Calibration of Non-Reciprocal Transceivers for Linearly Pre-Equalized MU-MISO-OFDM Systems in TDD Mode

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Abstract— **This paper deals with a relative calibration approach in terms of non-reciprocal transceivers in a multi-user MISO-OFDM TDD broadcast scenario where linear pre-equalization strategies are applied. The calibrated system, which is achieved by solving a total least squares problem, is compared with robust minimum mean square error pre-equalizer designs. Simulation results including channel estimation errors show a superior behavior of the calibrated system especially with a severe base station transceiver mismatch. However, by applying channel coding the advantage degrades with moderate reciprocity conditions.**

I. Introduction

ORTHOGONAL Frequency Division Multiplexing (OFDM) is an important transmission strategy due to its property of easily equalizing frequency-selective channels. In multi-user scenarios with decentralized non-cooperative mobile terminals one major focus is also on Space Division Multiple Access (SDMA) schemes by introducing multiple antennas at the base station (BS) [1]. The realization in terms of pre-equalization allows for less signal processing at the mobile terminals, which in return is generally based on channel state information (CSI) at the BS. In time division duplex (TDD) systems this can be achieved by relying on a channel estimate in the uplink transmission if the channel coherence time is large compared to one duplex phase and the system reciprocity theorem holds. This last requirement is usually not fulfilled due to non-reciprocal transmitters and receivers, e.g., coming from production tolerances. Thus, a process of calibration is necessary [2].

To avoid additional hardware costs due to RF calibration circuits the principle of so-called relative calibration presents a good method to track and estimate front-end influences during regular transmissions [3], [4]. In this procedure the whole calibration is done in signal space only. It simply relies on CSI coming from the uplink channel estimation and special knowledge of the corresponding downlink (DL) channel, which, e.g., can be realized by what is known as *analog feedback* [5]. The relative calibration approach is well-suited and less complex for narrowband flat-fading channels. In the OFDM context this motivates calibration per subcarrier. On the other hand, if no DL-CSI can be made available at the BS and only the order of reciprocity mismatch is known, a robust pre-equalization filter with respect to the non-reciprocal transmit and receive chains can be derived [6].

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In this contribution the downlink bit error rate (BER) performance of the multi-user multiple-input single-output (MISO) OFDM system with BS calibration is compared with robust linear minimum mean square error (MMSE) pre-equalizer designs. The robustness concerns the effects of channel estimation errors and non-reciprocal transceivers. The remainder of the paper is organized as follows. In Sec. II the system and the applied extended channel model are stated. Furthermore, the robust transmit pre-equalization filter design is described. The relative calibration approach is derived in Sec. III and simulation results are shown in Sec. IV. Finally, a conclusion is given in Sec. V.

II. System Description

A. Extended Channel Model

A downlink scenario of a system with N_B base station antennas and $N_M \leq N_B$ decentralized single-antenna mobile stations using OFDM with N_C subcarriers is considered. The vector of transmit symbols is obtained by preprocessing the M-QAM symbol vector $\mathbf{d} = [d_1, \ldots, d_{N_B}]^T$ with unit variance applying linear pre-equalization. To satisfy a total power constraint of N_B at the BS, the transmit symbols are scaled to ensure unit gain after pre-equalization. At the mobile stations complex Gaussian i.i.d. noise samples with variance σ_n^2 are added.

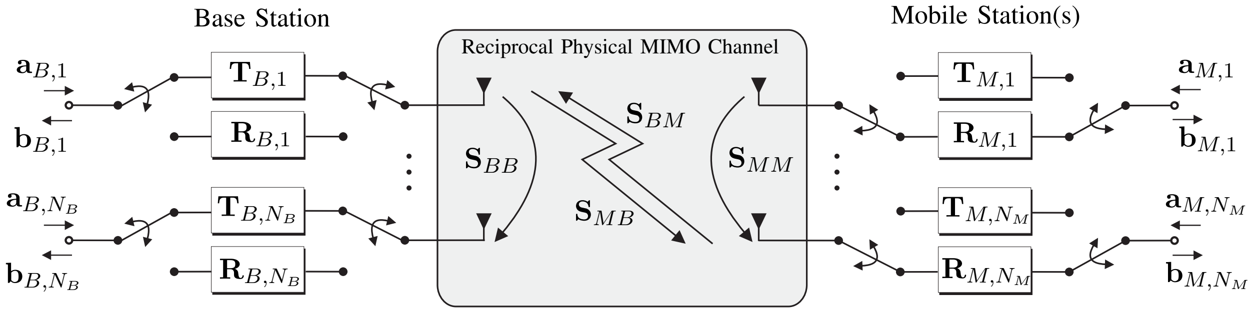

Fig. 1. Extended channel model using S-parameter description with BS and MS in downlink mode

Fig. 1 shows the extended channel model proposed in [6], [7], where $\mathbf{a}_{[B/M],[i/j]}$ and $\mathbf{b}_{[B/M],[i/j]}$ are auxiliary vectors for the transmit and receive signals in UL and DL direction, respectively. The i-th transmit and j-th receive antenna front-end is modeled as two-port device using a scattering matrix description [7]–[9],

$$\mathbf{T}_{[B/M],i} = \begin{bmatrix} 0 & 0 \\ \alpha_{T,[B/M],i} & \gamma_{T,[B/M],i} \end{bmatrix} \qquad (1)$$

and

$$\mathbf{R}_{[B/M],j} = \begin{bmatrix} 0 & \alpha_{R,[B/M],j} \\ 0 & \gamma_{R,[B/M],j} \end{bmatrix}, \qquad (2)$$

with complex gain factors $\alpha_{[T/R],[B/M],[i/j]}$ and input/output reflection coefficients $\gamma_{[T/R],[B/M],[i,j]}$. These factors are arranged in diagonal matrices, e.g.,

$$\mathbf{A}_{[T/R]B} = \operatorname{diag}\left\{\alpha_{[T/R]B,1}, \ldots, \alpha_{[T/R]B,N_B}\right\} \quad (3a)$$
$$\mathbf{\Gamma}_{[T/R]B} = \operatorname{diag}\left\{\gamma_{[T/R]B,1}, \ldots, \gamma_{[T/R]B,N_B}\right\}. \quad (3b)$$

Each gain factor, e.g., $\alpha_{[T/R]B,i} = 1 + \delta_{[T/R]B,i}$ is assumed to be slightly mismatched. Here, the statistically independent error terms $\delta_{[T/R]B,i}$ are zero mean complex Gaussian random variables with variance σ_δ^2 [6]. These factors are expected to change very slowly in time compared to the duplex phase and assumed to be equal per antenna on all subcarriers k.

Thus, the effective up- and downlink matrices in frequency-domain using the scattering matrix approach can be written as

$$\mathbf{G}(k) = \mathbf{H}_{\text{UL}}^T(k) = \mathbf{A}_{TM}\mathbf{W}_{TM}^T\mathbf{S}_{MB}(k)\mathbf{W}_{RB}^T\mathbf{A}_{RB} \qquad (4)$$

and

$$\mathbf{H}(k) = \mathbf{H}_{\text{DL}}(k) = \mathbf{A}_{RM}\mathbf{W}_{RM}\mathbf{S}_{MB}(k)\mathbf{W}_{TB}\mathbf{A}_{TB}, \qquad (5)$$

respectively. In (4) and (5) the matrices

$$\mathbf{W}_{T[B/M]} = \left(\mathbf{I}_{N_{[B/M]}} - \mathbf{\Gamma}_{T[B/M]}\mathbf{S}_{[BB/MM]}\right)^{-1} \quad (6a)$$
$$\mathbf{W}_{R[B/M]} = \left(\mathbf{I}_{N_{[B/M]}} - \mathbf{S}_{[BB/MM]}\mathbf{\Gamma}_{R[B/M]}\right)^{-1} \quad (6b)$$

describe the coupling and reflection at the transceivers. However, coupling and reflection effects can be neglected here as the system at least has to approximate channel reciprocity for well-matched transceivers to work properly [6]. Then, with $\mathbf{\Gamma}_{[\cdot]} \approx \mathbf{0}$ and $\mathbf{S}_{[BB/MM]}$ close to the all zero matrix, $\mathbf{W}_{T[B/M]}$ and $\mathbf{W}_{R[B/M]}$ become identity matrices. In [7] the assumption of neglecting the influence of the reflection coefficients in (3b) is justified by means of realistic matching. Finally, reasoning that during DL transmission the uplink chain at the mobile terminals is disconnected, meaning that $\mathbf{a}_{M,j} = \mathbf{0}$ in Fig. 1, the scattering matrix $\mathbf{S}_{MB}(k)$ can directly be replaced by the "extrinsic" downlink physical MIMO channel matrix $\mathbf{H}_{\text{FD}}(k)$ [9]. The channel matrix $\mathbf{H}_{\text{FD}}(k)$ in frequency-domain results from the frequency-selective time-domain channel matrix $\mathbf{H}_{\text{TD}}(\ell) \in \mathbb{C}^{N_M \times N_B}$, $0 \leq \ell \leq L_F - 1$, whose elements are i.i.d. complex Gaussian distributed. L_F denotes the number of uncorrelated equal power channel taps.

B. Robust Linear Pre-Equalization

Linear pre-equalization is applied using the uplink channel matrices $\mathbf{G}(k)$ such that the receive signal stacking all mobile stations reads

$$\mathbf{y}(k) = \beta(k)\mathbf{H}(k)\mathbf{F}(k)\mathbf{d}(k) + \mathbf{n}(k), \qquad (7)$$

with $\mathbf{F}_{\text{ZF}}(k) = \mathbf{G}^+(k)$ for the zero-forcing (ZF) or $\mathbf{F}_{\text{MMSE}}(k) = \mathbf{G}^H(k)\left(\mathbf{G}(k)\mathbf{G}^H(k) + \sigma_n^2\mathbf{I}_{N_B}\right)^{-1}$ for the MMSE case. The scalar

$$\beta(k) = \sqrt{\frac{N_B}{\operatorname{tr}\left\{\mathbf{F}(k)^H\mathbf{F}(k)\right\}}} \qquad (8)$$

is chosen such that the total sum power constraint per subcarrier is fulfilled. For convenience of a brief notation we drop the subcarrier index k for this section. As shown in [6] with the assumptions of perfect decoupling and the fact that the gain factors at the mobile terminals can be set to one (i.e. $\delta_{[T/R]M,j} = 0$) due to compensation, e.g., by pilot aided channel estimation, the effective downlink matrix in (5) can be rewritten with (4) to $\mathbf{H} = \mathbf{G}\mathbf{A}_{RB}^{-1}\mathbf{A}_{TB}$. As $\mathbf{A}_{RB} \neq \mathbf{A}_{TB}$ holds pre-equalization based on $\mathbf{G}$ leads to interference caused by imperfect transceivers. In case of $\left|\delta_{[T/R],B,i} \ll 1\right|$ the term $\mathbf{A}_{RB}^{-1}\mathbf{A}_{TB}$ can be approximated by $\mathbf{I}_{N_B} + \mathbf{\Delta}$. Then, the estimated receive data is

$$\tilde{\mathbf{d}} = \beta^{-1}\mathbf{y} = (\mathbf{G} + \mathbf{G}\mathbf{\Delta})\,\mathbf{F}\mathbf{d} + \beta^{-1}\mathbf{n}\,, \quad (9)$$

with $\mathbf{\Phi}_{\mathbf{\Delta}} = \mathrm{E}\left\{\mathbf{\Delta}\mathbf{\Delta}^H\right\} = 2 \cdot \sigma_\delta^2 \mathbf{I}_{N_B}$ being the covariance matrix of the reciprocity error. With (9) a robust MMSE pre-equalizer design with respect to non-reciprocal transceivers can be derived following the principles in [10] and [6]. Considering the same principle with channel estimation (CE) errors as in [1], a combined robust pre-equalizer reads

$$\mathbf{F}_{\text{rMMSE-ce}} = \quad (10)$$
$$\left(\mathbf{G}^H\mathbf{G} + (\sigma_n^2 + \underbrace{\sigma_e^2}_{\text{CE error}})\,\mathbf{I}_{N_B} + \underbrace{\mathrm{dg}\left\{\mathbf{\Phi}_{\mathbf{\Delta}}\mathbf{G}^H\mathbf{G}\right\}}_{\text{Imp. Tx/Rx error}}\right)^{-1}\mathbf{G}^H\,.$$

σ_e^2 is the variance of the estimation error and $\mathrm{dg}\left\{\cdot\right\} \mathrel{\hat{=}} \mathrm{diag}\left\{\mathrm{diag}^{-1}\left\{\cdot\right\}\right\}$ sets all off-diagonals of a matrix to zero [6].

III. Downlink Channel Calibration

For the purpose of estimating the reciprocity coefficients $\alpha_{[T/R],B,i}$ it is assumed that the uplink and downlink CSI at the BS are both disturbed by noise due to the assumption of imperfect channel estimation and erroneous analog feedback. Therefore, we assume a MMSE channel predictor model for the channel matrices to describe channel estimation errors. Then, the estimated uplink channel matrix $\hat{\mathbf{G}}(k)$ of one subcarrier can be realized by

$$\hat{\mathbf{G}}(k) = \sqrt{1-\sigma_e^2}\,\mathbf{G}(k) + \sqrt{\sigma_e^2\left(1-\sigma_e^2\right)}\,\mathbf{\Psi}(k)\,, \quad (11)$$

where $\mathbf{\Psi}(k)$ a Gaussian error matrix with an entry variance of one. The same holds for $\hat{\mathbf{H}}(k)$ with an independent noise term but here with identical error variance σ_e^2, which generally need not to be the same.

In general, with $\mathbf{W}_{T[B/M]} = \mathbf{W}_{R[B/M]} = \mathbf{I}_{N_{[B/M]}}$ equation (5) can be rewritten using (4) such that

$$\hat{\mathbf{H}}(k) = \underbrace{\mathbf{A}_{RM}\mathbf{A}_{TM}^{-T}}_{\mathbf{C}_M}\hat{\mathbf{G}}(k)\underbrace{\mathbf{A}_{RB}^{-T}\mathbf{A}_{TB}}_{\mathbf{C}_B}\,. \quad (12)$$

We define the vectors $\mathbf{c}'_B \triangleq \mathrm{diag}^{-1}\left\{\mathbf{C}_B^{-1}\right\}$ and $\mathbf{c}_M \triangleq \mathrm{diag}^{-1}\left\{\mathbf{C}_M^T\right\}$. Consequently, (12) can be reformulated with $\mathbf{c} \triangleq \left[\mathbf{c}_B'^T \mathbf{c}_M^T\right]^T$ to

$$\mathbf{E}\mathbf{c} = \mathbf{0}_{N_B N_M \times 1}\,, \quad (13)$$

where $\mathbf{E}$ is composed of the columns of $\hat{\mathbf{G}}^T(k) = \hat{\mathbf{H}}_{\mathrm{UL}}(k)$ and the rows of $\hat{\mathbf{H}}(k)$ (cf. [4]) such that

$$\mathbf{E} = \left[\mathbf{E}_1^T, \ldots, \mathbf{E}_K^T\right]^T \quad (14a)$$

with

$$\mathbf{E}_k = \begin{bmatrix} \mathrm{diag}\left\{\hat{\mathbf{h}}^{(1)}(k)\right\} & -\hat{\mathbf{h}}_{\mathrm{UL},1}(k) & & \mathbf{0} \\ \vdots & & \ddots & \\ \mathrm{diag}\left\{\hat{\mathbf{h}}^{(N_M)}(k)\right\} & \mathbf{0} & & -\hat{\mathbf{h}}_{\mathrm{UL},N_M}(k) \end{bmatrix}. \quad (14b)$$

Here, K defines the number of subcarriers used for calibration. A number $K > 1$ has the benefit of increasing the number of coefficients compared to the number of unknowns in $\mathbf{c}$.

As a result, (13) defines a special case of a total least squares (TLS) problem, where

$$\underset{\Delta\mathbf{E}}{\text{minimize}} \quad ||\Delta\mathbf{E}||_F \quad (15a)$$
$$\text{such that} \quad (\mathbf{E} + \Delta\mathbf{E})\,\mathbf{c} = \mathbf{0}_{N_B N_M \times 1} \quad (15b)$$

has to be solved. Here, $\Delta\mathbf{E}$ is the correction term of the TLS optimization problem. This specific problem is valid for narrowband flat-fading as in OFDM on each subcarrier and sparse filter matrices (3a) [4]. Mathematically, we want to find a perturbation matrix $\Delta\mathbf{E}$ with minimum Frobenius norm that lowers the rank of $\mathbf{E}$.

The solution lies in the right null space of $\mathbf{E}$ and can be computed with the singular value decomposition (SVD). Golub et al. showed the connection of the TLS solution to the SVD [11]. Then, if $\mathbf{E} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$ depicts the SVD and matrix $\mathbf{V} = \left[\mathbf{v}_1, \ldots, \mathbf{v}_{N_B+N_M}\right]$ the right singular vector space the estimated solution to $\mathbf{c}$ depends on the right singular vector corresponding to the smallest singular value in $\mathbf{\Sigma}$ such that

$$\mathbf{c}_0 = -\frac{1}{v_{N_B+N_M,N_B+N_M}}\,\mathbf{v}_{N_B+N_M}\,. \quad (16)$$

Thus, $\mathbf{c}$ can be fully determined (up to a scalar coefficient, which vanishes due to the reciprocal multiplication in (12)) if and only if $v_{N_B+N_M,N_B+N_M} \neq 0$ holds [3], [11]. With $\mathbf{c}_0$ the matrices $\hat{\mathbf{G}}(k)$ can be adjusted according to (12).

Fig. 2 shows the estimation performance of the TLS approach for different channel estimation error variances for a $N_B = N_M = 2$ system having $N_C = 256$ subcarriers and different numbers of calibration carriers. The reciprocity mismatch was set to $\sigma_\delta^2 = -20$ dB. The TLS calibration

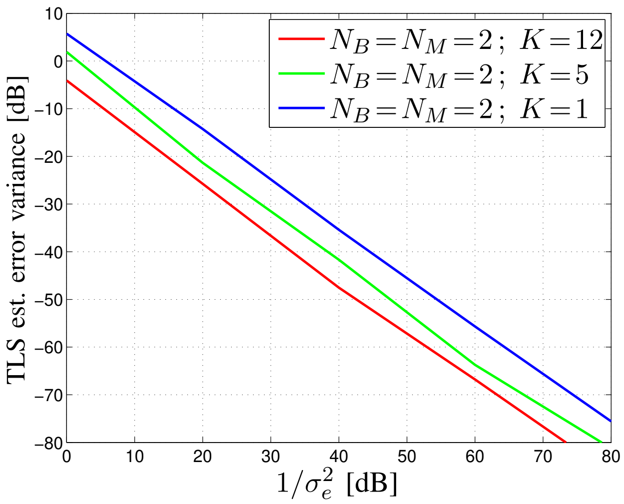

Fig. 2. Estimation performance of the TLS calibration approach depending on the channel estimation error σ_e^2

achieves quite good performace and shows almost linear behavior in the log-log scale. Using more than one subcarrier ($K > 1$) for calibration is only reasonable if the channel estimation error is large. For small channel estimation errors the gain in terms of accuracy is negligible compared to other influences in a communication system.

IV. Simulation Results

Fig. 3a) and b) show the bit error rate results versus E_b/N_0 for the different linear pre-equalization strategies in an uncoded and a coded $N_B = N_M = 4$ multi-user MISO-OFDM scenario applying $N_C = 256$ subcarriers and 16-QAM transmission. For the coded system the half-rate $(7,5)_8$ convolutional code with constraint length $L_c = 3$ and random interleaving is used. E_b denotes the average energy per information bit arriving at the receiver, thus $E_b/N_0 = N_M/(R_c \log_2(M)\sigma_n^2)$ holds. The guard interval has a length of $N_g = 6$, which is equal to the length of the applied Rayleigh channel taps L_F here. The guard loss is also considered in the curves.

The uncoded BER results indicate a small advantage for the MMSE pre-equalizer including the channel estimation error power (*MMSE-ce*) and the robust approach including both estimation and reciprocity error (*robust MMSE-ce*) compared to the ZF solution. Here, a gain of approximately 2 dB in the perfect reciprocity case is visible. At first, as the reciprocity error increases this gain is reduced at high signal-to-noise ratios, e.g., at $\sigma_\delta^2 = -30$ dB. This indicates a dominant part of interference coming from the estimation errors if moderate reciprocity conditions are existent. However, as the

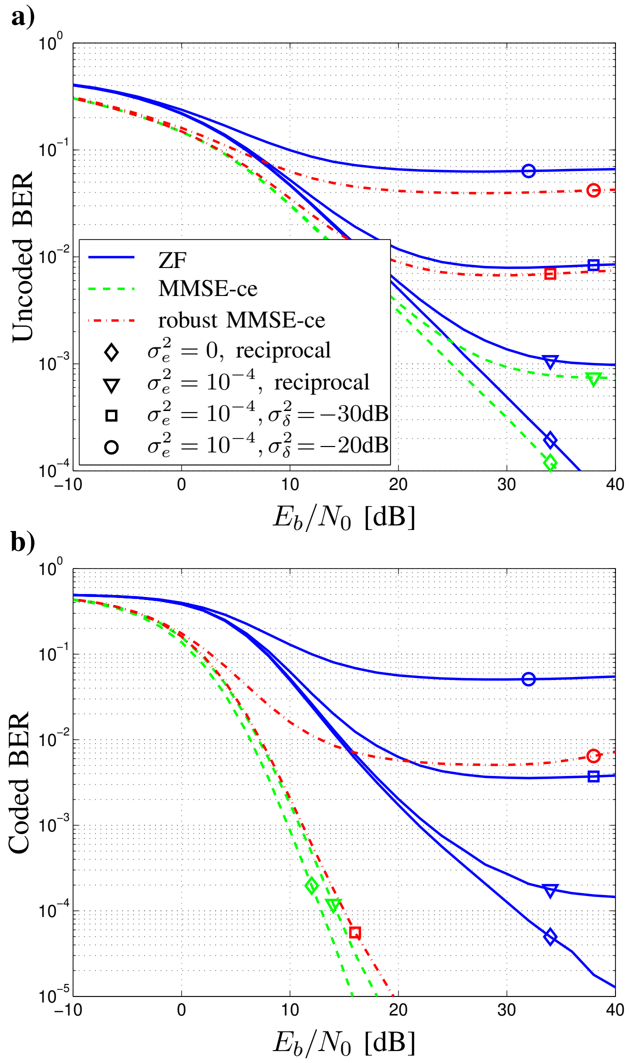

Fig. 3. **a)** Uncoded and **b)** coded BER versus E_b/N_0 for a system with $N_B = N_M = 4$, $N_C = 256$ subcarriers and 16-QAM with linear pre-equalization and different reciprocity mismatch conditions - channel coding with a half-rate conv. code ($L_c = 3$)

reciprocity error further increases to -20 dB the gain of the robust solution increases slightly. The coded results show a superior behavior for the robust MMSE pre-equalizers (cf. [12]). A gain of around 14 dB is obtained with a MMSE approach in the perfect reciprocal case without channel estimation errors. Using a robust approach according to (10) this perfect case is almost achievable with an estimation error of $\sigma_e^2 = 10^{-4}$ unless the reciprocity condition becomes too worse. With $\sigma_\delta^2 = -30$ dB the performance is excellent while the curve results in an error floor if the reciprocity mismatch is increased one decade. Nevertheless, the robust approach always outperforms the ZF approach.

In addition, Fig. 4 presents the uncoded and coded BER results if the TLS calibration approach is used in our scenario. Therefore, only up to twelve subcarriers ($K \in \{1, 5, 12\}$) are used in the calibration process, which in turn only adds a minor

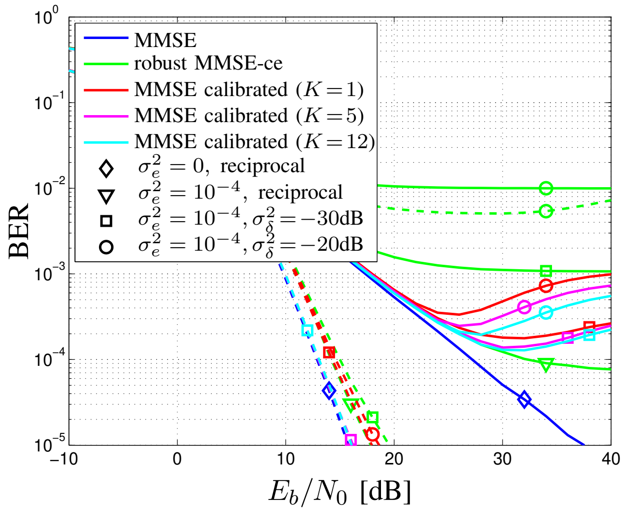

Fig. 4. BER versus E_b/N_0 for a system with $N_B = N_M = 4$, $N_C = 256$ subcarriers and $\eta = 2$ bit/s/Hz with MMSE precoding and different reciprocity conditions ((–) uncoded , (- -) with half-rate convolutional code ($L_c = 3$))

complexity to the base station. For comparison all curves have a spectral efficiency of 2 bit/s/Hz. It can be seen that with occurring reciprocity mismatches the calibrated ordinary MMSE solution clearly outperforms the robust pre-equalizers in terms of an uncoded transmission (*solid lines*) with channel estimation errors. The increasing error rates at high signal-to-noise ratios come from a remaining interference term, which results from imperfect CSI. The error rates can be slightly decreased with increasing K. An even better performance can be achieved if the channel estimation error is included in the MMSE approach during calibration. This is not considered here.

The coded results instead (*dashed lines*) show that with either using a robust approach or applying calibration excellent results can be obtained as long as the reciprocity mismatch remains small enough as for $\sigma_\delta^2 = -30$ dB. Almost the performance of a MMSE pre-equalizer with perfect reciprocity and without estimation error can be achieved. If the error is increased up to -20 dB the robust MMSE-ce pre-equalizer shows severe degradations, only the calibrated approach can deal with such a high reciprocity mismatch. This substantiates the need for calibration.

V. Conclusion

In this paper we discussed a relative calibration technique for multi-user MISO-OFDM systems with decentralized receivers applying TDD mode. This technique is able to combat imperfect transceiver calibration in terms of DL BER if (erroneous) instantaneous uplink and downlink CSI is available at the base station. Uncoded and coded results indicate the superiority of a robust MMSE design, which contains information of the channel estimation and reciprocity error variances, compared to the ZF pre-equalizer. In terms of the uncoded bit error rate ordinary MMSE pre-equalization with calibration using the proposed TLS approach outperforms this robust design especially for severe reciprocity conditions. Nevertheless, a robust design performs similar in coded scenarios with the drawback of necessary knowledge of estimation and reciprocity error variances.

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