Power Allocations for Adaptive Distributed MIMO Multi-hop Networks

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***Abstract*—Distributed MIMO multi-hop relaying is one of the most promising technologies that permits cost-effective improvement of coverage, data rate and end-to-end (e2e) user experience by utilizing distributed low-complexity space-time codes to overcome path losses and deep fades of wireless channels. However, an efficient transmission scheme and resource management are required to exploit these advantages. Specifically, low-complexity adaptive schemes and power control strategies should be designed, thereby achieving robust and cost-efficient e2e communications. In this paper an adaptive transmission scheme is presented, where one relay stops forwarding the message if it is in outage and other nodes adapt to a new space-time code. For this adaptive scheme, optimal as well as sub-optimal closed-form power allocation solutions are derived which minimize the total transmission power while satisfying a given e2e outage probability. The significant power savings due to the proposed approaches in comparison to a non-adaptive scheme is demonstrated by numerical results.**

I. Introduction

Recently, there has been increasing interest in combining traditional point-to-point MIMO techniques into multi-hop wireless relaying networks to support higher e2e data rates and to provide a better user experience. By the concept of virtual antenna array (VAA) spatially separated relaying nodes are allowed to utilize the capacity-enhancement approaches of MIMO techniques offering significant improvements for the data rate in multi-hop networks [1], e.g., by distributed space-time codes. Fig. 1 shows the concept of distributed MIMO multi-hop transmission, where one source communicates with one destination via a number of VAAs in multiple hops.

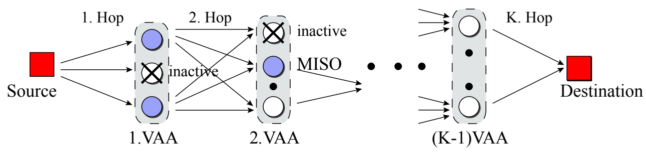

Fig. 1. Topology of adaptive distributed MIMO multi-hop relaying systems with active and inactive nodes.

In multi-hop communications, radio resources should be allocated to each hop efficiently in order to satisfying an e2e Quality-of-Service (QoS) requirement in terms of bit error rate, delay, throughout, as well as outage probability [2], [3], [4], [5]. Since the majority of real-world wireless applications happen over non-ergodic slow fading channels, the ergodic capacity is not applicable in strict sense. Therefore, we will consider the non-ergodic e2e outage probability as the measurement for the QoS in this paper.

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As pointed out in [6], the drawback of fixed decode-and-forward transmission is that it requires full decoding at all relays. The e2e connection is considered to be in outage if any relay can not decode the message correctly and the e2e performance is then determined by the worst relay link in the network. A similar assumption has also been made in [7], [8], [9] in terms of the e2e outage probability. This is clearly a pessimistic assumption; however, these investigations serve as a worst case in terms of system performance. In [10] closed-form capacity expressions of distributed MIMO multi-hop systems were derived for ergodic flat-fading Rayleigh and Nakagami channels and the outage probabilities were investigated by Monte-Carlo simulations.

The contribution of this paper is the derivation of a closed-form expression for the end-to-end outage probability of adaptive distributed MIMO multi-hop networks. Moreover, optimal as well as near-optimal closed-form power allocations that minimize the total power consumption while satisfying a given end-to-end outage probability are developed. The power allocation problem is formulated as a convex optimization problem, which can be solved by common optimization tools. In order to reduce complexity, a near-optimal but efficient power allocation algorithm is derived as well.

The remainder of this paper is organized as follows. In Section II the system model of the adaptive transmission scheme is introduced. The mathematical description of the outage probability will be given in Section III and the optimal power allocation problem is formulated in Section IV. A closed-form solution for an approximated optimization problem will be derived in Section VI. Finally, performance results and conclusions will be given in Section VII and VIII, respectively.

II. System Model

In the sequel, the adaptive transmission of the distributed MIMO multi-hop network as depicted in Fig. 1 is described. Here, a source node desires to communicate with a destination node via several relaying nodes in K hops. Several relaying nodes are composed to VAAs performing distributed space-time coding schemes. For simplicity, we assume that each node has only one antenna element and can only operate in half-duplex mode. Furthermore, the relays do not exchange data among each other when decoding the message due to the expense of additional complexity, power and time for information exchange, which leads to high system overheads.

The adaptive transmission procedure in TDMA mode is described as follows. The source broadcasts the information to the first VAA at the first time slot. Each node of the first VAA decodes the received information separately. We denote the relays successfully decoding the message (or being not in outage) as *active nodes* and the relays failing to decode the message (or being in outage) as *inactive nodes*, respectively. The inactive nodes will stop transmission at the next time slot. The active nodes will adapt to transmit the decoded message cooperatively according to a space-time code with respect to the number of active nodes. If all relays within one VAA fail to decode the message, the e2e connection is considered to be in outage. This adaptive transmission continues at each VAA until the message reaches the destination. The focus of this paper is on the power allocation strategy for the network, thus, for the further investigation a given fixed network topology is assumed. The task of grouping the VAAs is beyond the scope of this paper.

As described above, the nodes within the same VAA decode the information separately but re-encode the decoded information by using a spatial fraction of the space-time code word. Therefore, the transmission within one hop can be modeled as several multiple-input single-output (MISO) systems. It is assumed that each relay transmits signals with the same data rate R and equal duration. Furthermore, all hops use the total bandwidth W that is available to the network. Let k index the hop, t_k, r_k denote the number of transmit and receive nodes within hop k, respectively. Define $\mathbf{S}_k \in \mathbb{C}^{t'_k \times T_k}$ as the space-time encoded signal with length T_k from the t'_k active nodes at hop k, i.e., $0 \leq t'_k \leq t_k$. The received signal $\mathbf{y}_{k,j} \in \mathbb{C}^{1\times T_k}$ at the jth node at the kth VAA is given by

$$\mathbf{y}_{k,j} = \sqrt{\theta_k \mathcal{P}_k}\mathbf{h}_{k,j}\mathbf{S}_k + \mathbf{n}_{k,j} \ , \tag{1}$$

where $\mathbf{n}_{k,j} \sim \mathcal{N}_C(0, N_0) \in \mathbb{C}^{1\times T_k}$ denotes the Gaussian noise vector with power spectral density N_0. The transmit power of each active node at hop k is $\mathcal{P}_k$ and does not dependent on the number of inactive nodes. This allows simple power control and hardware implementation at each relaying node which is especially important for relays with minimal processing functionality. Thus, the total power of VAA k is at most $t_k\mathcal{P}_k$. The channel from the t'_k active nodes to the jth receive node within the kth hop is expressed as $\mathbf{h}_{k,j} \in \mathbb{C}^{1\times t'_k}$. The elements of $\mathbf{h}_{k,j}$ obey the same uncorrelated Rayleigh fading statistics with variance 1. The relaying nodes belonging to the same VAA are assumed be spatially sufficiently close as to justify a common path loss θ_k between two VAAs, i.e., the network is symmetric. It can be simply described as $\theta_k = d_k^{-\epsilon}$, where d_k is the distance between the transmit nodes and the receive nodes at the kth hop and ϵ is the path loss exponent within range of 2 to 5 for most wireless channels.

III. Outage Probability at Hop k

Before formulating the outage probability $P_{\text{out},k}$ of hop k, we first consider the outage probability $p_{\text{out},k,j}(t'_k)$ of a MISO system with t'_k active nodes at hop k as described in (1). The instantaneous achievable rate of the $t'_k \times 1$ link is given by

$$C_{k,j}(t'_k) = W \log\left(1 + \frac{\mathcal{P}_k}{d_k^{\epsilon} W N_0}\|\mathbf{h}_{k,j}\|^2\right) \ , \tag{2}$$

with $\|\mathbf{h}_{k,j}\|^2 = \sum_{i=1}^{t'_k} |h_{k,j,i}|^2$. The outage probability $p_{\text{out},k}(t'_k)$ can be expressed as the probability that the channel can not support an error-free transmission at rate R

$$\begin{aligned} p_{\text{out},k,j}(t'_k) &= \Pr(R > C_{k,j}(t'_k)) \\ &= \Pr\left(\|\mathbf{h}_{k,j}\|^2 < \underbrace{\frac{\left(2^{\frac{R}{W}}-1\right) W N_0 d_k^{\epsilon}}{\mathcal{P}_k}}_{x_k}\right) = \frac{\gamma(t'_k, x_k)}{\Gamma(t'_k)} \ . \end{aligned} \tag{3}$$

In order to simplify the notation the variables $x_k = Q_k/\mathcal{P}_k$ and $Q_k = (2^{\frac{R}{W}}-1)WN_0 d_k^{\epsilon}$ were introduced, where x_k is proportional to the inverse signal-to-noise-ratio (SNR), i.e., $x_k \sim 1/\text{SNR}_k$. Herein, $\|\mathbf{h}_{k,j}\|^2$ obeys a Gamma distribution [11], therefore its CDF can be described by the lower incomplete Gamma function $\gamma(t'_k, x_k) = \int_0^{x_k} e^{-u} u^{t'_k - 1}\,\mathrm{d}u$ normalized by the Gamma function $\Gamma(t'_k)$. Clearly, the outage probability $p_{\text{out},k,j}(t'_k)$ depends on the number of active t'_k at hop k, which depends itself on the outage probabilities of the nodes at the previous hop $k-1$.

Furthermore, the outage probability of receiving node j at hop k is denoted by $P_{\text{out},k,j}$. Under the assumption of symmetric networks the outage probabilities of the nodes within one VAA are equal, i.e., $P_{\text{out},k,1} = \cdots = P_{\text{out},k,r_k} = P_{\text{out},k,j'}$ where j' indexes an arbitrary $j \in [1, \cdots, r_k]$. The number of active nodes t'_k at hop k is a random number that depends on the outage probabilities in the previous hop $k-1$. As these probabilities $P_{\text{out},k-1,j'}$ are equal, the number of active nodes t'_k follows the binomial distribution $\mathcal{B}$ with parameters t_k and $P_{\text{out},k-1,j'}$ and we write [11]

$$t'_k \sim \mathcal{B}(t_k, 1 - P_{\text{out},k-1,j'}) \ . \tag{4}$$

The probability of i nodes being active at hop k is expressed by the probability mass function as

$$\Pr(t'_k = i) = \binom{t_k}{i}(1 - P_{\text{out},k-1,j'})^i P_{\text{out},k-1,j'}^{t_k - i} \ , \ \forall\, i \tag{5}$$

with $\binom{t_k}{i} = \frac{t_k!}{i!(t_k-i)!}$. As the outage probability of a MISO system with i active nodes is described by $\Pr(t'_k = i)\cdot p_{\text{out},k,j}(i)$, the outage probability $P_{\text{out},k,j'}$ is given by the sum of the outage probabilities over all possible i

$$\begin{aligned} P_{\text{out},k,j'} &= \sum_{i=1}^{t_k} \Pr(t'_k = i)\cdot p_{\text{out},k,j}(i) \\ &= \sum_{i=1}^{t_k} \binom{t_k}{i}(1 - P_{\text{out},k-1,j'})^i P_{\text{out},k-1,j'}^{t_k-i} \frac{\gamma(i, x_k)}{\Gamma(i)} \ . \end{aligned} \tag{6}$$

Clearly, an outage occurs in one hop if all the receive nodes within this hop can not decode the message, i.e., the outage probability of hop k corresponds to

$$P_{\text{out},k} = \prod_{j=1}^{r_k} P_{\text{out},k,j} = P_{\text{out},k,j'}^{r_k} \; . \tag{7}$$

The e2e outage probability for adaptive relaying transmission is finally expressed as

$$P_{\text{e2e}} = 1 - \prod_{k=1}^{K} (1 - P_{\text{out},k}) = 1 - \prod_{k=1}^{K} \left(1 - P_{\text{out},k,j'}^{r_k}\right) \; . \tag{8}$$

In the following investigation we use the end-to-end outage probability P_{e2e} as the measurement for the required QoS.

IV. Optimization Problem

The optimization problem to minimize the total transmit power for the adaptive multi-hop scheme while supporting a given end-to-end outage probability requirement e is formulated as

$$\text{minimize } \mathcal{P}_{\text{total}} = \sum_{k=1}^{K} \mathcal{P}_k t_k (1 - P_{\text{out},k-1,j'}) \tag{9a}$$

$$\text{subject to } P_{\text{e2e}} \le e \; . \tag{9b}$$

Here, the calculation of $\mathcal{P}_{\text{total}}$ considers the inactive nodes stopping the transmission to save power. The probability that one node at hop k transmits signal is exactly $1 - P_{\text{out},k-1,j'}$.

Generally, the optimization problem (9) is not convex with respect to the power $\mathcal{P}_k$ of each hop. Fortunately, similar to [12] it can be shown to be convex for low outage probability requirements by proving the Hessian matrix of $P_{\text{e2e}}(\mathcal{P}_k, \forall\, k)$ to be positive semi-definite. To this end, the optimal solution $\mathcal{P}_k^\star$ for (9) can be obtained by standard optimization tools leading to considerable complexity [13].

V. Problem Simplification

The optimization problem (9) is intricate. To simplify further analysis, some approximations to the outage probability are invoked that permit the derivation of a near-optimal closed-form power allocation solution. Following the approximation method given in [7], [14], the outage probability $p_{\text{out},k,j}(t'_k)$ in (3) is upper bounded for high SNRs as

$$p_{\text{out},k,j}(t'_k) = \frac{\gamma(t'_k, x_k)}{\Gamma(t'_k)} \lessapprox \frac{t_k'^{-1} x_k^{t'_k}}{\Gamma(t'_k)} = \frac{x_k^{t'_k}}{\Gamma(t'_k + 1)} \; . \tag{10}$$

Hence, the outage probability of node j' in hop k defined in (6) is approximated by $\tilde{P}_{\text{out},k,j'}$

$$P_{\text{out},k,j'} \lessapprox \sum_{i=1}^{t_k} \Pr(t'_k = i) \frac{x_k^i}{\Gamma(i+1)} \triangleq \tilde{P}_{\text{out},k,j'} \; . \tag{11}$$

The end-to-end outage probability (8) can be further approximated by the union bound [7]

$$P_{\text{e2e}} \le \sum_{k=1}^{K} P_{\text{out},k} = \sum_{k=1}^{K} P_{\text{out},k,j'}^{r_k} \le \sum_{k=1}^{K} \tilde{P}_{\text{out},k,j'}^{r_k} \triangleq \tilde{P}_{\text{e2e}} \; . \tag{12}$$

Finally, the objective function of the optimization problem (9) can be rewritten as $\mathcal{P}_{\text{total}} \approx \sum_{k=1}^{K} \mathcal{P}_k t_k$ for small $P_{\text{out},k-1,j'}$. Thus, the approximated optimization problem is obtained

$$\text{minimize } \mathcal{P}_{\text{total}} \approx \sum_{k=1}^{K} \mathcal{P}_k t_k \tag{13a}$$

$$\text{subject to } \tilde{P}_{\text{e2e}} = \sum_{k=1}^{K} \tilde{P}_{\text{out},k,j'}^{r_k} \le e \; . \tag{13b}$$

VI. Closed-form Power Allocation Solution (CF)

Clearly, the optimization problem (13) only leads to a near-optimal power allocation solution. However, from the complexity point of view, it is attractive to use (13) to derive efficient solutions. To solve the problem, the Lagrangian of the approximated optimization problem (13) is defined as

$$L(\mathcal{P}_k, \lambda) = \sum_{k=1}^{K} \mathcal{P}_k t_k + \lambda(\tilde{P}_{\text{e2e}} - e) \; . \tag{14}$$

To obtain the sub-optimal power allocation that yields minimum total power while meeting the e2e outage constraint e, the derivatives of $L(\mathcal{P}_k, \lambda)$ with respect to $\mathcal{P}_k$ has to be zero for all $1 \le k \le K$, i.e.,

$$\frac{\partial L(\mathcal{P}_k, \lambda)}{\partial \mathcal{P}_k} = 0, \quad \forall\, k \; . \tag{15}$$

Furthermore, for the optimum solution of (13) the constraint function in (13b) must be fulfilled with equality, i.e.,

$$\tilde{P}_{\text{e2e}} = \sum_{k=1}^{K} \tilde{P}_{\text{out},k} = e \; . \tag{16}$$

From (15) and (16), a closed-form power allocation can be achieved by using several further approximations as outlined in the Appendix. This leads to the following theorem.

Theorem 1 (Closed-form adaptive power allocation (CF)): To minimize the total transmission power while meeting a given e2e outage probability requirement e, the sub-optimal closed-form power allocation $\mathcal{P}_k^\star$ is given by

$$\mathcal{P}_k^\star = \frac{t_k^{\frac{2}{t_k+1}} Q_k}{\tilde{P}_{\text{out},k}^{\frac{2}{r_k(t_k+1)}}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} \left(1 - \tilde{P}_{\text{out},k-1}^{\frac{1}{r_{k-1}}}\right)^i \tilde{P}_{\text{out},k-1}^{\frac{t_k-i}{r_{k-1}}}}{\Gamma(i+1)} \right)^{\frac{2}{t_k(t_k+1)}} , \tag{17}$$

with approximated outage probability per hop

$$\tilde{P}_{\text{out},k} = \tilde{P}_{\text{out},k,j'}^{r_k} \approx \frac{a_k}{\sum_{k=1}^{K} a_k} \cdot e \quad \forall\, k \tag{18}$$

and parameter

$$a_k = \left(\frac{2 Q_k t_k^{\frac{t_k+3}{t_k+1}}}{r_k(t_k+1)} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} \left(1 - e^{\frac{1}{r_{k-1}}}\right)^i e^{\frac{t_k-i}{r_{k-1}}}}{\Gamma(i+1)} \right)^{\frac{2}{t_k(t_k+1)}} \right)^{\frac{(t_k+1) r_k}{2+(t_k+1) r_k}} . \tag{19}$$

Proof: See Appendix. ■

VII. Performance Evaluation

The performance of the proposed power allocation solution for adaptive distributed MIMO multi-hop schemes is assessed here for various network configurations. It is assumed that the e2e communication over $W = 5$ MHz should meet an e2e outage probability constraint of $e = 1\%$ where the path loss exponent ϵ is 3, the distance between two VAA d_k is 1 km and N_0 is -174 dBm/Hz.

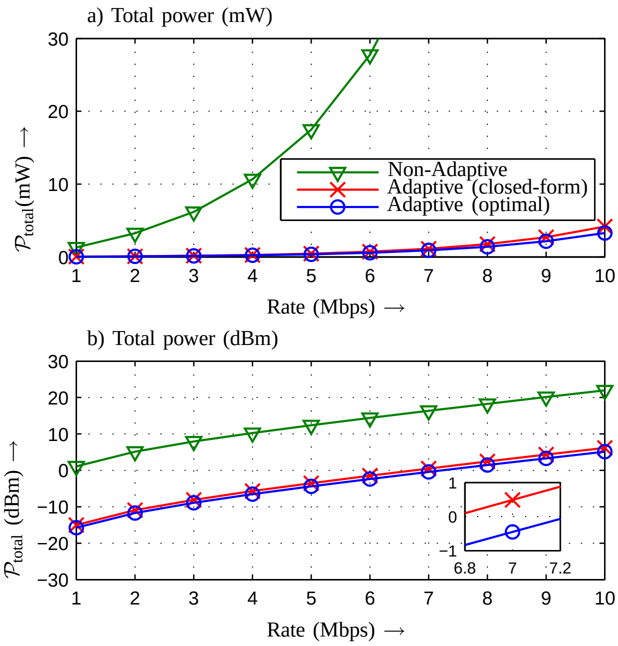

Fig. 2. $\mathcal{P}_{\text{total}}$ a) in mW and b) dBm for non-adaptive transmission, closed-form and optimal adaptive power allocation solutions.

In the first considered scenario the multi-hop network consists of $K = 3$ hops with the same number of relaying nodes $t_2 = t_3 = 3$ per VAA. Explicitly, Fig. 2 depicts the total power versus the data rate for non-adaptive and adaptive transmissions both with optimized power allocations in mW and dBm, respectively. Note that for non-adaptive transmission the optimal power allocation solution introduced in [7] is applied, where the e2e connection is considered to be in outage if any node in the system is in outage. As shown, the adaptive transmission scheme leads to power savings of more than 16 dBm comparing to the non-adaptive scheme. Moreover, the proposed closed-form solution yields near-optimum total power consumption. In comparison to the optimal solution it results only in a slightly increased power consumption of about 1 dBm which leads to a lower e2e outage probability than the required e.

Fig. 3 shows the outage probabilities occurring per hop versus data rate with optimized power allocation. For the non-adaptive scheme, almost all outage events happen at the first hop due to the lack of diversity. In contrast, in the adaptive schemes the most outages occur at the last hop. That is due to the fact, that the last hop (i.e., the destination) contains only one node that has to decode the message correctly. Otherwise, an outage event occurs.

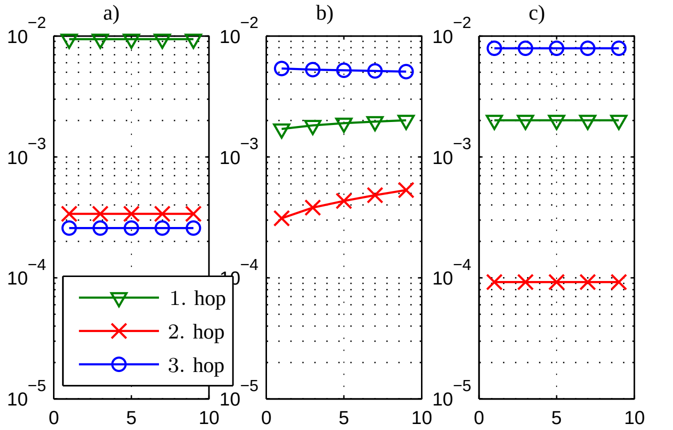

Fig. 3. The outage probabilities per hop for a) non-adpative transmission, b) closed-form (CF) and c) optimal adaptive power allocation solutions.

In the second scenario the same number of relaying nodes t_k in each VAA is varying from 1 to 6 for a 4-hop system with data rate 5 Mbps. Fig. 4 depicts the total power versus the number of relays per VAA. It can be observed, that the optimal number of relays per VAA for the non-adaptive scheme turns out to be 3 in this scenario. In contrast, the total power of the adaptive scheme is decreasing with increasing number of relays per VAA.

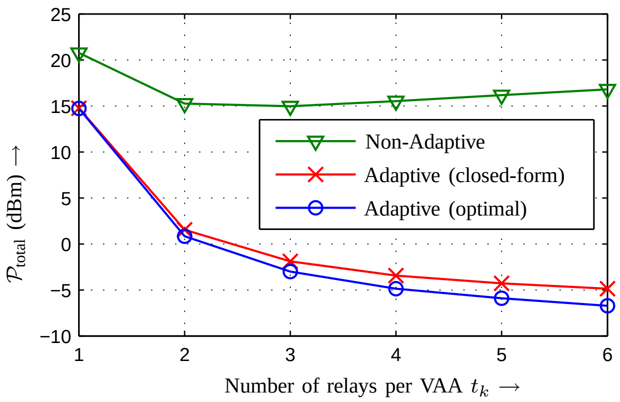

Fig. 4. $\mathcal{P}_{\text{total}}$ v.s. t_k for non-adpative transmission, closed-form (CF) and optimal power allocation solutions.

VIII. Conclusion

In this paper, we introduced an adaptive distributed MIMO multi-hop scheme. For this system, the convex power allocation problem that aims to minimize the total transmission power while satisfying a given e2e outage probability was formulated. It can be solved by standard optimization tools with considerable complexity. In order to derive a low-complexity sub-optimal closed-form solution, some approximations were employed. As shown by simulations results, significant power savings can be achieved by utilizing adaptive transmission schemes with optimized power allocation in comparison to non-adaptive transmission. The performance gap between the sub-optimal closed-form solution and the optimal solution is relatively small, while the complexity is reduced significantly. The extension of the presented approach to an joint optimization of power and time is represented in [15], [16].

APPENDIX

Proof of Theorem 1: Referring to (15), the first derivative of $L(\beta_k, \lambda)$ with respect to $\mathcal{P}_k$ relates to $\tilde{P}_{\text{out},k}$ as well as $\tilde{P}_{\text{out},k+1}$, given by

$$\frac{\partial L(\mathcal{P}_k, \lambda)}{\partial \mathcal{P}_k} = t_k + \lambda \left(\frac{\partial \tilde{P}_{\text{out},k}}{\partial \mathcal{P}_k} + \frac{\partial \tilde{P}_{\text{out},k+1}}{\partial \mathcal{P}_k} \right) = 0 \ , \quad (20)$$

which is due to the dependence between $\tilde{P}_{\text{out},k}$ and $\tilde{P}_{\text{out},k+1}$ indicted in (6). This makes the further analysis intricate. However, by recognizing that $\tilde{P}_{\text{out},k} = \tilde{P}^{r_k}_{\text{out},k,j'} < e, \forall\, k$ the dependency in (6) can be removed by replacing the outage probability $P_{\text{out},k-1,j'}$ by $e^{\frac{1}{r_{k-1}}}$. This relaxes (11) to

$$\tilde{P}_{\text{out},k,j'} \approx \sum_{i=1}^{t_k} \binom{t_k}{i} \left(1 - e^{\frac{1}{r_{k-1}}}\right)^i e^{\frac{t_k - i}{r_{k-1}}} \frac{x_k^i}{\Gamma(i+1)} \ . \quad (21)$$

Furthermore, (21) can be approximated by its geometric mean

$$\begin{aligned}\tilde{P}_{\text{out},k,j'} &\approx t_k \left(\prod_{i=1}^{t_k} \binom{t_k}{i} (1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}} \frac{x_k^i}{\Gamma(i+1)} \right)^{\frac{1}{t_k}} \quad (22)\\ &= x_k^{\frac{t_k+1}{2}} t_k \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} (1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}}}{\Gamma(i+1)} \right)^{\frac{1}{t_k}} \\ &= \left(\frac{Q_k}{\mathcal{P}_k} \right)^{\frac{t_k+1}{2}} t_k \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} (1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}}}{\Gamma(i+1)} \right)^{\frac{1}{t_k}} .\end{aligned}$$

Hence, $\mathcal{P}_k$ can be expressed by $\tilde{P}_{\text{out},k,j'}$ as

$$\mathcal{P}_k = Q_k \frac{t_k^{\frac{2}{t_k+1}}}{\tilde{P}_{\text{out},k,j'}^{\frac{2}{t_k+1}}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} (1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}}}{\Gamma(i+1)} \right)^{\frac{2}{t_k(t_k+1)}} . \quad (23)$$

As the dependence between $\tilde{P}_{\text{out},k}$ and $\tilde{P}_{\text{out},k+1}$ has been removed, the derivation (20) simplifies to

$$\frac{\partial L(\mathcal{P}_k, \lambda)}{\partial \mathcal{P}_k} = t_k + \lambda \frac{\partial \tilde{P}_{\text{out},k}}{\partial \mathcal{P}_k} = 0 \ . \quad (24)$$

Differentiating (22) along $\mathcal{P}_k$ yields

$$\begin{aligned} 0 &= t_k + \lambda r_k \tilde{P}^{r_k - 1}_{\text{out},k,j'} \frac{\partial \tilde{P}_{\text{out},k,j'}}{\partial \mathcal{P}_k} && (25a)\\ &= t_k - \frac{\lambda r_k (t_k+1) \tilde{P}^{r_k-1}_{\text{out},k,j'}}{2\mathcal{P}_k} \tilde{P}_{\text{out},k,j'} && (25b)\\ &= t_k - \frac{\lambda r_k (t_k+1)}{2\mathcal{P}_k} \tilde{P}^{r_k}_{\text{out},k,j'} && (25c)\\ &= t_k - \frac{\lambda r_k (t_k+1)}{2\mathcal{P}_k} \tilde{P}_{\text{out},k} \ . && (25d)\end{aligned}$$

Inserting (23) in (25d), $\tilde{P}_{\text{out},k,j'}$ is expressed as

$$\tilde{P}_{\text{out},k} = \tilde{P}^{r_k}_{\text{out},k,j'} = \lambda^{-\frac{(t_k+1) r_k}{2 + (t_k+1) r_k}} \cdot a_k \ , \quad (26)$$

where a_k is introduced to simply the notation

$$a_k = \left(\frac{2 Q_k t_k^{\frac{t_k+3}{t_k+1}}}{r_k (t_k+1)} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} (1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}}}{\Gamma(i+1)} \right)^{\frac{2}{t_k(t_k+1)}} \right)^{\frac{(t_k+1) r_k}{2 + (t_k+1) r_k}} . \quad (27)$$

Since $\lambda^{-\frac{(t_k+1) r_k}{2 + (t_k+1) r_k}}$ can be approximated by λ^{-1} for large t_k, the insertion of (26) in (16) yields the approximation

$$\lambda^{-1} \approx \frac{e}{\sum_{k=1}^{K} a_k} \ . \quad (28)$$

Hence, the sub-optimal outage probability is given by

$$\tilde{P}_{\text{out},k} = \tilde{P}^{r_k}_{\text{out},k,j'} \approx \frac{a_k}{\sum_{k=1}^{K} a_k} \cdot e \ . \quad (29)$$

By inserting this relation into (23) and replacing e by $\tilde{P}_{\text{out},k-1}$ we finally achieve (17). This concludes the proof. ■

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