ADAPTIVE BICM-OFDM SYSTEMS

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Abstract Link Adaptation for Orthogonal Frequency Division Multiplex (OFDM) systems is an important topic to achieve higher spectral efficiencies or better error rate performance. However, channel coding has been neglected previously, only gaining some attention recently. In this paper, an overview over existing techniques and algorithms to adapt Bit Interleaved Coded Modulation (BICM) systems are given and new results regarding the extension of known approaches to multiple antenna OFDM systems are presented.

1. Introduction

The adaptation of power and modulation of a set of parallel channels, e.g., using Orthogonal Frequency Division Multiplex (OFDM), is an important topic in research. Early works focused on uncoded BER optimisation [1–3], but nowadays the consideration of channel coding into bit and power loading schemes has become a major aspect. Several contributions to this field have been made, especially considering the influence of Bit Interleaved Coded Modulation (BICM) on such systems. With regard to the channel capacity of BICM systems, respectively the average mutual information (AMI) of a code word, strategies have been devised to either maximise the transmittable data rate [4–6] or minimise the bit or frame error rate at a fixed data rate [7–10]. In order to compare and analyse these approaches, the respective optimisation problems for coded systems will be pointed out and different solutions will be discussed with regard to their complexity, performance and specific differences of the given approaches.

One very interesting extension of the coded bit and power loading field is the application to multiple-antenna systems. Besides the approach of diagonalisation, which requires perfect channel state information (CSI), few contributions have been made. Especially, regarding suboptimal receivers (e.g, linear receivers), new results will be discussed.

The remainder of this paper is organised as follows. In Section 2 the system model and important parameters are shortly stated, which are then used in Section 3 to formulate the examined optimisation problems and discuss the respective solutions. In Section 4 the single antenna results are then extended to multiple antenna systems. Finally, Section 5 concludes this paper.

2. System Model

We consider an OFDM system with N_T transmit and N_R receive antennas, which is assumed to be perfectly synchronised and free of intersymbol and inter-channel interference (ISI, ICI). Furthermore, CSI at both transmitter and receiver side shall be perfectly known. Thus, the equivalent baseband system in frequency domain is given by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{P}_k^{1/2} \mathbf{d}_k + \mathbf{n}_k \, , \tag{1}$$

where $\mathbf{H}_k \in \mathbb{C}^{N_R \times N_T}$ and $\mathbf{P}_k^{1/2} = \mathrm{diag}\{[\sqrt{p_{1,k}}, \cdots, \sqrt{p_{N_T,k}}]\}$ denote the channel and power allocation matrix on subcarrier $k = 1, \ldots, N_C$ and $\mathbf{y}_k \in \mathbb{C}^{N_R}$, $\mathbf{d}_k \in \mathbb{C}^{N_T}$, $\mathbf{n}_k \in \mathbb{C}^{N_R}$ are the receive vector, transmit vector and noise vector, respectively. The total transmit power is given by $\mathcal{P} = \sum_{i=1}^{N_T} \sum_{k=1}^{N_C} p_{i,k}$ and the power of the i.i.d. noise $\mathbf{n}_k \sim \mathcal{N}_C(0, \sigma_n^2 \mathbf{I}_{N_R})$ is fixed to $\sigma_n^2 = 1$. To each subcarrier k and antenna i an individual alphabet (M-QAM) of cardinality $M_{i,k}$ may be assigned. In terms of forward error correction (FEC) non-systematic non-recursive convolutional encoders of rates $R_C \in \{1/4, 1/3, 1/2, 2/3, 3/4\}$ and constraint length $L_C = 3$ are applied throughout this paper leading to a variety of possible *transmission modes* (code/modulation combinations).

As a special case, the system model for a single antenna at receiver and transmitter $N_T = N_R = 1$ results

$$y_k = h_k \cdot \sqrt{p_k} \cdot d_k + n_k \, , \tag{2}$$

which will be used throughout Section 3.

3. Link Adaptation for Single Antenna Systems

In general two classes of link adaptation problems can be defined: rate maximisation and error rate enhancement. The latter can be achieved directly by optimisation of some measure of the bit or frame error rate (BER/FER) or by optimising the required power for a certain error constraint. The specific approach to each of these optimisation problems, however, varies according to the assumptions on performance measures and constraints. Two widely used quality indicators are the capacities and signal-to-noise-ratios (SNRs) of a set of parallel channels. SNR requirements can be formulated for the allowed *modes* and capacity results are generally a good indicator for the performance of strong channel codes. Considering single-antenna systems both measures are exchangeable to some extend. However, regarding finite input alphabets in contrast to Gaussian input alphabets, a saturation at the maximum rate of the input alphabet occurs. Thus, higher SNRs may provide no gain in terms of capacity, whereas lower BER/FER may still be achieved if the actual performance of a coded system is considered. Furthermore, the resulting capacity expression can no longer be solved in closed form, even though an efficient numerical solution via Gauss-Hermite quadrature [11] is possible. Nevertheless, considering the bitwise capacity the BER/FER performance of a system can be described independently from the applied alphabet by the AMI of a code word [4][6]. This observation has motivated a class of algorithms, which will be described in Section 3.2.

On the other hand, the issue of saturation still remains, especially for non-capacity approaching codes, justifying another approach, which utilises the AWGN performance of an equivalent coded system to describe the SNR requirements of certain *modes*. The following section will discuss this idea in more detail.

3.1 Simulation Supported Methods

In the following the description will be focused on the rate maximisation problem as the problem solutions of both rate optimisation and error enhancement are almost identical except for some realisation aspects. Aiming at the maximisation of the data rate, we have to solve

$$\text{maximize} \quad R_{\text{Total}} = \sum_{k=1}^{N_C} r_k$$

$$\text{subject to} \quad \sum_{k=1}^{N_C} p_k = \mathcal{P} \quad \text{and} \quad \mathrm{P}_{b/f} < \mathrm{P}_{\text{Target}}\,, \tag{3}$$

where r_k and p_k denote the rate and power on subcarrier k, respectively. $\mathcal{P}$ is the overall available power and $\mathrm{P}_{\text{Target}}$ denote a service of quality target in form of a minimum BER P_b or FER P_f, which should be obtained.

One important question arises in this context: how are the rates and powers of the parallel subchannels connected? In order to carry out the optimisation, the rates have to be formulated as explicit functions of the powers, such that $r_k = f(p_k)$, which in case of coded systems with finite symbol alphabets is a non-trivial problem. If channel coding is neglected, analytical BER/FER expressions, e.g., for M-QAM modulations, can be used [1–3]. Due to the difficulties in finding analytical expressions for the FER of coded systems, however, the simulated performance of different code and modulation combinations is often utilised to quantify the performance [8, 10]. The error rate constraint $\mathrm{P}_{\mathrm{Target}}$ provides the necessary connection of power and rate to calculate a look-up table of the rate-power function, where only discrete points can be achieved. This provides an operating point allowing for some greedy or heuristic procedure to solve the optimisation problem approximately.

Eq. (3) has been solved by the authors via expansion to coded systems of the well known bisection approach used by Krongold et al. [1]. The set of all rate-power pairs at a given BER/FER $\mathrm{P}_{\mathrm{Target}}$ of all allowed transmission modes, employing SNR thresholds obtained from AWGN simulation as well as analytical expressions for uncoded M-QAMs, has been used to find a convex set allowing for a convex optimisation solution of (3) via Lagrangian multipliers [10]. The important difference to previous approaches are a fine grained power allocation and the optimality of the bisection solution under the given constraints.

Stiglmayr et al. [8] developed a very similar approach, however, solving a capacity maximisation problem via linearisation, which resulted in a simple heuristic. Here, also a look-up table of SNR thresholds from AWGN simulations of equivalent frame length has been applied. One important difference to the previously mentioned solution is, that out of all available *modes*, only these with the same rate but higher SNR treshold than the minimum one at that rate are excluded, whereas a convex set over all possible points may exclude modes viable in terms of the SNR treshold.

3.2 Capacity Based Approaches

The derivation of bit and power loading algorithms employing mutual information (MI) with Gaussian signalling as a quality indicator is widely used in literature and capacity losses due to finite symbol alphabets have been incorporated by application of the well known gap approximation, e.g. [12]. However, this approach still neglects the BICM system structure widely adapted in communication systems [13]. Recent research, hence, considers the BICM or parallel decoding capacity of a system promising a better performance estimate due to consideration of the interleaving and negligence of dependencies between the bits of a symbol. An equivalent optimisation problem to (3) in capacity terms is

$$\begin{aligned} &\text{maximize} \quad R_{\text{Total}} = \sum_{k=1}^{N_C} r_k \\ &\text{subject to} \quad \sum_{k=1}^{N_C} p_k = \mathcal{P} \qquad \text{and} \quad \text{MI}_{\text{avg}} \leq \text{MI}_{\text{req}}\,, \end{aligned} \tag{4}$$

where MI_{avg} and MI_{req} denote the AMI of one code word and the minimum required AMI, respectively. Furthermore, here the r_k are limited to integer number of bits as available through M-QAM constellations.

For nearly capacity achieving codes (Turbo codes, LDPC) it was found out, that the AMI of a code word is a good indicator for the error rate performance and approximately independent of the applied modulation, e.g. [4]. Even if the assumptions of perfect codes and infinitely long code words used to derive the corresponding capacity expression are never exactly achievable with practical codes, the mean BER/FER can be described quite accurately also for weaker codes and shorter code word lengths. Unfortunately, the BICM capacity is a nonlinear function of the SNR and has to be calculated and stored as look-up tables beforehand. Then, interpolation can be used to calculate the capacities of the subchannels. To ease memory consumption, the number of different entries in such a look-up table can be limited by thresholding at the cost of accuracy. Also, similar to the previous section and indicated by (4) a look-up table of SNR thresholds can be derived from AMI requirements using a certain target BER/FER.

Based on these observations, e.g., Li et al. [4] proposed three algorithms, each determining power and bit allocations for a set of parallel channels in order to maximise the rate given a BER/FER requirement. The BER/FER requirement is translated to a minimum AMI requirement, which is employed to allocate suitable modulations and powers to the subchannels in a greedy fashion. Exemplary, considering Algorithm 1 of [4] a subchannel's allocated bits are incremented if it has the smallest mutual information loss of all subcarriers. The overall AMI requirement has to be checked after each allocation to ensure the constraints. This approach employs the construction of SNR look-up tables as pointed out before. A related solution has been proposed for MIMO systems by the authors [6].

Another idea has been developed by Sankar et al. [5], who approximated the BICM capacities by simple exponential functions allowing the closed form derivation of a waterfilling scheme, which can be used to adapt the rate and power of a set of parallel channels. However, their approach is hard to apply to real systems as optimised code rates may

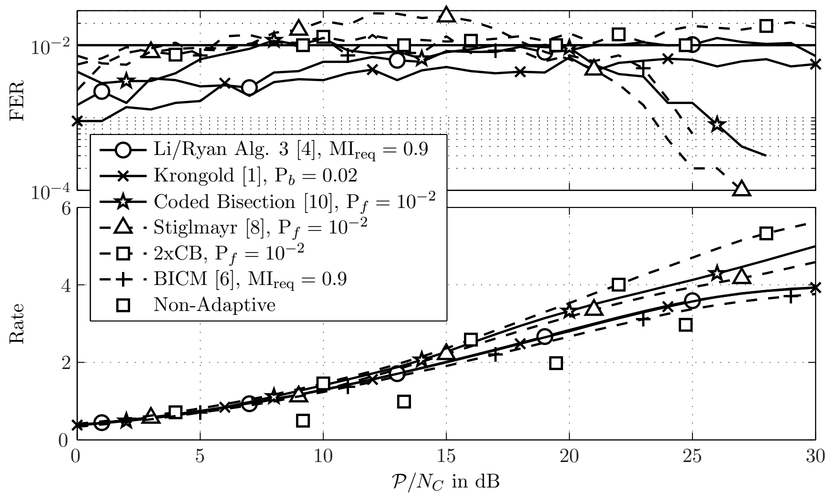

Figure 1. FER and Rate vs average power per subcarrier $\mathcal{P}/N_C$ for convolutional codes of constraint length $L_C = 3$; $N_C = 1024$ subcarriers, $L_F = 10$ channel taps, target FER of $\mathrm{P}_{\mathrm{Target}} = 10^{-2}$

vary strongly, which leads to severe performance losses if rounding to available code rates is exercised. Similarly, Lozano et al. [14] developed a closed form waterfilling solution dependent on the MMSE functions of the finite alphabets, which have to be numerically determined. This so called mercury waterfilling provides the optimal power allocation for a fixed bit allocation complementing schemes that neglect power loading.

The equivalent problem of error rate minimisation has been tackled by the authors with an algorithm that maximises the AMI of a code word for a single-input-single-output (SISO) system given an overall data rate [9]. Through this approach, no error rate requirement is needed as no power allocation beyond the reallocation of power from switched off subchannels is applied. Independently, Stiersdorfer et al. [7] presented a fast bit loading algorithm using a coarse approximation of the bit level capacities to distribute bits, which is very fast, but also results in slight to sever performance losses depending on the application scenario.

3.3 Results

Fig. 1 shows the Monte Carlo simulations of several algorithms for rate optimisation, including their respective constraints. As can be observed, the FER requirement $\mathrm{P}_{\mathrm{Target}} = 10^{-2}$ is roughly fulfilled by all approaches, with the tightest fulfilment up to medium SNR by the coded bisection solution. Only the "Coded Bisection" and "Stiglmayr" algorithms apply code rate adaptation, thus, achieving an overall better

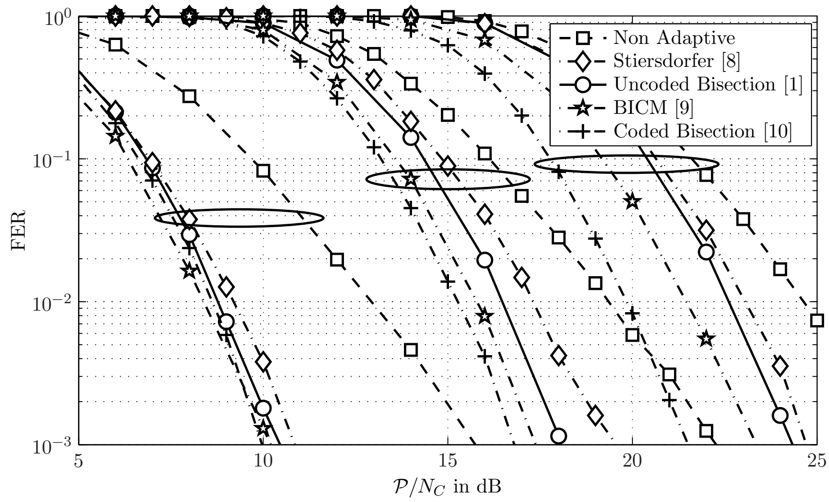

Figure 2. FER vs average power per subcarrier $\mathcal{P}/N_C$ for convolutional codes of constraint length $L_C = 3$ at spectral efficiencies of 1, 2 and 3 bit/s/Hz (left to right); $N_C = 1024$ subcarriers, $L_F = 10$ channel taps

performance than all fixed code rate solutions in terms of the supported rate. "Li/Ryan", "BICM" and "Krongold" show very similar results. At some point all methods reach saturation - near the maximum achievable rate no further improvement is possible - thereby overfulfilling the FER constraint with growing power. In case of the "Coded Bisection" the performance can be further enhanced by a two step process, running the algorithm once, fixing the code rate of the outer code, then running it a second time considering the fixed code rate, which is shown in "2xCB". The additional gain due to the new degree of freedom, the choice of the code rate, which is offered by the simulation supported methods, obviously allows for even higher transmission rates.

Fig. 2 shows results regarding the error rate enhancement at different spectral efficiencies of 1, 2 and 3 bit/s/Hz. Similar to the previous results, the additional adaptation of code rates allows for far greater performance gains. For this example the approximation used for "Stiersdorfer" is obviously not valid. Applying stronger codes, however, will lessen the overall loss considerably as will more diversity, i.e. more channel taps.

4. Extension to Multiple Antenna Systems

MIMO-OFDM is a promising technique considered for future communication systems. The extension of the aforementioned link adaptation

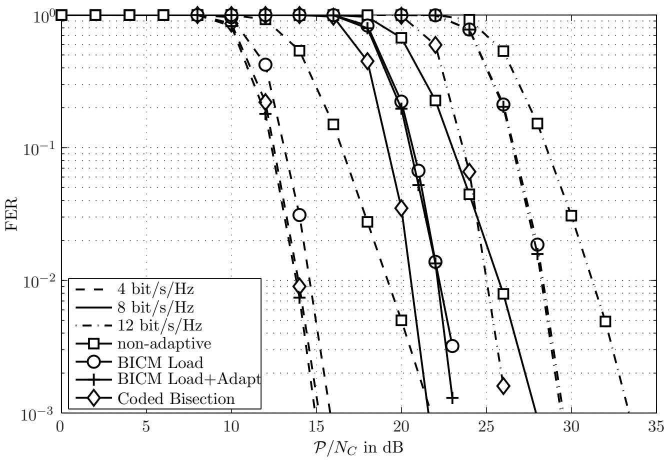

Figure 3. FER vs average power per subcarrier $\mathcal{P}/N_C$ for ZF receive filtering and convolutional codes of constraint length $L_C = 3$; $N_C = 1024$ subcarriers, $N_T = N_R = 4$ antennas, $L_F = 10$ channel taps

schemes to multiple antennas, however, is not straight forward and depends on the MIMO transmission scheme. If perfect transmit CSI can be assumed, channel diagonalisation via singular value decomposition (SVD) is a well known approach shown to be almost optimal utilising waterfilling with the gap approximation [15] under the assumption of an uncoded transmission. Independent parallel spatial channels would again allow for the application of all previously discussed approaches.

However, achieving full transmit CSI in a MIMO-OFDM system requires vast amounts of feedback. Thus, the authors proposed a solution for spatial multiplexing without spatial power and bit loading, that shows good performance [6]. Considering MIMO channels poses the problem, that the BICM capacity is not only depended on the transmit power, but also the channel matrix $\mathbf{H}_k$, ruling out efficient look-up table solutions. Instead the capacity has to be numerically calculated for every subcarrier by Monte-Carlo integration, which is computationally prohibitive. Using approximations, however, allows for a near optimal efficient solution [6]. Recent results on the capacity of MIMO systems with finite alphabets, e.g., [16] and [17], may either be used to achieve lower complexity solutions adapting [6] to suboptimal receivers or to provide the necessary BICM capacities to apply SISO link adaptation schemes.

Another quite straightforward approach is the consideration of linear zero-forcing (ZF) receivers leading to N_T equivalent AWGN channels with noise power $\sigma^2_{k,\ell} = \sigma^2_n\{(\mathbf{H}_k^H\mathbf{H}_k)^{-1}\}_{\ell,\ell}\,;\, \ell = 1, \cdots, N_T$ on each subcarrier k. Simply applying one of the SISO loading strategies, though, neglects the colouring of the noise and assumes independence of the equivalent AWGN channels on a subcarrier. Due to this, the solution will inherently be suboptimal, as antennas will be switched off by the algorithm, which is unconsidered by the receive filter. A joint receive filter design and loading should be developed in such a case.

Fig. 3 shows results for ZF receive filtering, where the author's BICM loading approach has been used to adapt the resulting set of "parallel" channels. Even though channel dependencies are neglected, huge performance gains can be achieved. "BICM Load+Adapt" additionally shows results for adapted ZF receive filters with respect to switched off antennas. As can be seen, only a relatively small enhancement is possible, which may be expanded by a few iterations of link and filter adaptation. A similar approach regarding rate optimisation is presented in [18].

5. Conclusion

A variety of solutions to the resource allocation problem of coded adaptive systems has been discussed in this contribution, where two principle approaches have been identified. The BICM capacity and simulated FER rates both offer multiple algorithmic solutions, which show performance differences due to their heuristic nature. The best approaches to the knowledge of the authors, however, adapt modulation, power and channel code jointly. Application of the discussed approaches is restricted to parallel SISO channels, necessitating some form of channel diagonalisation in the case of multiple antennas. Depending on the available transmitter side CSI, this may be carried out as a singular value decomposition or some suboptimal linear filter leading to self-interference. Regarding multi-antenna OFDM systems, a vast amount of feedback would be needed, so that alternative solutions are required. To this end, an approach for MIMO systems utilising ZF receive filters has been presented. Even though, dependencies of the spatial channels are neglected, good performance can be achieved.

References

[1] B.S. Krongold, K. Ramchandran, and D.L. Jones. Computationally Efficient Optimal Power Allocation Algorithms for Multicarrier Communication Systems. *IEEE Trans. on Communications*, 48(1):23–27, January 2000.

[2] R.F.H. Fischer and J.B. Huber. A New Loading Algorithm for Discrete Multitone Transmission. In *IEEE Global Telecommunications Conference*, pages 724–728, London, England, November 1996.

[3] H. Hartogs. Ensemble modem structure for imperfect transmission mead. US Patent No. 4,731,816, March 1988.

[4] Y. Li and W.E. Ryan. Mutual-Information-Based Adaptive Bit-Loading Algorithms for LDPC-Coded OFDM. *IEEE Trans. on Wireless Communication*, 6(5):1670–1680, May 2007.

[5] H. Sankar and K.R. Narayanan. Design of Near-Optimal Coding Schemes for Adaptive Modulation with Practical Constraints. In *Int. Conference on Communications*, Istanbul, Turkey, June 2006.

[6] C. Bockelmann, D. Wübben, and K.-D. Kammeyer. Mutual Information based Rate Adaptation for MIMO-OFDM System with Finite Signal Alphabets. In *Int. ITG Workshop on Smart Antennas*, Darmstadt, Germany, February 2008.

[7] C. Stierstorfer and R.F.H. Fischer. Rate Loading in OFDM Based on Bit Level Capacities . In *Int. Symposium on Information Theory*, Toronto, Canada, July 2008.

[8] S. Stiglmayr, M. Bossert, and E. Costa. Adaptive Coding and Modulation in OFDM Systems using BICM and Rate-Compatible Punctured Codes. In *European Wireless*, Paris, France, April 2007.

[9] C. Bockelmann and K.-D. Kammeyer. Adaptive Interleaving and Modulation for BICM-OFDM. In *Int. OFDM-Workshop (InOWO 2008)*, pages 40–44, Hamburg, Germany, August 2008.

[10] C. Bockelmann, D. Wübben, and K.-D. Kammeyer. Efficient Coded Bit and Power Loading for BICM-OFDM. In *IEEE Vehicular Technology Conference (VTC2009-Spring)*, Barcelona, Spain, April 2009.

[11] W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery. *Numerical Recipes 3rd Edition: The Art of Scientific Computing*. Cambridge University Press, 2007.

[12] P.S. Chow, J.M. Cioffi, and J.A.C. Bingham. A practical discrete multitone transceiver loading algorithm for data transmission over spectrally shaped channels. *IEEE Trans. on Communications*, 43(234):773–775, Feb/Mar/Apr 1995.

[13] G. Caire, G. Taricco, and E. Biglieri. Bit-Interleaved Coded Modulation. *IEEE Trans. on Information Theory*, 44(3):927–946, 1998.

[14] A. Lozano, A.M. Tulino, and S. Verdu. Optimum Power Allocation for Parallel Gaussian Channels With Aribitrary Input Distributions. *IEEE Trans. on Information Theory*, 52(7):3033–3051, July 2006.

[15] D.P. Palomar and S. Barbarossa. Designing MIMO Communication Systems: Constellation Choice and Linear Transceiver Design. *IEEE Trans. on Signal Processing*, 53(10):3804–3818, October 2005.

[16] S. Stiglmayr, J. Klotz, and M. Bossert. Mutual Information of V-BLAST Transmission. In *IEEE Int. Symposium on Wireless Communication Systems*, Reykjavik, Iceland, October 2008.

[17] P. Fertl, J. Jaldén, and G. Matz. Capacity-based performance comparison of MIMO-BICM demodulators. In *IEEE Workshop on Signal Processing Advances in Wireless Communications*, pages 166–170, Recife, Brasilien, July 2008.

[18] S. Stiglmayr, J. Klotz, and M. Bossert. Adaptive Coding and Modulation in MIMO OFDMA Systems. In *IEEE Int. Symposium on Personal, Indoor and Mobile Radio Communications*, Cannes, France, September 2008.