knowledge or at least a synchronization of all relays, respectively. This is only possible if large signaling overhead is spent. To circumvent these problems, delay-diversity is applied in order to exploit the spatial diversity. Synchronization and channel knowledge at the relays are not needed but, nevertheless, delay-diversity enables to achieve full diversity. An equivalent frequency selective channel is observed at the receiver, which can be exploited similar to the multi-path case [4]. For IDMA this can be easily incorporated into the iterative detection and the complexity grows only linearly with the number of paths [1]. Recently, a different approach based on IDMA was proposed for a single-user relay assisted network in [5] in order to avoid the signaling overhead needed for space-time cooperative protocols. IDMA was used to distinguish the signals from different relays supporting only one user without the need of channel knowledge of other relays or synchronization. In contrast to this, we will use the properties of IDMA to support several users in a relay system at the same time and separate the signals from different relays with the help of an iterative detector for the multi-path scenario introduced by delay-diversity. Thus, the combination of IDMA and delay diversity provides a flexible asynchronous relay network with small signaling overhead.

II. System Description

We consider a the uplink of a system with multiple users U_ν, $1 \leq \nu \leq N$, several relays R_ℓ, $1 \leq \ell \leq L$, and one destination D as shown in Figure 1. All users transmit to all relays in a broadcast manner and the channels are assumed to be Rayleigh flat fading and constant within one transmission block. The complex channel coefficient between user U_ν and relay R_ℓ is $h_{\nu\ell}$ and between relay R_ℓ and the destination the channel coefficient is denoted by g_ℓ. All channel coefficients are assumed to be independent of each other.

A. IDMA Transmitter

A block diagram of the transmitting side incorporating IDMA is shown in Figure 2. Each user U_ν encodes his data $b_\nu[i]$ at bit index i with a rate $1/S$ code (denoted as CC in Figure 2) and interleaves the resulting BPSK-modulated code bits with the user-specific random interleaver Π_ν yielding $c_\nu[k]$ at time instant k. To support many users, the code rate should be sufficiently small, e.g., $1/8$ is chosen in this paper. Although it is known that unequal receive powers can increase the number of supportable users [6], all users and relays have the same transmit power P_U and P_R in this paper, respectively for simplicity. The relays receive the sum of all user signals at time instant k weighted with the corresponding channel coefficient and transmit power constraint and the additive white Gaussian noise

$$y_{R_\ell}[k] = \sum_{\nu=1}^{N} \sqrt{P_U} \cdot h_{\nu\ell} \cdot c_\nu[k] + n_{R_\ell}[k] \qquad (1)$$

with $\mathrm{E}\{|h_{\nu\ell}|^2\} = 1$. The variance of the additive white Gaussian noise $\sigma^2_{R_\ell}$ is assumed to be the same for all relays.

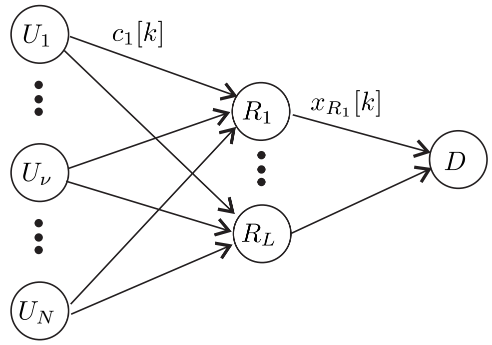

Fig. 1. System model for IDMA relay system with multiple users and relays

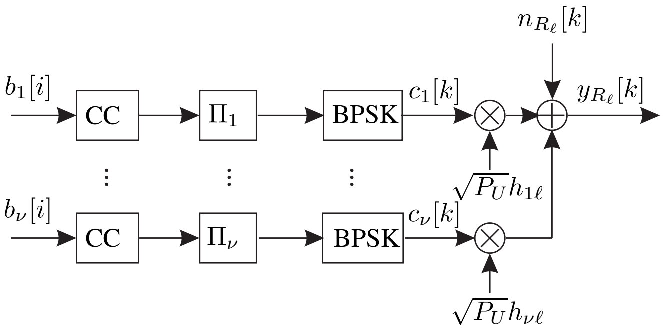

Fig. 2. Block diagram of an IDMA transmitter

B. Relay Protocol with Delay Diversity

Several relay protocols defining the relays' functionality were proposed. The most common ones are Decode-and-Forward (DF) and Amplify-and-Forward (AF) [2]. DF requires complete decoding of the received signals at the relays which means in our system setup an application of iterative multi-user detection (MUD) at each relay resulting in a huge computational complexity. Therefore, we only consider Amplify-and-Forward in this paper, i.e., each relay amplifies its received signal while fulfilling the individual power constraint. So the signal transmitted by the ℓ-th relay can be written as

$$x_{R_\ell}[k] = \alpha_\ell \cdot y_{R_\ell}[k] \quad \text{with} \quad \alpha_\ell = \sqrt{\frac{P_R}{\mathrm{E}\{|y_{R_\ell}|^2\}}} \ . \tag{2}$$

Since no additional signal processing is needed, the relays can be kept very simple.
Now the question is, how the relays should transmit their information to the destination. If all relays would transmit simultaneously over a Rayleigh fading channel and their distances to the destination are nearly the same, the resulting channel would be a flat Rayleigh fading channel and there would be no diversity gain. Applying a TDMA scheme would result in a large rate loss which increases with the number of relays. Hence all relays should transmit at the same time to avoid this rate loss. A good strategy in terms of performance is given by beamforming, which leads to constructive addition of all signal parts at the receiver, but this approach requires exact channel state information (CSI) at the transmitters also about all other relays' channels. Distributed space-time block codes show good performance without channel state information but require at least synchronous transmission of all relays. As these requirements are very hard to fulfill in a relay network, we consider delay diversity to provide spatial diversity. The main idea of delay diversity is the conversion of spatial diversity into frequency diversity which can be exploited, e.g., by a viterbi equalizer or a frequency domain equalizer (FDE). This scheme can achieve full diversity [4]. In the case of distributed relays each relay starts the transmission at a different time instant. The delays should be in the range of some code bit durations (for IDMA it is called chip). In a practical system these delays could be chosen randomly at each relay in a decentralized manner. In this case it is not guaranteed that all relays choose different delays but with a high probability at least some of the relays choose different values. Another possibility is to choose these delays centralized at the cost of some signaling. In this paper the latter approach is assumed without taking the signaling into account. It is assumed that the first relay transmits with delay $\tau_1 = 0$ and the other relays have delay $\tau_\ell = \ell - 1$, for $\ell = 2, .., L$. The effective channel between each user and the destination is then turned into a multi-path channel. Although the impact of the coefficients g_ℓ is the same for all transmit signals of the relays, the overall effective channel is different for all users due to the user-specific channel coefficients $h_{\nu\ell}$.
As shown in [1], an iterative IDMA detector can simply be extended to the multi-path case and is able to exploit the diversity at moderate cost of order $\mathcal{O}(NL)$. Therefore, we will apply this so-called *soft rake* (SR) detector to the proposed delay-diversity system for multi-user detection at the destination. A detailed description of the SR detector is given in the next section following the derivation in [1].

C. IDMA Receiver for Multi-path Scenarios

The received signal at the destination at time instant k is given by

$$\begin{aligned} y_D[k] &= \sum_{\ell=1}^{L} g_\ell \cdot x_{R_\ell}[k-\ell+1] + n_D[k] \\ &= \sum_{\ell=1}^{L} g_\ell \cdot \alpha_\ell \cdot y_{R_\ell}[k-\ell+1] + n_D[k] \\ &= \sum_{\ell=1}^{L} g_\ell \alpha_\ell \left(\sum_{\nu=1}^{N} \left(\sqrt{P_U} h_{\nu\ell} c_\nu[\kappa] + n_{R_\ell}[\kappa] \right) \right) + n_D[k] \end{aligned} \tag{3}$$

with $\kappa = k - \ell + 1$. The ℓ-th effective channel coefficient of user U_ν can be expressed as

$$h_n[\ell] = h_{\nu\ell} \cdot g_\ell \cdot \alpha_\ell \cdot \sqrt{P_U} \ . \tag{4}$$

The simplified block diagram of the iterative detection is shown in Figure 3. The basic principle of the interference canceler (IC) will be explained based on the effective channel in (4). If we focus on a specific user, e.g. $\nu = 1$, and assume only $L = 1$ relay, the signal can be split into a useful part,

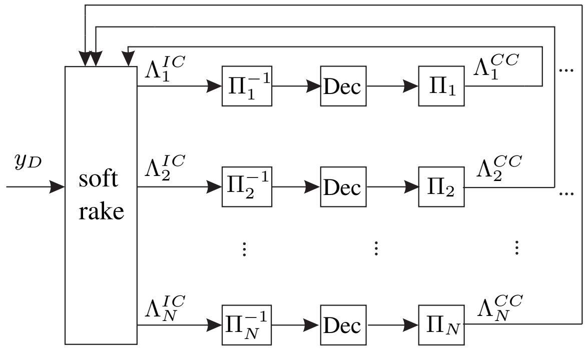

Fig. 3. Receiver structure for IDMA

interference and noise

$$\begin{aligned} y_D[k] &= h_\nu c_\nu[k] + \sum_{\mu\neq\nu} \left(h_\mu c_\mu[k] + \alpha_\ell g_\ell n_{R_\ell}[k]\right) + n_D[k] \\ &= h_\nu c_\nu[k] + \eta_\nu[k] \end{aligned} \quad (5)$$

with $\eta_\nu[k]$ containing the interference of all other users and the noise. The aim of the iterative detector is to estimate the interference and subtract this estimation denoted as $\overline{\eta}_\nu[k]$ from the received signal. This estimation of $\eta_\nu[k]$ is based on the estimation of all other users' code bits provided e.g. by the channel decoder in the last iteration by weighting the expectation values of the code bits of all interfering users with the corresponding effective channel coefficients. In the case of BPSK the expectation value is given by the $\tanh(x/2)$ of the extrinsic Log-Likelihood Ratio (LLR) at the output of the channel decoder denoted as Λ^{CC}

$$\mathrm{E}\{c_\nu[k]\} = \tanh(\Lambda_\nu^{CC}[k]/2) \quad (6)$$

and the error variance of this estimation is denoted as

$$\mathrm{Var}(c_\nu[k]) = 1 - (\mathrm{E}\{c_\nu[k]\})^2 \ . \quad (7)$$

At the output of the interference cancellation we obtain several variables used for LLR calculation. With

$$\mathrm{Var}(y_D[k]) = \sum_{\nu=1}^{N} |h_\nu|^2 \mathrm{Var}(c_\nu[k]) + \sigma_D^2 \quad (8)$$

$$\overline{y}_D[k] = \sum_{\nu=1}^{N} h_\nu \mathrm{E}\{c_\nu[k]\} \quad (9)$$

$$\begin{aligned} \overline{\eta}_\nu[k] &= \sum_{\mu\neq\nu}^{N} h_\mu \mathrm{E}\{c_\mu[k]\} \\ &= \overline{y}_D[k] - h_\nu \mathrm{E}\{c_\nu[k]\} \end{aligned} \quad (10)$$

we obtain the variance of $\eta_n[k]$ as

$$\mathrm{Var}(\eta_\nu[k]) = \mathrm{Var}(y_D[k]) - |h_\nu|^2 \mathrm{Var}(c_\nu[k]) \ . \quad (11)$$

Equations (11) and (10) are used to calculate the LLR of the k-th code bit of user ν

$$\Lambda_\nu^{IC}[k] = \mathrm{Re}\left\{4h_\nu^* \frac{y_D[k] - \overline{\eta}_\nu[k]}{\mathrm{Var}(\eta_\nu[k])}\right\} \quad (12)$$

where h_ν^* denotes the conjugate of h_ν. Equation (12) is similar for the AWGN case since the distribution of the residual interference is well approximated by a Gaussian distribution. This LLR is fed back to the SISO (soft input soft output) channel decoder for the next iteration.

The extension of this detector to the multi-path case is straight forward. Each path ℓ of each user's signal is evaluated separately and their corresponding LLRs are then summed up. Equations (5)-(12) are extended as follows:

$$y_D[k] = h_\nu[\ell] c_\nu[\kappa] + \eta_{\nu\ell}[k] \quad (13)$$

$$\mathrm{Var}(y_D[k]) = \sum_{\ell=1}^{L}\sum_{\nu=1}^{N} |h_\nu[\ell]|^2 \mathrm{Var}(c_\nu[\kappa]) + \sigma_D^2 \quad (14)$$

$$\overline{y}_D[k] = \sum_{\ell=1}^{L}\sum_{\nu=1}^{N} h_\nu[\ell] \mathrm{E}\{c_\nu[\kappa]\} \quad (15)$$

$$\overline{\eta}_{\nu\ell}[k] = \overline{y}_D[k+\ell-1] - h_\nu[\ell] \mathrm{E}\{c_\nu[k]\} \quad (16)$$

$\overline{\eta}_{\nu\ell}[k]$ now contains not only the interference introduced by the other users but also Inter-Symbol Interference (ISI) caused by himself.

$$\mathrm{Var}(\eta_{\nu\ell}[k]) = \mathrm{Var}(y_D[k+\ell-1]) - |h_\nu[\ell]|^2 \mathrm{Var}(c_\nu[k]) \quad (17)$$

$$\Lambda_{\nu\ell}^{IC}[k] = \mathrm{Re}\left\{4h_\nu^*[\ell] \frac{y_D[k+\ell-1] - \overline{\eta}_{\nu\ell}[k]}{\mathrm{Var}(\eta_{\nu\ell}[k])}\right\} \quad (18)$$

The output LLR of the IC is then simply the sum over all paths which corresponds to a Maximum Ratio Combiner (MRC).

$$\Lambda_\nu^{IC}[k] = \sum_{\ell=1}^{L} \Lambda_{\nu\ell}^{IC}[k] \quad (19)$$

D. SNR evaluation

Each effective channel coefficient in (4) results from a multiplication of two complex Gaussian variables which degrades the performance. As shown in [7] the performance of a two-hop Rayleigh fading relay system with AF protocol is worse than direct transmission if the same average SNR at the destination is assumed. Only in the asymptotic case of transmit power going to infinity the diversity is equal to the number of relays. For moderate SNR values our scheme will perform slightly worse than for one hop Rayleigh fading. Nevertheless, the performance will be shown to be still good for system loads $\beta = N/S$ up to about 2.5. To get an insight into the behavior of our relay system, we derive the Single-User Bound (SUB) of the SNR in the case of all channels are Rayleigh fading and the case of AWGN channels on the second hop. The SUB is reached, if the entire multiple access interference is known and canceled. The received signal at the destination concerning user U_ν can then be written as

$$y_D[k] = \sum_{\ell=1}^{L} g_\ell \alpha_\ell \left(\sqrt{P_U} h_{\nu\ell} c_\nu[\kappa] + n_{R_\ell}[\kappa]\right) + n_D[k] \quad (20)$$

with $\kappa = k - \ell + 1$. In the following we assume $P_R = P_U/L$ to ensure a meaningful comparison for systems with different number of relays in terms of diversity gains as the aim of our investigations is not the SNR gain due to increased number of relays but the diversity gain. The scaling factor α_ℓ in (2) can be approximated for a large number of users ($N \gg L$) by

$$\alpha_\ell \approx \sqrt{\frac{P_R}{NLP_R + \sigma^2_{R_\ell}}} \tag{21}$$

Now the output of the interference canceler simplifies to

$$y_D[k] - \overline{\eta}_\nu[k] \approx \sum_{\ell=1}^{L} g_\ell \sqrt{\frac{1}{NL + \frac{\sigma^2_{R_\ell}}{P_R}}} \left(\sqrt{P_U} h_{\nu\ell} c_\nu[\kappa] + n_{R_\ell}[\kappa]\right) + n_D[k] \tag{22}$$

with $\kappa = k - \ell + 1$. To derive the asymptotic SNR for the multi-path case we follow the approach of LLR combining as done in (19). Since the SNR after Maximum Ratio (or LLR) Combining is equal to the sum of all individual SNRs [8], we first derive the SNR for one distinct path from relay R_ℓ for one user U_ν.

$$\mathrm{SNR}_\ell \approx \frac{\frac{LP_R}{NL+\sigma^2_{R_\ell}/P_R} |g_\ell h_{\nu\ell}|^2}{\frac{1}{NL+\sigma^2_{R_\ell}/P_R} |g_\ell|^2 \sigma^2_{R_\ell} + \sigma^2_D} \, . \tag{23}$$

As all noise variances are assumed to be equal, i.e., $\sigma^2_{R_\ell} = \sigma^2_D$, the SNR can be simplified to

$$\mathrm{SNR}_\ell \approx \frac{|g_\ell h_{\nu l}|^2 LP_R}{|g_\ell|^2 \sigma^2_D + \left(NL + \frac{\sigma^2_D}{P_R}\right) \sigma^2_D} \, . \tag{24}$$

For the high SNR regime, i.e., $P_R \gg \sigma^2_D$ the part $\frac{\sigma^2_D}{P_R}$ tends to zero and we get the asymptotic SNR

$$\mathrm{SNR}_\ell \approx \frac{|g_\ell h_{\nu l}|^2 LP_R}{\sigma^2_D (|g_\ell|^2 + NL)} \, . \tag{25}$$

The approximated SNR after combining reads

$$\mathrm{SNR} = \sum_{\ell=1}^{L} \mathrm{SNR}_\ell \approx \sum_{\ell=1}^{L} \frac{|g_\ell h_{\nu l}|^2 LP_R}{\sigma^2_D (|g_\ell|^2 + NL)} \, . \tag{26}$$

The distribution of the SNR is difficult to obtain, because the distribution of $\frac{|g_\ell h_{\nu l}|^2}{\sigma^2_D(|g_\ell|^2+NL)}$ is unknown. The distribution of $|g_\ell h_{\nu l}|$ is known to be a Bessel function of the second kind [7], also known as double-Rayleigh distribution, but the derivation of the distribution of (26) is not available to the authors knowledge. In the case of AWGN channels on the second hop, i.e., $|g_\ell|^2 = 1$, the approximated SNR simplifies to

$$\mathrm{SNR} = \sum_{\ell=1}^{L} \frac{|h_{\nu l}|^2 LP_R}{\sigma^2_D (1 + NL)} \, . \tag{27}$$

In Figure 4 the impact of the distributions of the channel coefficients can be observed. The case of Rayleigh distributed channels on all hops is compared with the case of Rayleigh fading on the first hop but AWGN channels on the second hop. The loss of the Rayleigh fading case is obvious but the slope of the BER curves is similar, i.e., the diversity degree is

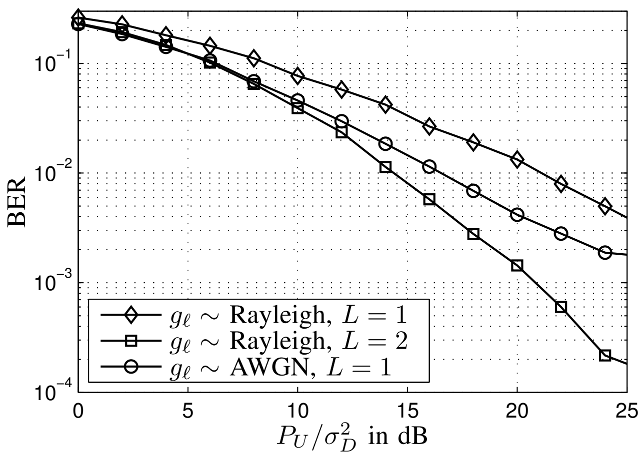

Fig. 4. BER for IDMA relay system with 8 users, Rayleigh fading and AWGN channel for g_ℓ

nearly the same in both cases. Furthermore, if we assume two relays, i.e. $L = 2$, our relay scheme with all hops Rayleigh fading outperforms the case of one relay with AWGN channel on the last hop.

III. Simulation Results

In this section we will show Monte-Carlo simulation results for different numbers of users and relays. As mentioned already in Section II, the sum power of all relays is kept constant even if the number of relays increases. A direct transmission between the users and the destination is not considered here.

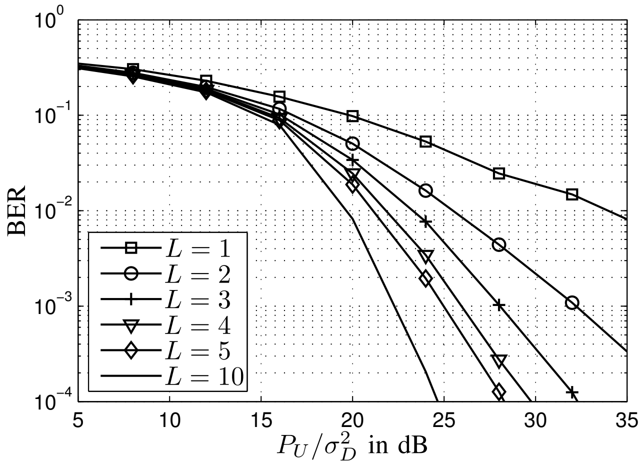

Fig. 5. BER for IDMA relay system with 16 users and different number of relays

In Figure 5 and 6 the bit error rate (BER) performance of an IDMA system with rate $1/8$ repetition coding, 16 and 20 users is shown corresponding to a user load of $\beta = 2$ and $\beta = 2.5$, respectively. All users transmit with the same power P_U and at the destination 10 iterations for detection are performed. The variance of the noise is the same at all relays and at the destination. It can be observed that increasing the number of relays improves the performance significantly due to the

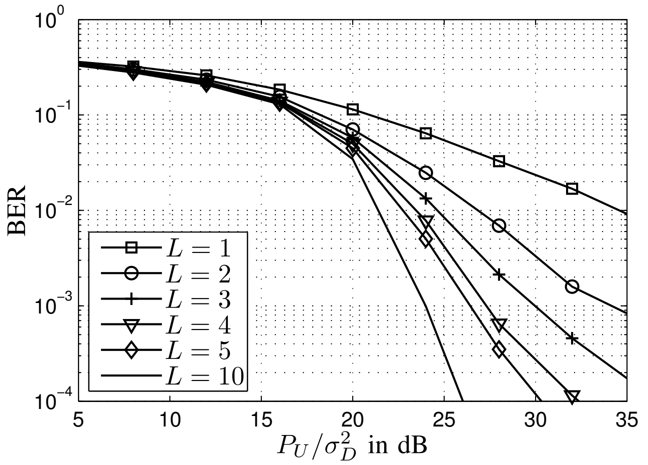

Fig. 6. BER for IDMA relay system with 20 users and different number of relays

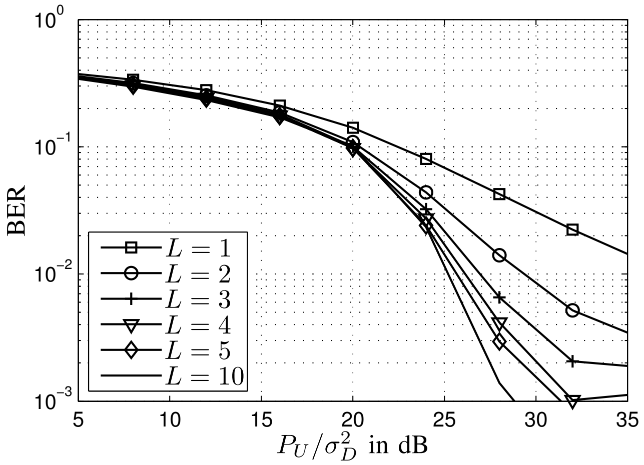

Fig. 7. BER for IDMA relay system with 24 users and different number of relays

diversity increasing with the number of relays. Especially the difference between one and two relays is significant while the difference between four and five users is smaller. The larger the number of relays, the less diversity gain is achieved by one additional relay. Due to the total power constraint over all relays, an SNR gain can not be observed here. Nevertheless, in a practical network, additional relays with fixed individual power constraint will additionally introduce an SNR gain.

It should be mentioned that on the relay-destination channels all users have to share the power provided by the relays, i.e., the total amount of $L \cdot P_R = P_U$ is shared by up to 24 users. Therefore, the performance of the 24-user system shows an SNR loss in comparison to the 16-user system. By increasing the transmit power of each relay, the performance can be increased further.

As can be seen in Figure 7 the performance at high values of P_U gets worse for $\beta = 3$. This is due to convergence behavior of the iterative detection at the destination for high system loads. This is a basic property of IDMA: for equal transmit powers and repetition code the possible supportable load is ≈ 3 which is approximately also valid for independent Rayleigh fading channels. Only by optimizing the received power distribution over the users the supportable load can be increased significantly [9]. Nevertheless, the advantage of delay diversity still remains.

IV. Conclusions

In this paper, an IDMA system with multiple users is considered when relays are applied to support the communication. In order to preserve the advantages of IDMA systems, as robustness against asynchronism and multi-path channels, we proposed a delay-diversity scheme to exploit the spatial diversity offered by the relays. In contrast to other possible approaches like beamforming and distributed space-time block codes, delay diversity does not require any synchronization or global channel knowledge, which would lead to a huge signaling overhead. Therefore, this approach is especially well suited for networks with mobile terminals serving as relays. Our proposed scheme provides diversity gain increasing with the number of relays at moderate computational cost only increasing with $\mathcal{O}(N \cdot L)$ while still providing good multiple access capability and error rate performance. The impact of channel characteristics on the overall performance was investigated via simulations and analysis. Although the SNR distribution due to application of an AF scheme introduces a loss compared to Rayleigh fading end-to-end channels, the cooperative diversity can still be achieved.

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