Near-Optimum Power Allocation for Outage Restricted Distributed MIMO Multi-Hop Networks

Dirk Wübben and Yidong Lang
Department of Communications Engineering, University of Bremen, Otto-Hahn-Allee, D-28359 Bremen, Germany
Email: {wuebben, lang}@ant.uni-bremen.de, Phone: +49 421 218 2407

***Abstract*—The throughput of multi-hop communication systems can significantly be increased by the application of MIMO concepts. To utilize the physical resources in an efficient way while meeting the Quality-of-Service (QoS) constraints appropriate power allocation strategies are desired. In this paper the total transmit power of a MIMO multi-hop system is minimized under the constraint of a given end-to-end outage probability. The optimum power allocation corresponds to a convex optimization problem. In order to achieve an analytical solution, the original task is relaxed by stringent approximations and a simple closed-form near-optimum solution is proposed. As this Improved Approximative Power Allocation (IAPA) achieves excellent performance results this new approach is also very useful for investigating outage restricted multi-hop systems analytically.**

I. Introduction

In wireless mesh networks the source communicates with the destination via a number of nodes to enhance the link performance. For the concept of distributed MIMO adjacent nodes are combined into virtual antenna arrays (VAAs), so that transmission techniques known from multiple antenna systems can be applied [1]–[4]. In each hop the nodes of the transmit VAA serve as virtual antennas of a distributed space-time code and the nodes of the receiving VAA perform independent data detection. In order to achieve a given QoS constraint while saving physical resources, the optimized allocation of transmit power to the distinct VAAs becomes one of the most important issues in the design of such a multi-hop system. Ressource allocation strategies for maximizing the end-to-end (e2e) ergodic capacity and for minimizing the error-rate have been presented in [3]–[5] under the assumption of a fixed total transmit power. However, as the majority of today's wireless communications happen over slow-fading channels, i.e., non-ergodic in the capacity sense, the consideration of the e2e outage probability is of higher practical relevance and in addition approaches for minimizing the transmit power are desired. The task of minimizing the total power while meeting a given e2e outage probability has been analyzed in [6]–[8]. As the resulting convex optimization problem can only be solved by numerical optimization tools, some tight approximations have been introduced to achieve a near-optimum formulation for the power allocation. The general solution for this problem and a closed-form solution valid for systems with many nodes per VAA have been proposed in [6]. In this paper we extend this work by presenting a closed-form solution for an arbitrary number of nodes per VAA. This novel approach is called IAPA (Improved Approximative Power Allocation) and results in an almost optimal power allocation for general system configurations. Thus, a very helpful tool for further analytical investigations of outage restricted networks is achieved [7].

The remainder of this paper is organized as follows. The system model is introduced in Section II and the optimization problem is formulated in Section III. The near-optimal allocation problem and its analytical solutions from [8] are presented in Section IV before the novel approach IAPA is derived in Section V. The performance analysis is given in Section VI and the paper is summarized in Section VII.

II. System description

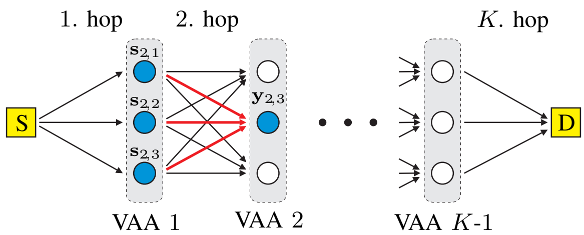

Fig. 1. System model for a distributed MIMO multi-hop network with highlighted MISO link in the second hop.

We consider a distributed MIMO multi-hop system where communication takes place in K hops using K-1 VAAs as shown in Fig. 1. The nodes within the same VAA decode the information separately but re-encode the decoded information by using a spatial fraction of the chosen space-time code. Therefore, the transmission within one hop can be modeled as several multiple-input single-output (MISO) systems. It is assumed that each VAA transmits signals with the same rate R and all hops use the total bandwidth W of the network. Let k index the hop, then t_k, r_k denote the number of transmit and receive nodes within the kth hop, respectively. With the space-time encoded signal $\mathbf{S}_k \in \mathbb{C}^{t_k \times T}$ of length T transmitted from the t_k nodes at hop k, the corresponding received signal $\mathbf{y}_{k,j} \in \mathbb{C}^{1 \times T}$ at the node j of VAA k is given by

$$\mathbf{y}_{k,j} = \sqrt{\frac{\theta_k \mathcal{P}_k}{t_k}} \mathbf{h}_{k,j} \mathbf{S}_k + \mathbf{n}_{k,j} \ , \qquad (1)$$

where $\mathbf{n}_{k,j}$ denotes the Gaussian noise vector with power spectral density N_0 and the total transmit power $\mathcal{P}_k$ of hop k is equally allocated to its nodes. The channel from the t_k transmit nodes to the jth receive node within hop k is expressed as $\mathbf{h}_{k,j} \in \mathbb{C}^{1 \times t_k}$ and contains complex zero-mean circular symmetric Gaussian distributed elements with variance 1. It is assumed that the relaying nodes belonging to the same VAA are spatially sufficiently close as to justify a common path loss $\theta_k = d_k^{-\epsilon}$ between the nodes of two adjacent VAAs, where d_k is the distance between the transmit and receive nodes at hop k and ϵ is the path loss exponent within range of 2 to 5.

III. Optimum Power Allocation

According to the capacity of a MIMO channel [9], the instantaneous rate supported by the MISO link (1) is given by

$$C_{k,j} = W \log_2 \left(1 + \frac{\mathcal{P}_k}{d_k^\epsilon t_k W N_0} \|\mathbf{h}_{k,j}\|^2 \right) . \quad (2)$$

In the sequel the outage probability $P_{\mathrm{out},k,j} = \Pr(R > C_{k,j})$ of receiving node j in hop k is investigated, which describes the probability that the transmit rate R is greater than the supported rate $C_{k,j}$. It is expressed as a cumulative distribution function (CDF) and depends on the fixed transmission parameters and the channel condition within the hop

$$P_{\mathrm{out},k,j} = \Pr\left(\|\mathbf{h}_{k,j}\|^2 < x_k\right) = \frac{\gamma(t_k, x_k)}{\Gamma(t_k)} . \quad (3)$$

To ease notation in (3), the system parameters of the kth hop have been collected in the variable

$$x_k = (2^{\frac{R}{W}} - 1) d_k^\epsilon t_k W N_0 / \mathcal{P}_k = Q d_k^\epsilon t_k / \mathcal{P}_k \quad (4)$$

being inversely proportional to the signal-to-noise-ratio. Furthermore, the system-wide parameters bandwidth, noise power and data rate have been combined in $Q = (2^{\frac{R}{W}} - 1) W N_0$. As $\|\mathbf{h}_{k,j}\|^2$ obeys a Gamma distribution [9], the CDF is given by the incomplete Gamma function $\gamma(t_k, x_k) = \int_0^{x_k} e^{-u} u^{t_k - 1} \, \mathrm{d}u$ normalized by the Gamma function $\Gamma(t_k)$.

As the same path loss is assumed within a hop, each MISO system of the hop has the same outage probability and hence the outage probability of the kth hop is given by

$$P_{\mathrm{out},k} = 1 - \prod_{j=1}^{r_k} (1 - P_{\mathrm{out},k,j}) = 1 - (1 - P_{\mathrm{out},k,j'})^{r_k} , \quad (5)$$

where j' indexes an arbitrary $j \in \{1, \ldots, r_k\}$. Since D&F is applied, the signals are completely decoded at each VAA, so that the outage probabilities per hop $P_{\mathrm{out},k}$ are mutually independent. For simplicity, it is assumed that the e2e communication is in outage if any of the MISO systems can not correctly decode the information [2]. With this pessimistic assumption the e2e outage probability corresponds to

$$P_{\mathrm{e2e}} = 1 - \prod_{k=1}^{K} (1 - P_{\mathrm{out},k}) = 1 - \prod_{k=1}^{K} (1 - P_{\mathrm{out},k,j'})^{r_k} \quad (6)$$

and is used as the indicator for the QoS in the sequel.

The optimization problem that minimizes the total transmit power $\mathcal{P}_{\mathrm{tot}}$ of the network while meeting the end-to-end outage probability requirement e can be formulated as

$$\text{minimize} \quad \mathcal{P}_{\mathrm{tot}} = \sum_{k=1}^{K} \mathcal{P}_k \quad (7a)$$

$$\text{subject to} \quad P_{\mathrm{e2e}} = 1 - \prod_{k=1}^{K} (1 - P_{\mathrm{out},k,j'})^{r_k} \le e . \quad (7b)$$

By proving that the Hessian matrix of P_{e2e} is positive semidefinite with respect to all $\mathcal{P}_k$, it can be shown that (7) is convex for low outage probability constraints e [10]. However, due to the rather complex product description of the e2e outage probability (6), no closed-form solution in terms of the power $\mathcal{P}_k$ per hop for the exact optimization problem (7) is available. Thus, standard optimization tools, as presented in [11], have to be used. In order to simplify the calculation and to achieve analytical expressions, several approximations as well as solutions for the resulting near-optimal power allocation problem have been presented in [6]. As our new approach IAPA is based on these solutions, we summarize them in the following section.

IV. Near-Optimum Power Allocation

A. Approximative Optimization Problem

As the low outage probability region is of interest for practical systems (e.g., $e = 1\%$), the SNR has to be sufficiently large and consequently we have $x_k \to 0$. Considering the series expansion of the incomplete Gamma function [12, §6.5.29]

$$\gamma(t_k, x_k) = \frac{x_k^{t_k}}{t_k} + \sum_{m=1}^{\infty} \frac{(-1)^m \, x^{t_k + m}}{(t_k + m) \, m!} , \quad (8)$$

it can be shown that $\gamma(t_k, x_k) \approx t_k^{-1} x_k^{t_k}$ for $x_k \to 0$. Thus, the simple approximation for (3) is achieved [6], [13]

$$P_{\mathrm{out},k,j} = \frac{\gamma(t_k, x_k)}{\Gamma(t_k)} \approx \frac{t_k^{-1} x_k^{t_k}}{\Gamma(t_k)} = \frac{x_k^{t_k}}{\Gamma(t_k + 1)} \triangleq \tilde{P}_{\mathrm{out},k,j} , \quad (9)$$

where symbols labeled by tilde indicate approximated terms in the sequel, i.e., $\tilde{P}_{\mathrm{out},k,j}$ denotes the approximation of $P_{\mathrm{out},k,j}$. Similar to [14] the product representation of the outage probability can further be approximated by a sum expression. Thus, the outage probability per hop (5) is expressed as

$$P_{\mathrm{out},k} \approx \sum_{j=1}^{r_k} \tilde{P}_{\mathrm{out},k,j} = r_k \tilde{P}_{\mathrm{out},k,j'} = \frac{r_k x_k^{t_k}}{\Gamma(t_k + 1)} \triangleq \tilde{P}_{\mathrm{out},k} \quad (10)$$

and the e2e outage probability (6) can be approximated by

$$P_{\mathrm{e2e}} \approx \sum_{k=1}^{K} P_{\mathrm{out},k} \approx \sum_{k=1}^{K} \frac{r_k x_k^{t_k}}{\Gamma(t_k + 1)} \triangleq \tilde{P}_{\mathrm{e2e}} . \quad (11)$$

With a derivation similar to [14] it is straightforward to show, that the approximated form (11) serves as an upper bound for the exact outage probability (6), i.e., $P_{\mathrm{e2e}} \le \tilde{P}_{\mathrm{e2e}}$. Thus, if $\tilde{P}_{\mathrm{e2e}}$ is considered for distributing the power within the multi-hop system, the applied e2e probability constraint is more stringent. Consequently, each power allocation strategy satisfying $\tilde{P}_{\mathrm{e2e}}$ fulfills the original constraint P_{e2e} automatically, so that the original optimization problem (7) can be simplified to

$$\text{minimize} \quad \mathcal{P}_{\mathrm{tot}} = \sum_{k=1}^{K} \mathcal{P}_k \quad (12a)$$

$$\text{subject to} \quad \tilde{P}_{\mathrm{e2e}} = \sum_{k=1}^{K} \tilde{P}_{\mathrm{out},k} = \sum_{k=1}^{K} \frac{r_k x_k^{t_k}}{\Gamma(t_k + 1)} \le e . \quad (12b)$$

Note that the solution of problem (12) leads only to a near-optimal power allocation with increased power consumption compared to the exact solution formulated in (7). However, this form permits the construction of relatively simple or even closed-form solutions. In addition, it can be shown, that the solution of the near-optimum problem (12) with respect to an outage constraint $e+4\,e^4$ serves as a lower bound on the total transmit power of the exact solution (7) [14], so that the power increment can be appraised.

B. Near-Optimum Power Allocation (NOPA)

The near-optimal solution can be obtained by solving the constrained optimization problem (12) using Lagrange multipliers. Since the e2e outage probability $\tilde{P}_{\mathrm{e2e}}$ is a monotonic decreasing function of the power $\mathcal{P}_k$, $\forall\, k$, the optimal power allocation is attained when $\tilde{P}_{\mathrm{e2e}} = e$. By defining the Lagrangian $L(\mathcal{P}_k,\lambda)=\sum_{k=1}^{K}\mathcal{P}_k+\lambda(\tilde{P}_{\mathrm{e2e}}-e)$, the near-optimal power allocation solutions $\mathcal{P}_k^\star$ satisfy

$$\frac{\partial L(\mathcal{P}_k,\lambda)}{\partial \mathcal{P}_k} = 1+\lambda\frac{\partial \tilde{P}_{\mathrm{e2e}}}{\partial \mathcal{P}_k} = 0\ , \quad \forall\, k \tag{13a}$$

$$\tilde{P}_{\mathrm{e2e}}(\mathcal{P}_k^\star) = \sum_{k=1}^{K} r_k \tilde{P}_{\mathrm{out},k,j'}(\mathcal{P}_k^\star) = e\ . \tag{13b}$$

From (4) the relation $x_k = Qd_k^\epsilon t_k/\mathcal{P}_k$ follows and the first derivative of $\tilde{P}_{\mathrm{e2e}}$ (11) is given by [6]

$$\frac{\partial \tilde{P}_{\mathrm{e2e}}}{\partial \mathcal{P}_k} = -\frac{r_k x_k^{t_k+1}}{\Gamma(t_k+1)Qd_k^\epsilon} = -\frac{r_k \tilde{P}_{\mathrm{out},k,j'}^{\frac{t_k+1}{t_k}}\Gamma(t_k+1)^{\frac{1}{t_k}}}{Qd_k^\epsilon}\ . \tag{14}$$

In order to fulfill (13a) this derivative has to be equal for all k. Let A denote the constant value of the derivative for all k normalized by $-Q$, then we obtain from (14)

$$A = -Q\frac{\partial \tilde{P}_{\mathrm{e2e}}}{\partial \mathcal{P}_k} = \frac{r_k \tilde{P}_{\mathrm{out},k,j'}^{\frac{t_k+1}{t_k}}\Gamma(t_k+1)^{\frac{1}{t_k}}}{d_k^\epsilon}\ , \quad \forall\, k\ . \tag{15}$$

Note that this expression is independent of the system wide parameters collected in $Q=(2^{\frac{R}{W}}-1)WN_0$ and depends only on the number of nodes t_k per hop, the distances d_k and the required outage probability e [7]. As the lower region of the outage probability as well as distances $d_k \geq 1000$ m between the VAAs are considered within the paper, the value of A will be $0<A\ll 1$.

By rewriting (15) with respect to $\tilde{P}_{\mathrm{out},k,j'}$, the outage probability per hop $\tilde{P}_{\mathrm{out},k} = r_k\tilde{P}_{\mathrm{out},k,j'}$ is given by

$$\tilde{P}_{\mathrm{out},k} = r_k\left(\frac{A\,d_k^\epsilon}{r_k\Gamma(t_k+1)^{\frac{1}{t_k}}}\right)^{\frac{t_k}{t_k+1}} = a_k\cdot A^{\frac{t_k}{t_k+1}}\ , \tag{16}$$

where the parameters of hop k are combined in the coefficient

$$a_k = r_k\left(\frac{d_k^\epsilon}{r_k\Gamma(t_k+1)^{\frac{1}{t_k}}}\right)^{\frac{t_k}{t_k+1}} = \left(\frac{r_k\,d_k^{\epsilon\cdot t_k}}{\Gamma(t_k+1)}\right)^{\frac{1}{t_k+1}}\ . \tag{17}$$

Thus, the relation $e=\sum_{k=1}^{K}\tilde{P}_{\mathrm{out},k} = \sum_{k=1}^{K} a_k A^{\frac{t_k}{t_k+1}}$ is achieved to determine A by inserting (16) in (13b). Solving the optimization problem (12) is therefore equivalent to calculating the constant A that fulfills this equation. To this end, we rewrite the relation as a polynomial in the variable A

$$f_a(A) = \sum_{k=1}^{K} a_k\cdot A^{\frac{t_k}{t_k+1}} - e \tag{18}$$

and search for the real-valued and positive root A_a of (18), which can be determined by applying standard root-finding algorithms. By using the root A_a of $f_a(A)$ in (16), the outage probability per hop $\tilde{P}_{\mathrm{out},k}$ as well as the approximated e2e outage probability

$$\tilde{P}_{\mathrm{e2e}}^{\mathrm{NOPA}} = \sum_{k=1}^{K} a_k\,A_a^{\frac{t_k}{t_k+1}} \leq e \tag{19}$$

are determined. The near-optimal power allocation $\mathcal{P}_k^\star$ can then be derived from (10) using the definition of x_k in (4).

Theorem 1 (Near-Optimum Power Allocation (NOPA)): With the assumptions of Section IV-A, an arbitrary number of nodes t_k per VAA and an e2e outage probability requirement e, the near-optimal power allocation $\mathcal{P}_k^\star$ is given by

$$\begin{aligned}\mathcal{P}_k^\star &= \frac{Qd_k^\epsilon t_k}{x_k} = Qd_k^\epsilon t_k\left(\frac{r_k}{\tilde{P}_{\mathrm{out},k}^\star\Gamma(t_k+1)}\right)^{1/t_k} \\ &= Qt_k\left(\frac{r_k d_k^{\epsilon\cdot t_k}}{\Gamma(t_k+1)}\right)^{\frac{1}{t_k+1}} A_a^{-\frac{1}{t_k+1}} = Qt_k a_k A_a^{-\frac{1}{t_k+1}}\ .\end{aligned} \tag{20}$$

The constant A_a is a real-valued positive root of the higher-order equation (18) and the coefficients a_k are given by (17).

C. Approximative Power Allocation (APA)

A simple closed-form solution can be achieved under the assumption, that the number of transmit nodes per hop t_k for $k\geq 2$ is so large that the approximation $\frac{t_k}{t_k+1}\approx 1$ is valid. Using this approximation and the fact that the source contains only one antenna, the polynomial (18) is replaced by

$$f_b(A) = b_1\cdot A^{\frac{1}{2}} + A\cdot\sum_{k=2}^{K} b_k - e \tag{21}$$

with coefficients achieved from (17) using $t_1=1$ and $\frac{t_k}{t_k+1}\approx 1$

$$b_1 = (r_1 d_1^\epsilon)^{\frac{1}{2}} \quad \text{and} \quad b_k = d_k^\epsilon\,\Gamma(t_k+1)^{-\frac{1}{t_k}} \quad \text{for } k\geq 2\ . \tag{22}$$

Equation (21) is a polynomial of degree two with positive root

$$A_b^{\frac{1}{2}} = \frac{-b_1+\sqrt{b_1^2+4e\sum_{k=2}^{K}b_k}}{2\sum_{k=2}^{K}b_k}\ . \tag{23}$$

This solution for A_b determines again the outage probability $\tilde{P}_{\mathrm{out},k}$ and the power allocation $\mathcal{P}_k^\star$ per hop.

Theorem 2 (Approximative Power Allocation (APA)): With the assumptions of Section IV-A, a large number of nodes t_k per VAA and an e2e outage probability requirement e, the near-optimal power allocation $\mathcal{P}_k^\star$ is given by

$$\mathcal{P}_k^\star = Qd_k^\epsilon t_k\left(\frac{r_k}{\tilde{P}_{\mathrm{out},k}^\star\Gamma(t_k+1)}\right)^{1/t_k} \tag{24}$$

with approximated outage probabilities

$$\tilde{P}^{\star}_{\mathrm{out},1} = b_1 \, A_b^{\frac{1}{2}} \quad \text{and} \quad \tilde{P}^{\star}_{\mathrm{out},k} = b_k \, A_b \;\; \text{for } k \geq 2 \;, \qquad (25)$$

where the variables b_k and the real-valued positive root A_b are given by (22) and (23), respectively. The approximated e2e outage probability is $\tilde{P}^{\mathrm{APA}}_{\mathrm{e2e}} = b_1 A_b^{\frac{1}{2}} + A_b \sum_{k=2}^{K} b_k$.

V. Improved Approximative Power Allocation

As shown in the subsequent performance analysis, the analytical solution NOPA leads to almost the optimum transmit power for any number of nodes per hop. However, as the solution of the higher-order polynomial (18) is required, it leads to a substantial complexity. In contrast to this, a closed-form power allocation is given by APA with the drawback of an inaccurate solution for small t_k leading to an increased total transmit power. Thus, the question arises, if the advantages of both approaches can be combined leading to a closed-form near-optimum solution.

In order to achieve an accurate but closed-form solution for the approximated optimization problem (12), the e2e outage probability $\tilde{P}^{\mathrm{NOPA}}_{\mathrm{e2e}}$ (19) is factorized with respect to $A_a^{1/2}$

$$\tilde{P}^{\mathrm{NOPA}}_{\mathrm{e2e}} = A_a^{\frac{1}{2}} \left(a_1 + \sum_{k=2}^{K} a_k \cdot A_a^{\frac{t_k-1}{2t_k+2}} \right) . \qquad (26)$$

Here, the relation $\frac{t_k}{t_k+1} - \frac{1}{2} = \frac{t_k-1}{2t_k+2}$ was used for brevity. From the fact that the roots fulfill $0 < A_a \leq A_b \ll 1$, we conclude that $0 \leq A_a^{(t_k-1)/(2t_k+2)} - A_b^{(t_k-1)/(2t_k+2)} \ll 1$ holds. Using $A_a^{(t_k-1)/(2t_k+2)} \approx A_b^{(t_k-1)/(2t_k+2)}$ in (26), the e2e outage probability can hence be approximated as

$$\tilde{P}^{\mathrm{NOPA}}_{\mathrm{e2e}} \approx A_a^{\frac{1}{2}} \left(a_1 + \sum_{k=2}^{K} a_k \cdot A_b^{\frac{t_k-1}{2t_k+2}} \right) . \qquad (27)$$

By finally setting the right hand side of (27) equal to the required e2e outage probability e, an approximated closed-form solution for calculating the optimization parameter A_a of NOPA using the solution A_b of APA is achieved

$$\tilde{A}_a = e^2 \cdot \left(a_1 + \sum_{k=2}^{K} a_k \cdot A_b^{\frac{t_k-1}{2t_k+2}} \right)^{-2} . \qquad (28)$$

Note that this approximative solution is always smaller than the exact root A_a of $f_a(A)$ and thus the e2e outage requirement e is fulfilled in general. Thereby, an approximation for the root of the higher-order equation (18) has been found, which can then be used for the power allocation as presented in Theorem 1.

In order to further improve the estimate for the root of $f_a(A)$ and thereby minimizing the total transmit power, the approximative solution $\tilde{A}_a$ from (28) can be used in iterative root-finding algorithms as an excellent initial estimate. Applying for example Newtons's root-finding method, an improved estimate for the root is obtained by

$$\tilde{A}_a^{+} = \tilde{A}_a - \frac{f_a(\tilde{A}_a)}{f'_a(\tilde{A}_a)} , \qquad (29)$$

with the first-order derivative of $f_a(A)$

$$f'_a(A) = \frac{\partial f_a(A)}{\partial A} = \sum_{k=1}^{K} a_k \frac{t_k}{t_k+1} \cdot A^{\frac{-1}{t_k+1}} . \qquad (30)$$

As observed in the subsequent performance evaluation, it is in general sufficient to apply one Newton iteration in order to achieve the near-optimal power allocation (20).

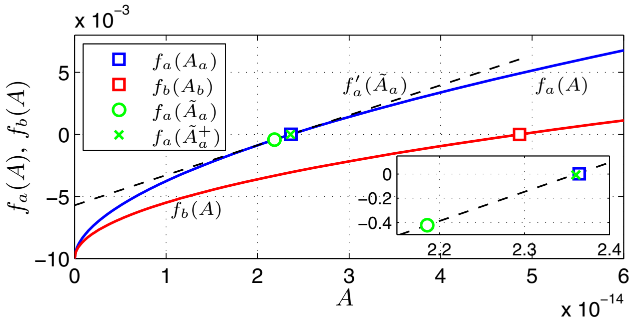

Fig. 2. Polynomials $f_a(A)$ and $f_b(A)$ for the 5-hop system with $t_k = 2$ nodes per VAA defined in Section VI.

Fig. 2 shows the behavior of the polynomials $f_a(A)$ and $f_b(A)$ for the multi-hop system with $t_k = 2$ nodes for all hops $k = 2, \ldots, 5$ as specified in Section VI in detail. In addition, the roots of the higher-order polynomial $f_a(A)$ and the second-order polynomial $f_b(A)$ are labeled. First of all it is important to notice, that all values A that satisfy $f_a(A) \leq 0$ or $f_b(A) \leq 0$ fulfill the e2e outage requirement e for NOPA or APA, respectively. Thus, the straightforward approach of using the parameter A_b for the near-optimum power allocation in Theorem 1 would not achieve the required QoS as $f_a(A_b) > 0$. However, the approximation $\tilde{A}_a$ given by (28) fulfils the e2e outage probability as $f_a(\tilde{A}_a) < 0$ and is relatively close to the exact root A_a resulting in nearly the same power allocation as NOPA. A tighter estimate $\tilde{A}_a^{+}$ for the root of $f_a(A)$ can be achieved by applying (29). To demonstrate this, we added the tangent line with slope $f'_a(\tilde{A}_a)$ in Fig. 2 indicating that the corresponding root is close to the exact root of $f_a(A)$. In this example, further Newton iterations would not lead to additional savings with respect to the total transmit power.

The improved approximative power allocation approach is summarized in the following theorem.

Theorem 3 (Improved Approx. Power Allocation (IAPA)): With the assumptions of Section IV-A, an arbitrary number of nodes t_k per VAA and an e2e outage probability requirement e, the near-optimal power allocation $\mathcal{P}_k^{\star}$ is given by

$$\mathcal{P}_k^{*} = Q \cdot t_k \left(\frac{r_k d_k^{\epsilon \cdot t_k}}{\Gamma(t_k+1)} \right)^{\frac{1}{t_k+1}} \cdot \tilde{A}_a^{-\frac{1}{t_k+1}} \qquad (31)$$

where the optimized system parameter

$$\tilde{A}_a = e^2 \left(\sqrt{r_1 d_1^{\epsilon}} + \sum_{k=2}^{K} \left(\frac{r_k d_k^{\epsilon \cdot t_k}}{\Gamma(t_k+1)} \right)^{\frac{1}{t_k+1}} \cdot A_b^{\frac{t_k-1}{2t_k+2}} \right)^{-2} \qquad (32)$$

is determined by

$$A_b^{\frac{1}{2}} = \frac{\sqrt{r_1 d_1^{\epsilon} + 4e \sum_{k=2}^{K} d_k^{\epsilon} \cdot \Gamma(t_k+1)^{-\frac{1}{t_k}}} - \sqrt{r_1 d_1^{\epsilon}}}{2 \sum_{k=2}^{K} d_k^{\epsilon} \cdot \Gamma(t_k+1)^{-\frac{1}{t_k}}} . \qquad (33)$$

To clarify the impact of the system parameters, the relations for A_b and $\tilde{A}_a$ have been derived from (23) and (28) by replacing the coefficients a_k and b_k using (17) and (22), respectively.

VI. Performance Analysis

In the sequel the performance of the proposed closed-form solution IAPA is compared to the optimum power allocation (OPT) (7) obtained by numerical optimization and the analytical approaches NOPA and APA. For the investigation a distributed MIMO multi-hop network with $K = 5$ hops and the same number of relaying nodes $t = t_k$, $k = 2, \cdots, K$ per hop is considered. Furthermore it is assumed that the e2e communication over $W = 5$ MHz should meet an e2e outage probability constraint $e = 1\%$ for noise power $N_0 = -174$ dBm according to the UMTS standard. The distances between each VAA are $d_k = 1$ km and the path-loss exponent is $\epsilon = 3$.

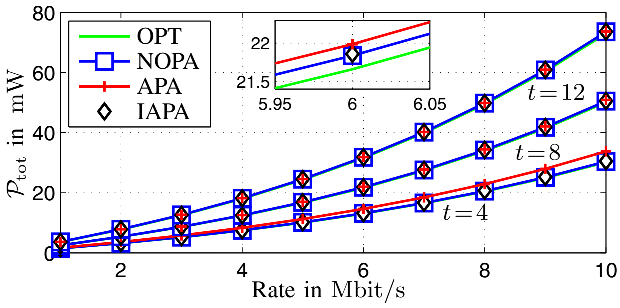

Fig. 3. Total allocated transmit power by OPT, NOPA, APA and IAPA for varying numbers of nodes per hop $t = 4, 8, 12$ and $e = 1\%$.

Fig. 3 shows for multi-hop systems with $t = \{4, 8, 12\}$ nodes per VAA and varying data rate R the total transmit power $\mathcal{P}_{\text{tot}}$ for the different power allocation solutions. Notice, that the increase of the total power with t is caused by the pessimistic model for the e2e outage probability (6). It can be observed that the developed analytical solutions achieve almost the same performance as the exact one. To even see the difference, a zoom into the graph for $t = 8$ has been added. However, if the number of nodes is small, e.g., $t = 4$, the simple closed-form solution APA leads to an increased power consumption.

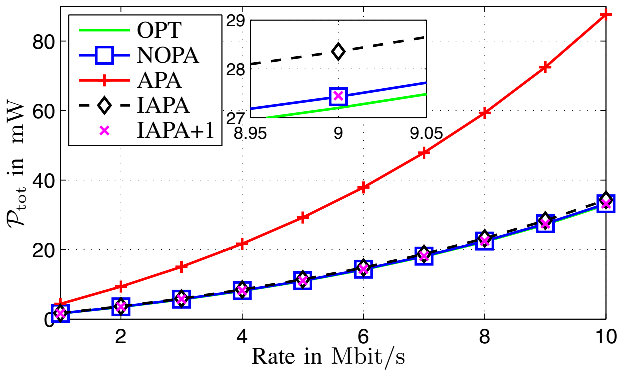

Fig. 4. Total allocated transmit power by OPT, NOPA, APA and IAPA for $t = 2$ nodes per hop and e2e outage probability $e = 1\%$.

The total power consumption of the different allocation approaches for a system with only $t = 2$ nodes per hop are depicted in Fig. 4. Obviously, the APA approach requires significantly more transmit power to satisfy the required QoS, whereas NOPA results in an almost negligible increase of the total power. Almost the same result can be achieved by the novel allocation method IAPA, but with the advantage of a closed-form solution of low complexity. By the additional application of Newton's root-finding method (IAPA+1) this gap can be closed. The excellent performance of our new approach has also been verified for other system setups and analytical investigations for symmetric relaying networks (i.e. same number of nodes per VAA and equal distance) have been presented in [7].

VII. Summary and Conclusions

In this paper a novel power allocation solution for outage restricted distributed MIMO multi-hop networks was presented by the application of some stringent approximations. The closed-form solution IAPA (Improved Approximative Power Allocation) achieves nearly the minimum total power in order to fulfill the required end-to-end outage probability. The presented solution is not only favorable due to its low complexity, but in addition it enables further insights for the optimum design of multi-hop networks.

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