IMPROVING THE PERFORMANCE OF BICM-OFDM SYSTEMS IN PRESENCE OF HPA NONLINEARITIES BY EFFICIENT BIT AND POWER LOADING

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ABSTRACT

The usage of Orthogonal Frequency Division Multiplexing (OFDM) allows to adapt the available time-frequency grid to the current channel conditions. To exploit the offered degrees of freedom many bit and power loading algorithms were introduced. However, they do not include channel coding in the optimization. Similar to conventional multicarrier schemes in such link adaptation scenarios the peak-to-average power-ratio (PAPR) problem occurs. The combination of active constellation extension (ACE) with additional tone reservation (TR) is a promising approach to reduce this ratio without sacrificing the bit error rate (BER). Hence, in this contribution we propose an extended bisection method for coded bit and power loading in combination with ACE/TR techniques to jointly improve the BER and PAPR performance in high-rate OFDM systems. Results indicate that in presence of a memoryless nonlinear high power amplifier (HPA) device the BER can be lowered compared to existing loading algorithms.

1. INTRODUCTION

OFDM has become a popular modulation technique for broadband wireless communication systems [1]. The transmitter side adaptation of these systems to the current channel state is a crucial step towards higher spectral efficiencies and more robust systems. Although offering the use of adaptive algorithms OFDM also suffers from disadvantages like the high PAPR. Obviously, the whole system has to be considered in the optimization, where this paper deals with a combined approach of bit and power loading and PAPR reduction.

The bit and power allocation problem includes common parameters like modulation and power, but also the applied channel code needs to be considered to satisfy specific bit error rate (BER) or rate requirements. This, however, has usually been neglected in previous works, e.g., [2] and only recently some attention has been spent to the consideration of channel coding in bit and power loading algorithms with respect to the capacity of Bit Interleaved Coded Modulation (BICM) [3, 4] systems [5–7]. Especially BICM is suited for our purposes and allows for flexible allocation of code rate and modulation. Still, the information theoretical measure capacity does not describe the performance of a coded system completely and only holds for perfect capacity achieving codes. Therefore, we propose an efficient extension of the original loading approach of Krongold et al. [2], which uses the bisection method to solve the resulting convex optimization problem. Instead of analytical error rate expressions, in this paper the simulated AWGN performance of a set of code and modulation combinations, also known as transmission modes, will be used to form a look-up table describing the required signal-to-noise-ratio (SNR) for a specific subcarrier rate. The codes may be chosen from a common code family, e.g., convolutional codes with a fixed constraint length, realizing different code rates. Adapting the code rate additionally allows for higher flexibility in comparison to fixed code rate scenarios. A similar approach has been proposed by Stiglmayr et al. [8]. The authors solve the rate optimization problem formulated in terms of the BICM capacity by linearization, however they neglect a finer grained power control. Further investigations dealing with SNR mapping strategies based on clustered time-frequency grids (chunks) also neglect the power allocation [9, 10].

In OFDM systems potentially high peak values of the time-domain signal pose a big problem. As the transmit signal may be distorted by a high-power amplifier (HPA) nonlinear distortions and a spectral regrowth occur. This especially holds for adaptive scenarios as previous results showed a performance decrease of link adaptation algorithms if different modulation schemes are used for a certain output back-off (OBO) of the HPA [11]. Hence, several techniques were proposed to reduce the PAPR, a good overview can be found in [12]. The ideas of tone reservation (TR) to exploit unused subcarriers and tone injection (TI) to extend constellation points on used carriers motivate the combination of both for further PAPR reduction [13]. An efficient subclass of TI is the active constellation extension (ACE) method, which ex-

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clusively alters outer constellation points of the used modulation alphabets on the subcarriers [14].

In this contribution we propose a combination of an extended bisection method for coded bit and power loading [15] with ACE/TR techniques [16] to jointly improve the BER and PAPR performance in high-rate OFDM systems.

The remainder of the paper is organized as follows. In Section 2 the OFDM system is described and the PAPR problem is stated. Section 3 deals with the performance of coded OFDM systems and the solution of the convex optimization problem using the proposed coded bisection method. Afterwards, Section 4 introduces a the new PAPR reduction approach without sacrificing the BER. Simulation results are shown in Section 5 and, finally, conclusions are given in Section 6.

2. SYSTEM MODEL AND PAPR PROBLEM

The considered OFDM system is assumed to be perfectly synchronized and free of intercarrier interference (ICI), i.e., the duration of the guard interval (GI) is sufficiently long. Furthermore, the channel state shall be perfectly known at both the transmitter and receiver side. The general system model including channel coding and interleaving is shown in Fig. 1.

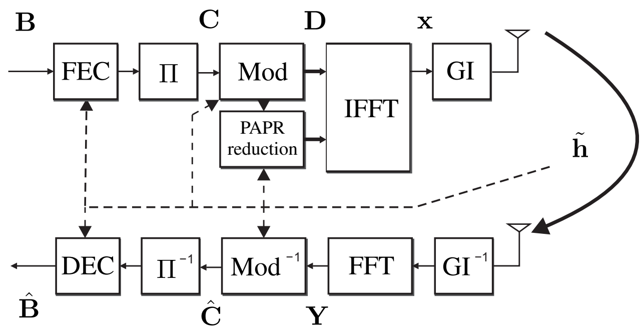

Fig. 1. Considered coded OFDM system model with additional PAPR reduction

The equivalent baseband system can be described in frequency domain as

$$Y_n = H_n \cdot \sqrt{P_n} \cdot D_n + N_n \,, \tag{1}$$

where H_n denotes the channel coefficient in frequency domain on subcarrier $n = 0, \cdots, N_C - 1$. P_n, D_n, N_n and Y_n denote the transmit power, transmit symbol, Gaussian noise and receive symbol, respectively. The total transmit power is given by $P_{\text{Total}} = \sum_{n=0}^{N_C-1} P_n$ and the power of the noise $N_n \sim \mathcal{N}_C(0, \sigma_N^2)$ is fixed to $\sigma_N^2 = 1$. The N_C frequency domain channel coefficients are determined by

$$H_n = \sum_{\ell=0}^{L_F-1} \tilde{h}(\ell) e^{-j2\pi \frac{n\ell}{N_C}} \,, \tag{2}$$

where the L_F taps of the time domain channel are defined as $\tilde{h}(\ell) \sim \mathcal{N}_C(0, 1/L_F)$.

Throughout this paper transmit symbols stemming from M-QAM modulation alphabets with Gray mapping are considered. To each subcarrier n an individual alphabet of cardinality $|M_n|$ will be assigned. Soft-Demapping via a-posteriori-probability (APP) detection is used to supply soft information to the decoder. In terms of forward error correction (FEC) non-systematic non-recursive convolutional codes of rates $R_C \in \{1/4, 1/3, 1/2, 2/3, 3/4\}$ and constraint length $L_C = 3$ are applied, leading to a variety of possible transmission modes, i.e. combinations of modulation and code. In all cases, the code word length is fixed to the number of bits in one OFDM symbol, leading to longer code words for higher data rates. Thus, no time diversity is exploited. A BCJR algorithm is used for soft-decoding and random interleaving is applied.

The time-domain transmit baseband signal for one OFDM symbol following (1) can be obtained using the IFFT

$$x_k = \frac{1}{\sqrt{N_C}} \sum_{n=0}^{N_C-1} X_n \exp\left(j2\pi \frac{kn}{N_C}\right) \,, \tag{3}$$

for time index $k = 0, \ldots, N_C - 1$, where $X_n = \sqrt{P_n} \cdot D_n$. Then, $\mathbf{x} = [x_0, \ldots, x_{N_c-1}]^T$ is the time-domain symbol vector at the IFFT output, whose elements, due to the central limit theorem, can be modeled as truncated zero-mean Gaussian random variables. This leads to a large probability of high peak values especially for an increasing number of subcarriers. Usually, an IFFT of a zero-padded input data vector of length wN_c is applied, where w is the oversampling factor. All peaks of the time-domain signal can be captured if an oversampling factor of $w \geq 4$ is used. Then, its corresponding discrete PAPR closely approximates that of the continuous-time signal. In this work all algorithms perform at Nyquist sampling rate, whereas the PAPR of the oversampled signal is measured in the investigations. Consequently, the PAPR for such an OFDM system is defined as [12]

$$\text{PAPR}\,(\mathbf{x}) = \frac{\|\mathbf{x}\|_\infty^2}{\text{E}\left\{\|\mathbf{x}\|_2^2\right\}/wN_C} \,, \tag{4}$$

where $\text{E}\{\cdot\}$ denotes the expectation and $\|\cdot\|_\alpha$ is the α-norm. In (4) $\|\mathbf{x}\|_\infty^2$ defines the peak value $\max_{\mathbf{x}} |x_k|^2$. This PAPR can be significantly reduced, if the transmit signal $\mathbf{x}$ is modified by an additive time-domain signal $\mathbf{c} \in \mathbb{C}^{N_c \times 1}$, which is optimized with respect to ACE and TR constraints such that the corresponding PAPR

$$\text{PAPR}\,(\mathbf{x}, \mathbf{c}) = \frac{\|\mathbf{x} + \mathbf{c}\|_\infty^2}{\text{E}\left\{\|\mathbf{x} + \mathbf{c}\|_2^2\right\}/wN_c} \tag{5}$$

is minimized. The principles of ACE and TR are explained in Section 4.1. Following [14] the optimization problem using the ACE and/or TR technique can be written as

$$\underset{\mathbf{c}}{\text{minimize}}\ \|\mathbf{x}+\mathbf{c}\|_\infty = \underset{\mathbf{C}}{\text{minimize}}\ \|\mathbf{x}+\mathbf{FC}\|_\infty\ , \quad (6)$$

where $\mathbf{C}$ is the corresponding frequency-domain vector of the additive signal $\mathbf{c}$ and $\mathbf{F}$ denotes the IDFT matrix of size $N_c \times N_c$ with elements $f_{n,k} = \left(1/\sqrt{N_c}\right)\exp\left(j2\pi kn/N_c\right)$. This convex problem can be interpreted as minimizing the maximum squared magnitude of the resulting signal with respect to $\mathbf{c}$. Unfortunately, this is a minimization task in the complex plane and results in a quadratically constraint quadratic program (QCQP) [17]. Thus, a less complex solution is required.

As any square M-QAM can be represented by two $\sqrt{M}$-ASK without loss, we will restrict to ASK constellations in the following. This means the optimization of the bit and power loading and the PAPR optimization are carried out with an equivalent real-valued representation. For bit and power loading an expansion to QAM constellations is obtained by simply halving the power constraint and the maximum rate, while doubling the resulting powers and rates. Taking into account the ASK symbol representation, assume the real-valued system variables

$$\mathbf{X}_\text{R} = \left[\Re\{\mathbf{X}\}\,\Im\{\mathbf{X}\}\right]^T \in \mathbb{R}^{2N_c\times 1} \quad (7a)$$

$$\mathbf{x}_\text{R} = \left[\Re\{\mathbf{x}\}\,\Im\{\mathbf{x}\}\right]^T \in \mathbb{R}^{2N_c\times 1} \quad (7b)$$

$$\mathbf{C}_\text{R} = \left[\Re\{\mathbf{C}\}\,\Im\{\mathbf{C}\}\right]^T \in \mathbb{R}^{2N_c\times 1} \quad (7c)$$

$$\mathbf{F}_\text{R} = \begin{bmatrix}\Re\{\mathbf{F}\} & -\Im\{\mathbf{F}\}\\ \Im\{\mathbf{F}\} & \Re\{\mathbf{F}\}\end{bmatrix} \in \mathbb{R}^{2N_c\times 2N_c}\ , \quad (7d)$$

where $\mathbf{X}_\text{R}$ is a real-valued ASK symbol vector, $\mathbf{x}_\text{R}$ the equivalent time-domain represenation, $\mathbf{C}_\text{R}$ the frequency-domain correction term and $\mathbf{F}_\text{R}$ the real-valued representation of the IDFT matrix $\mathbf{F}$. Then, the PAPR optimization problem is given by

$$\underset{\mathbf{C}_\text{R}}{\text{minimize}}\ \|\mathbf{x}_\text{R}+\mathbf{F}_\text{R}\mathbf{C}_\text{R}\|_\infty\ . \quad (8)$$

In the next section the coded bit and power loading is explained, whereas the PAPR reduction is addressed in Section 4.

3. CODED BIT AND POWER LOADING

3.1. Coded System Performance

The bit error rate performance of an uncoded $\sqrt{M}$-ASK transmission is well-known. It can be described as a function of the SNR γ by

$$\mathrm{P}_{b,\sqrt{M}-\text{ASK}} = \frac{2}{\log_2(M)}\left(1-\frac{1}{\sqrt{M}}\right)\operatorname{erfc}\left(\sqrt{\frac{3}{M-1}\gamma}\right). \quad (9)$$

Accordingly, the frame error rate (FER) given a certain frame length L_N - the number of channel uses - is defined as

$$\mathrm{P}_{f,\sqrt{M}-\text{ASK}} = 1-\left(1-\mathrm{P}_{b,\sqrt{M}-\text{ASK}}\right)^{L_N\log_2(\sqrt{M})} \quad (10)$$

and can be used to derive the SNR γ required to achieve a given frame error rate for a certain modulation. This is the basis for many known bit loading algorithms, e.g., [2]. However, these results are limited to uncoded systems. To capture the behavior of the whole system - including an appropriate channel code - (9) and (10) are not sufficient. One way to obtain a quality indicator is to simulate the performance of the coded system.

In order to characterize the SNR, which is necessary to achieve a given bit or frame error rate performance on a single subcarrier, a system with equivalent block length of $L_N = N_C$ and AWGN noise is simulated. The motivation for this approach is, that given the SNR of a single subcarrier γ_n, we assume all symbols of the code word to have the SNR γ_n. The error rate performance of a specific code and modulation disturbed by AWGN at γ_n indicates, which transmission mode may be chosen to guarantee an error rate constraint. Using this heuristic, the overall error rate of an OFDM symbol with different SNRs and properly chosen subcarrier modes can be assumed to fulfill the error rate constraint.

Still, the AWGN assumption is optimistic in the sense that individual channel states have been compensated for properly. Nevertheless, it offers a good indication of the SNR which is necessary to support a target error rate on a single subcarrier at a specific data rate. Even though the applied channel code could cope with SNR variations over subcarriers, an ergodic Rayleigh fading channel would lead to far too pessimistic performance measures because of existing subcarriers with very low SNR. Such subcarriers will be compensated for in a perfect adaptive system by the assignment of more power, different modulation and stronger coding.

Fig. 2 shows the BER results of Monte-Carlo simulations for $\sqrt{M}$-ASK constellations up to $\sqrt{M} = 2^4$ and a variety of code rates versus E_b/N_0. It is quite clear, that only a subset of combinations will actually be used due to the fact that at a given rate one of the code-modulation combinations will lead to the best performance, e.g., 4-ASK with a half rate code compared to 16-ASK with a quarter rate code achieves the same spectral efficiency of 1 bit/s/Hz at a much lower E_b/N_0. Based on these simulation results, the system performance can be characterized as will be discussed in the next section.

3.2. Coded Bisection Approach

Consider the well-known optimization problem to enhance the error rate performance

$$\text{minimize} \quad P_\text{Total} = \sum_{n=0}^{N_C-1} P_n \quad (11a)$$

$$\text{subject to} \quad \sum_{n=0}^{N_C-1} r_n = R_\text{Total} \quad (11b)$$

$$\mathrm{P}_b < \mathrm{P}_\text{Target}\ . \quad (11c)$$

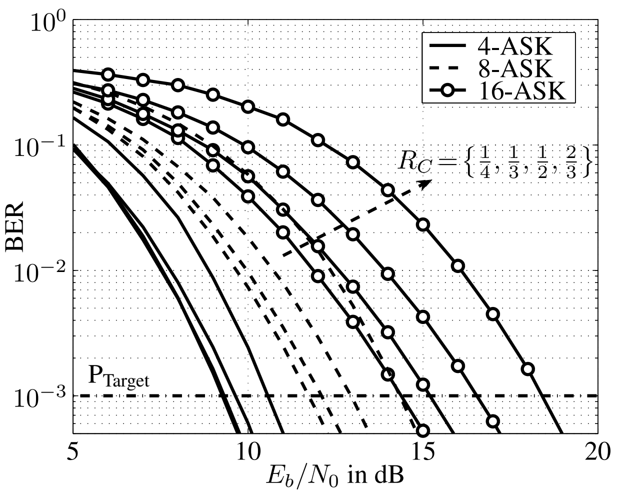

Fig. 2. BER vs. E_b/N_0 comparison of all combinations of $\sqrt{M}$-ASK with $\log_2(\sqrt{M}) = 2, 3, 4$ and convolutional codes $R_C \in \{1/4, 1/3, 1/2, 2/3\}$ (left to right) with $L_C = 3$ and frame length $L_N = 256$, $\text{P}_{\text{Target}} = 10^{-3}$

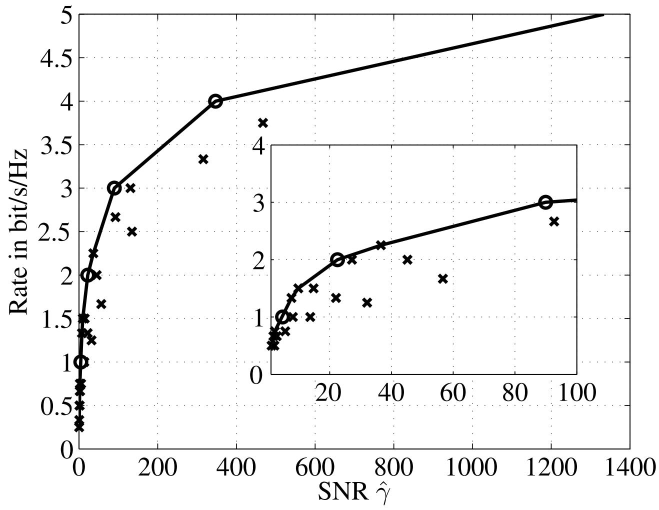

Fig. 3. Set of all rate-power points (x - coded modes; o - uncoded modes) and its convex hull (solid line) at $\text{P}_{\text{Target}} = 10^{-3}$, $L_N = 256$ and $L_C = 3$.

With this approach the transmit power P_{Total} is minimized given an overall target rate R_{Total} and error rate P_{Target}. A local code rate $R_{C,n}$ in conjunction with the applied modulation $\sqrt{M_n}$ define the bit rate $r_n = \log_2(\sqrt{M_n})R_{C,n}$ on subcarrier n. Note that the optimum solution at one target BER/FER scaled to the available transmit power can be used to show performance gains in terms of the error rate.

The error rate constraint P_{Target} determines the power that is necessary to achieve a certain rate requirement. The results in Fig. 2 at the target BER lead to rate-power pairs defining potential subcarriers modes. Accordingly, Fig. 3 shows all rate-power points up to a maximum rate $R_{\text{max}} = 5$ bit/s/Hz at a target BER of $\text{P}_{\text{Target}} = 10^{-3}$ found by Monte-Carlo simulations, stating the AWGN performance of an equivalent system with frame length N_C and the analytical bit error rate expression for uncoded ASK constellations in (9). In a perfect adaptive system subcarrier channel variations would be exploited or compensated by assignment of power, modulation alphabet and code rate, making the AWGN performance a good quality indicator to identify the SNR requirement of each mode.

The set of all rate-power points is therefore defined as $\mathcal{S} = \{(R_{C,i}, \hat{\gamma}_i) | f_{M_i,R_{C,i}}(p_i) = \text{P}_{\text{Target}}\}$, where $f_{M_i,R_{C,i}}$ denotes the utilized error rate function, e.g., the simulated BER in Fig. 2, parametrized by the transmission parameters and the power. The SNR $\hat{\gamma}_i$ for the real valued system is defined as

$$\hat{\gamma}_i = R_{C,i} \log_2(\sqrt{M_i}) \frac{E_{b,i}}{N_0/2} \,. \tag{12}$$

where i is the index over the elements of set $\mathcal{S}$.

To solve (11a) efficiently, convexity has to be ensured. To this end, a convex set of rate-power points $\mathcal{C} \subset \mathcal{S}$ has to be found, which can be constructed by the convex hull of the whole set as shown by the solid line in Fig. 3, where [18] has been used. A look-up table has to be generated once for a set of system parameters, i.e., N_C, code ensemble and maximum allowed rate. Based on such an easily storable look-up table the rate of each OFDM symbol can be optimized. More specifically, the look-up table has to return the mode with the greatest rate still being feasible. Feasibility in the bisection approach is connected to the slope of the rate-power curve, given by $\delta\mathbf{R}/\delta\hat{\mathbf{\Gamma}}$ with $\mathbf{R}$ being the vector of all rates on the convex hull and $\hat{\mathbf{\Gamma}}$ the vector of the respective SNRs $\hat{\gamma}$.

One very efficient way to find the optimal solution given the Lagrangian formulation of the convex optimization problem (11a)

$$J(\lambda) = \sum_{n=1}^{N_C} P_n + \lambda \left(\sum_{n=1}^{N_C} r_n - R_{\text{Total}} \right) \tag{13}$$

is the bisection approach, which has been applied to the uncoded bit and power loading problem by Krongold et. al [2]. Considering the unconstrained problem, i.e., neglecting R_{Total}, each λ corresponds to an optimal power and rate distribution, which minimizes the cost function $J(\lambda)$. Accordingly, if $\delta J(\lambda)/\delta r_n = 0, \ \forall n$, the condition

$$\frac{\delta P_n}{\delta r_n} = -\lambda \quad \forall n \,, \tag{14}$$

has to be fulfilled, meaning that the optimal rates and powers have to be chosen through that point on the rate-power curve with slope λ.

The (scaled) look-up table, which provides the rate and power at a specific slope $\eta = \lambda/|H_n|^2$, has to be constructed from $\mathcal{C}$ to calculate the optimal rates and powers on all subcarriers for a given λ. The optimal λ^*, which minimizes the power taking the target rate R_{Total} into account, can then be found iteratively. A pseudo-code description of the algorithm and more details are given in [15].

This bisection approach efficiently solves the convex optimization problem of the coded system by the previously explained steps, which is carried out once (1-step). However, the optimization results in local subcarrier code rates, whose applicability, e.g., by variable puncturing techniques is beyond the scope of this paper. Instead, a mean code rate $\bar{R}_C$ is calculated and chosen such that the code rate $\bar{R}_C$ fulfills

$$\bar{R}_C < \frac{1}{N_C} \sum_{n \in \mathcal{T}}^{N_C} R_{C,n} \,. \tag{15}$$

This code is used as the outer code for one OFDM symbol, where $\mathcal{T}$ denotes the set of all nonzero local code rates $R_{C,n}$. Due to this solution, though, the target rate cannot be guaranteed. As the subcarrier rates are changed by the application of a fixed global code rate, an overall rate loss is introduced, which means that stronger error protection than required is applied. This in turn violates the target rate constraint. A solution to this problem is a 2-step process, fixing the code rate in the first step followed by the optimization over a convex set of all modes applying this code rate in the second step. This procedure is used in our simulations as well.

The proposed bisection method used here is a very efficient and fast algorithm as simple calculations are necessary to obtain a feasible solution.

4. COMBINING THE CODED BISECTION METHOD WITH PAPR REDUCTION

4.1. Active Constellation Extension and Tone Reservation

The bit and power loading approach from Section 3.2 may decrease the error rate, whereas the possibly large PAPR value after the IFFT operation remains. Therefore, the principles of active constellation extension and tone reservation are combined in one common optimization approach to reduce the PAPR. The idea of ACE is to extend the outer constellation points of the utilized modulation alphabet. For higher order modulation the inner constellation points must remain at their positions, otherwise the Euclidean distance between these symbols is decreased. Fig. 4 depicts an ACE example of 16-QAM, where the points at the edges only have one degree of freedom (real *or* imaginary part) and the corner points have two (both real and imaginary part). The inner points are identical to the original constellation. Basically, such an extended constellation does not increase the bit error rate due to the constant minimum Euclidean distance. Instead, an increased Euclidean distance at the outer constellation points may decrease the error rate.

Furthermore, if bit and power loading procedures are applied free subcarriers may occur, i.e., no modulation is applied on these carriers. Now, tone reservation (TR) can be used to exploit these unused subcarriers. They can also be found in the origin of the left I/Q diagram in Fig. 4. Using TR methods means to find a complex additive signal on these unused subcarriers, where compared to ACE subcarriers no restrictions to the optimum additive components are made. According to (8), the QCQP problem in (6) is now stated as a real-valued problem with $2N_C$ optimization variables by separating real and imaginary part. However, as real and imaginary part are dependent optimization variables, this separation leads to a suboptimal solution with a remaining small error. The right part of Fig. 4 shows how the approach approximates the optimal solution circle of the power restricted complex envelope, which depicts the maximum achievable peak power reduction resulting from the optimum complex optimization problem. The real-valued approach packs all time-domain samples in an outer square, where the maximum possible error is indicated by the two dots. The variable t will be explained later. Furthermore, the circle can be closer approximated by using

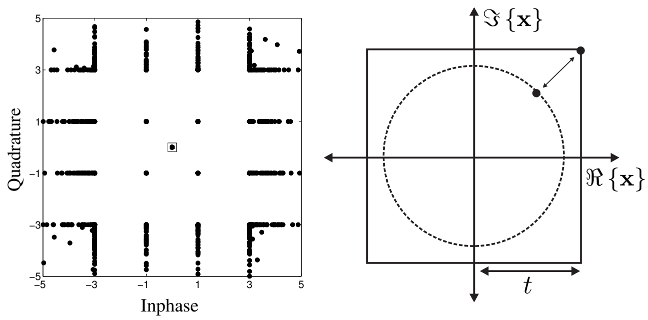

Fig. 4. (left) Exemplary active constellation extension for 16-QAM with zero symbols exploitable by TR techniques; (right) Approximation of the power restricted complex envelope

additional phase-shifted versions of the time-domain signal block. This in turn escalates the amount of constraint equations especially in a link adaption scenario and slows down convergence speed of the resulting algorithm [17]. That is quite an important aspect as the link adaption algorithms already introduce processing delays at the transmitter.

4.2. Joint ACE/TR Optimization for Bit Loading

Motivated by the fact that the QAM symbols are separated into two ASK symbols in our system model, the magnitudes of the real and imaginary parts are now optimized independently. Therefore, (8) can be written as a linear programming (LP) problem [19]. The combination of ACE and TR introduces the following constraints to this optimization problem. For all non-data bearing subcarriers that can be exploited by tone reservation, there are no restrictions on the feasible region of the symbols after optimization. For all other active carriers only those symbols corresponding to a corner or edge point of the selected complex symbol alphabet are considered. In summary, this implies that only outer constellation symbols of the ASK signal $\mathbf{X}_\mathrm{R}$ are incorporated in the following optimization process. The determined set of ASK symbol indices prior to the optimization is indicated by $\mathcal{I}_c = \left[i_1, \ldots, i_{|\mathcal{I}_c|}\right]$. Only those elements contained in this set are used for optimization purposes. It is worth mentioning that the zero amplitude symbols of the TR subcarriers are also included in $\mathcal{I}_c$. Hence, by optimizing with respect to the constraints on $\mathcal{I}_c$ the additive signal $\mathbf{C}$ including ACE and TR subcarriers is optimized. If the columns of the disregarded ASK symbols are excluded from the IDFT matrix such that $\tilde{\mathbf{F}}_\mathrm{R} = \left[\mathbf{f}_{i_1}, \mathbf{f}_{i_2}, \ldots, \mathbf{f}_{i_{|\mathcal{I}_c|}}\right] \in \mathbb{R}^{\,2N_c \times |\mathcal{I}_c|}$ and similarly arrange all frequency-domain correction symbols of the considered symbols in vector $\tilde{\mathbf{C}}_\mathrm{R} = \left[C_{i_1}, C_{i_2}, \ldots, C_{i_{|\mathcal{I}_c|}}\right]^T \in \mathbb{R}^{\,|\mathcal{I}_c| \times 1}$, it is easy to see that the relation $\mathbf{F}_\mathrm{R}\mathbf{C}_\mathrm{R} = \tilde{\mathbf{F}}_\mathrm{R}\tilde{\mathbf{C}}_\mathrm{R}$ holds. Then, if the objective function in (8) is upperbounded by a certain value t, the optimization problem can be reformulated by

$$\text{minimize} \quad t \tag{16a}$$

$$\text{subject to} \quad \left| x_{R,k} + \mathbf{F}_R^{(k)} \tilde{\mathbf{C}}_\mathrm{R} \right| \leq t \quad \forall\, k \in \mathcal{I}_c \;, \tag{16b}$$

where $\mathbf{F}_R^{(k)}$ denotes the k-th row of matrix $\mathbf{F}_\mathrm{R}$. The element-wise inequality constraints for the absolute values in (16b) can be written in matrix form

$$\begin{bmatrix} \tilde{\mathbf{F}}_\mathrm{R} & \mathbf{I}_{2N_c \times 1} \\ -\tilde{\mathbf{F}}_\mathrm{R} & \mathbf{I}_{2N_c \times 1} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{C}}_\mathrm{R} \\ t \end{bmatrix} \geq \begin{bmatrix} -\mathbf{x}_\mathrm{R} \\ \mathbf{x}_\mathrm{R} \end{bmatrix} \;, \tag{17}$$

whereas the set limitation in (16b) can be stated with respect to the feasible regions of the transmit symbols

$$S_k C_{R,k} \geq 0 \quad \forall \quad k \in \mathcal{I}_c \tag{18a}$$

$$C_{R,k} \stackrel{!}{=} 0 \quad \forall \quad k \notin \mathcal{I}_c \;, \tag{18b}$$

where $S_k = \mathrm{sgn}\left\{X_{R,k}\right\} \in \{-1, 0, +1\}$ is the sign of the kth ASK symbol. The constraints of inner constellation points and the unconsidered real and imaginary parts of edge points of the QAM symbols are given in (18b). These constraints can be easily fulfilled if they are excluded from the optimization problem. The inequality constraints in (18a) can again be written in matrix form with element-wise inequality

$$\mathbf{S}\tilde{\mathbf{C}}_\mathrm{R} \geq \mathbf{0}_{|\mathcal{I}_c| \times 1} \;. \tag{19}$$

In (19) the sign variables are arranged in a diagonal matrix $\mathbf{S} = \mathrm{diag}\left\{S_1, \ldots, S_{|\mathcal{I}_c|}\right\}$. At last, by defining the vectors $\mathbf{d} = \left[\mathbf{0}_{1 \times |\mathcal{I}_c|}\ 1\right]^T \in \mathbb{R}^{|\mathcal{I}_c|+1 \times 1}$ and $\mathbf{y}_\mathrm{R} = \left[\tilde{\mathbf{C}}_\mathrm{R}\ t\right]^T \in \mathbb{R}^{|\mathcal{I}_c|+1 \times 1}$ the optimization problem in (16) becomes

$$\text{minimize} \quad \mathbf{d}^T \mathbf{y}_\mathrm{R} \tag{20a}$$

$$\text{subject to} \quad \mathbf{A}_\mathrm{R}\mathbf{y}_\mathrm{R} \geq \mathbf{b}_\mathrm{R} \;, \tag{20b}$$

with matrix $\mathbf{A}_\mathrm{R}$ and vector $\mathbf{b}_\mathrm{R}$ given by

$$\mathbf{A}_\mathrm{R} = \begin{bmatrix} \tilde{\mathbf{F}}_\mathrm{R} & \mathbf{I}_{2N_c \times 1} \\ -\tilde{\mathbf{F}}_\mathrm{R} & \mathbf{I}_{2N_c \times 1} \\ \mathbf{S} & \mathbf{0}_{|\mathcal{I}_c| \times 1} \end{bmatrix} \quad \text{and} \quad \mathbf{b}_\mathrm{R} = \begin{bmatrix} -\mathbf{x}_\mathrm{R} \\ \mathbf{x}_\mathrm{R} \\ \mathbf{0}_{|\mathcal{I}_c| \times 1} \end{bmatrix} . \tag{21}$$

This is a LP problem [19] with $2N_c + 1$ variables and $4N_c + |I_c|$ constraints that can be solved with standard tools, e.g., an efficient iterative Newton method [20], which works considerably well with a large number of constraints. The minimum solution value of t describes the size of the approximate square as shown in Fig. 4 [16].

One possible alternative method for approximating the previously described LP with less complexity is to use a gradient-project approach like in [14] for the ACE component and extend it with an update rule for the additive correction term related to the TR subcarriers [21]. This combines the properties of both iterative schemes in a single procedure, which is called enhanced gradient-project algorithm (EGPA) in the following. The corresponding update rule for the time-domain transmit vector at iteration i is [16]

$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} + \mu\mathbf{c}_\mathrm{ACE} - \nu\mathbf{c}_\mathrm{TR} \;, \tag{22}$$

where the scalars μ and ν are smart-gradient step sizes chosen as suggested for the smart gradient method in [14] and the algorithm in [21]. For more details refer to [16]. It is worth mentioning that this algorithm introduces additional IFFT and FFT operations, which in turn slightly increases the complexity compared to the original OFDM system. For high spectral efficiencies all subcarriers are data bearing subcarriers especially if a maximum modulation alphabet size is set. Then this algorithm inherently reduces to the one in [14].

5. SIMULATION RESULTS

For the investigations the number of subcarriers was set to $N_C = 256$, the channel has $L_F = 6$ taps, the spectral efficiency is 3 bit/s/Hz with a non-adaptive average code rate of 0.5, whereas the maximum modulation size is $\sqrt{M_{n,\max}} =$ 32 ASK (1024-QAM)[1]. The error rate constraint was set to a BER of $\mathrm{P}_\mathrm{Target} = 10^{-3}$. For the consideration of the memoryless nonlinear HPA, the well-known Rapp model with smoothing parameter $p = 2$ and an input power backoff (IBO) of 3 dB is used [22]. The EGPA method uses a clipping ratio of 3 dB and the maximum iteration number was set to 3. In Fig. 5 it is shown that for a high spectral efficiency the proposed coded bit and power loading outperforms the original biscetion method of Krongold, where a gain of approximately 1.5 dB is obtained at a BER of $2 \cdot 10^{-3}$. By applying PAPR reduction with ACE/TR techniques using a direct solution of the LP, the BER performance gain can be further improved up

[1]Higher order modulation is generally more often used in DSL as in wireless channels. Nevertheless, although occurring rarely (cf. Fig. 6), they allow for higher flexibility of the algorithm.

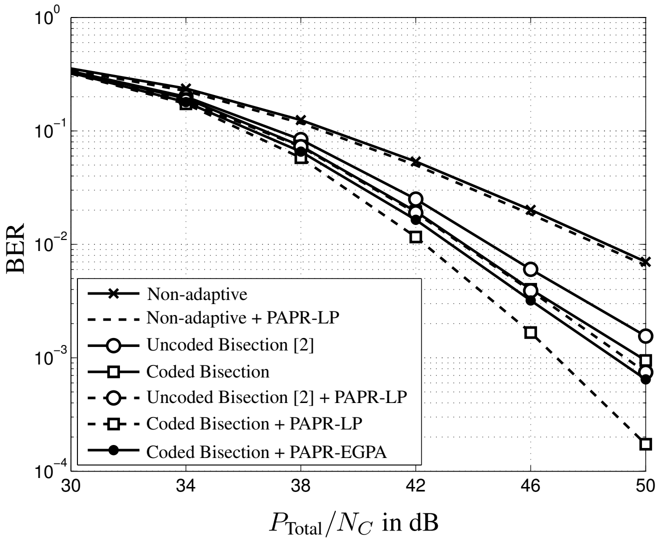

Fig. 5. BER vs. subcarrier SNR for 3 bit/s/Hz for $N_c = 256$ using the Rapp HPA model with $p = 2$ and an IBO of 3 dB

to approximately 4 dB at the same BER. This improvement follows from the increased Euclidean distance of outer constellation points and is larger for the coded bisection method. This is due to the fact that the coded bisection method deactivates subcarriers more often by trend. In addition, lower order modulation is chosen more frequently compared to the uncoded bisection method. Instead, larger code rates are used to compensate for the rate loss. This behavior is depicted in a histogram in Fig. 6 recorded for 1000 channel realizations with a maximum modulation size of $M_{n,\max} = 64$ QAM. The difference of the coded bisection methods (1-step or 2-step) was explained in Section 3.2, whereas the 2-step algorithm is applied in the BER curves in Fig. 5. Hence, as lower order modulation is used more often the ACE reduction algorithm has more degrees of freedom to decrease the PAPR and inherently increases the Euclidean distance of more constellation points. Furthermore, the proposed coded method needs less power to achieve the same target BER. Thus, SNR gains can be obtained.

The performance of the EGPA is degraded due to the imperfect constellation extension but still achieves better performance compared to the uncoded bisection method with PAPR reduction. For a spectral efficiency of 3 bit/s/Hz the OFDM scheme without loading suffers from the commonly chosen 64-QAM alphabet, which strongly degrades the BER performance under HPA influences. The ACE/TR scheme cannot compensate this behavior as for 64-QAM only a small number of symbols at the constellation edges can be used for improvements.

Fig. 7 shows the complementary cumulative distribution function (CCDF) of the uncoded and coded bisection methods with different PAPR reduction techniques. There, the curves indicate the probability that the PAPR of one OFDM symbol

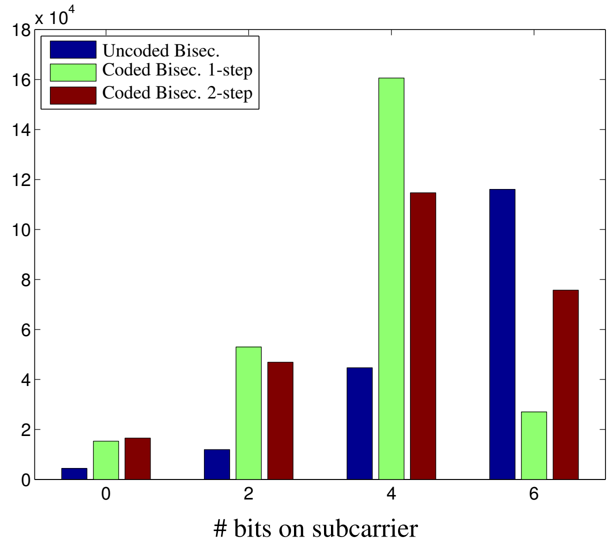

Fig. 6. Histogram of different bit allocation algorithms for 1000 channel realizations with $N_C = 256$ subcarriers, a maximum modulation of $M_{n,\max} = 64$ (QAM), $L_F = 6$ and $P_{\text{Target}} = 10^{-3}$

exceeds a certain threshold, i.e.,

$$\text{CCDF}\left(\text{PAPR}\left(\mathbf{x},\mathbf{c}\right)\right) = P\left(\text{PAPR}\left(\mathbf{x},\mathbf{c}\right) > \beta\right) \ , \qquad (23)$$

where $P\left(\cdot\right)$ denotes probability and β is the threshold in dB. For the chosen IBO of the HPA the PAPR does neither increase nor decrease for different loading algorithms compared to the non-adaptive OFDM scheme. Hence, loading has no impact on the PAPR performance. Furthermore, applying additional ACE/TR techniques at this rate does not lead to any gains compared to the non-adaptive case as in the original bisection method too many subcarriers obtain a high order modulation. In contrast, due to the above-mentioned reasons the coded bisection method is able to reduce the PAPR. Here, a gain of 0.5 dB in terms of PAPR at 10^{-4} is visible for the proposed combination of coded bit and power loading and ACE/TR techniques. The EGPA algorithm instead is able to achieve a good compromise between PAPR/BER performance and complexity.

The common principle of clipping and filtering [23] usually achieves better PAPR performance with less complexity compared to our approach. Nevertheless, the increased complexity comes with a large BER gain in terms of efficient loading strategies and no increase in the out-of-band radiation. However, it is possible to apply another clipping device after our approach to further decrease the PAPR as the BER gain can be exploited for further PAPR improvements.

6. CONCLUSIONS

In this contribution a combination of a modified bisection approach and PAPR reduction using ACE and TR techniques is

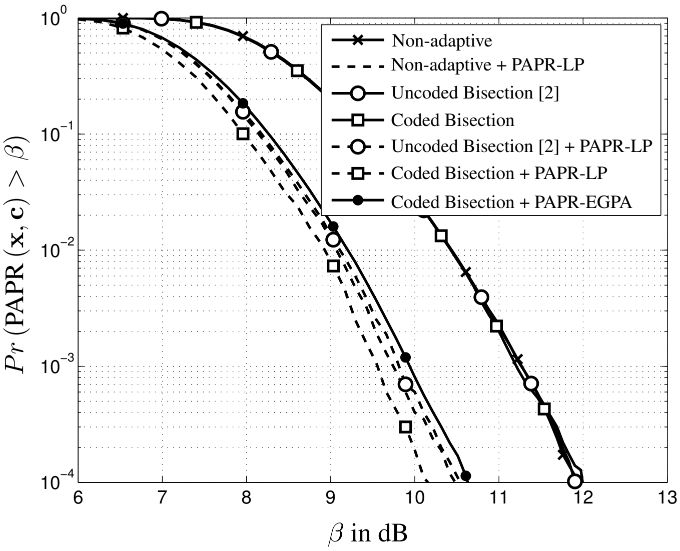

Fig. 7. PAPR CCDFs of different schemes with $N_c = 256$

proposed. Firstly, the channel code is included in the optimization of bit and power loading for multicarrier communication systems. In addition, the ACE technique can exploit the smaller modulation alphabets on the subcarriers allocated by the coded bisection method. Then, the rate loss is compensated for by larger code rate assignments. The proposed combination efficiently enhances the bit error rate performance for high-rate systems especially in presence of memoryless nonlinear distortions. Furthermore, the PAPR reduction is significantly improved with less complexity by an enhanced gradient-project algorithm, which offers a better trade-off between BER and PAPR performance.

7. REFERENCES

[1] J. A. C. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Commun. Mag.*, vol. 28, pp. 5–14, May 1990.

[2] B. S. Krongold, K. Ramchandran, and D. L. Jones, "Computationally efficient optimal power allocation algorithms for multicarrier communication systems," *IEEE Trans. Commun.*, vol. 48, pp. 23–27, Jan. 2000.

[3] G. Caire, G. Taricco, and E. Biglieri, "Capacity of bit-interleaved channels," *IEE Electronics Letters*, vol. 32, no. 12, pp. 1060–1061, June 1996.

[4] G. Caire, G. Taricco, and E. Biglieri, "Bit-Interleaved Coded Modulation," *IEEE Trans. Inf. Theory*, vol. 44, no. 3, pp. 927–946, May 1998.

[5] C. Bockelmann and K.-D. Kammeyer, "Adaptive Interleaving and Modulation for BICM-OFDM," in *International OFDM-Workshop (InOWo)*, Hamburg, Germany, August 2008.

[6] Y. Li and W. Ryan, "Mutual-information-based adaptive bit-loading algorithms for LDPC-coded OFDM," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1670–1680, May 2007.

[7] C. Stierstorfer and R. F. H. Fischer, "Rate Loading in OFDM Based on Bit Level Capacities," in *International Symposium on Information Theory (ISIT)*, Toronto, Canada, July 2008.

[8] S. Stiglmayr, M. Bossert, and E. Costa, "Adaptive Coding and Modulation in OFDM Systems using BICM and Rate-Compatible Punctured Codes," in *European Wireless*, Paris, France, April 2007.

[9] K. Brueninghaus, D. Astely, T. Sölzer, S. Visuri, A. Alexiou, S. Karger, and G. Seraji, "Link performance models for system level simulations of broadband radio access systems," in *IEEE Intern. Symposium on Personal, Indoor and Mobile Radio Commun. (PIMRC)*, Berlin, Germany, Sept. 2005.

[10] S. Stiglmayr, M. Bossert, and E. Costa, "Mutual-information based adaptive coding and modulation in bit-interleaved OFDM systems using punctured LDPC codes," *European Trans. on Telecommun.*, vol. 19, no. 7, pp. 801–811, Sept. 2008.

[11] S. S. Das, F. Tariq, M. I. Rahman, F. B. Frederiksen, E. De Carvalho, and R. Prasad, "Impact of nonlinear power amplifier on link adaptation algorithm of OFDM systems," in *IEEE Vehicular Technology Conf. (VTC) Fall*, Baltimore, MD, USA, Oct. 2007.

[12] S. H. Han and J. H. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission," *IEEE Wireless Commun. Mag.*, vol. 12, no. 2, pp. 56–65, Apr. 2005.

[13] Y. J. Kou, W.-S. Lu, and A. Antoniou, "New peak-to-average power-ratio reduction algorithms for multicarrier communications," *IEEE Trans. Circuits Syst. I*, vol. 51, no. 9, pp. 1790–1800, Sept. 2004.

[14] B. S. Krongold and D. L. Jones, "PAR reduction in OFDM via active constellation extension," *IEEE Trans. Broadcast.*, vol. 3, pp. 258–268, Sept. 2003.

[15] C. Bockelmann, D. Wübben, and K.-D. Kammeyer, "Efficient coded bit and power loading for BICM-OFDM," in *IEEE Vehicular Technology Conf. (VTC) Spring*, Barcelona, Spain, Apr. 2009.

[16] M. Petermann, D. Wübben, and K.-D. Kammeyer, "Joint constellation extension and tone reservation for PAPR reduction in adaptive OFDM systems," *submitted to IEEE Intern. Workshop on Signal Proc. Advances for Wireless Comm. (SPAWC)*, 2009.

[17] B. S. Krongold and D. L. Jones, "An active-set approach for OFDM PAR reduction via tone reservation," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 495–509, Feb. 2004.

[18] C. Bradford Barber, D. P. Dobkin, and H. Huhdanpaa, "The quickhull algorithm for convex hulls," *ACM Trans. on Mathematical Software*, vol. 22, pp. 469–483, 1996.

[19] S. P. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[20] O. L. Mangasarian, "A Newton method for linear programming," *Journal of Optimization Theory and Applications*, vol. 121, no. 1, pp. 1–18, Apr. 2004.

[21] H. Schmidt and K.-D. Kammeyer, "Reducing the peak to average power ratio of multicarrier signals by adaptive subcarrier selection," in *IEEE Int. Conf. Universal Personal Commun. (ICUPC)*, Florence, Italy, Oct. 1998.

[22] C. Rapp, "Effects of HPA-nonlinearity on a 4-DPSK/OFDM-signal for a digital sound broadcasting signal," in *Second European Conference on Satellite Communications (ECSC-2)*, Luettich, Belgium, Oct. 1991.

[23] J. Armstrong, "Peak-to-average power reduction for OFDM by repeated clipping and frequency domain filtering," *IEE Electronics Letters*, vol. 38, no. 5, pp. 46–247, Feb. 2002.