# Link Adaptation for MIMO-OFDM with partial CSI

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*Abstract*— **The discussion of bit and power loading algorithms for Multiple Input Multiple Output (MIMO) systems often neglects the requirement of perfect channel state information to perform singular value decomposition. Especially in multi-carrier systems like Orthogonal Frequency Division Multiplex (OFDM) a vast amount of feedback is necessary to facilitate such a scheme. In this paper the combination of simple linear and successive interference cancellation receivers and a bit and power loading scheme that includes the channel coding into the optimization is investigated for MIMO-OFDM systems to achieve good performance with reasonable complexity and feedback.**

## I. Introduction

Multiple Input Multiple Output (MIMO) Orthogonal Frequency Division Multiplex (OFDM) systems gained much interest in recent years. In order to achieve ever higher spectral efficiencies, link adaptation has been introduced for multi carrier as well as multiple antenna systems. The application of so called bit and power loading schemes, however, is mostly considered for perfect channel state information (CSI) at the transmitter allowing a decomposition of the channel matrix in the MIMO case. Considering the vast amount of feedback that is necessary for MIMO-OFDM systems even with a few subcarriers renders such a transmission scheme infeasible. In contrast to this, feeding back a set of modulation and power indices may be feasible, especially in combination with grouping of subcarriers to so called “chunks”. MIMO systems with imperfect CSI at the transmitter, though, introduce a new problem, namely the definition of a suitable quality metric, which can be used to optimize the bit and power allocation, whereas for single antenna systems the subcarrier SNR is sufficient. One such approach is based on the mutual information with finite alphabets of a MIMO channel [1]. While requiring less feedback, complexity is quite high due to the fact that the mutual information has to be calculated either numerically or by approximation. Another contribution similar to our approach analyzes the performance of a loading scheme with Minimum Mean Square Error (MMSE) equalization [2].

In this contribution a combination of simple linear and successive interference cancellation (SIC) receivers and “coded” bit and power loading schemes for MIMO-OFDM systems will be discussed with respect to the interdependence of equalizer design and loading schemes. Furthermore, a reduction in feedback by subcarrier grouping to chunks, employing the well known mutual information based SINR averaging will be considered. It will be shown, that just by signaling of the employed code rate and the modulation on a group of subcarriers to the transmitter, good performance can be achieved at reasonable complexity and feedback.

The remainder of this work is structured as follows. Section II details the system model, while Section III shortly discusses the applied MIMO equaliziers. In Section IV the applied bit and power loading scheme will be presented with regard to possible feedback reductions and implications to the equalizer design. Finally, simulation results will be shown in Section V followed by a conclusion in Section VI.

### *Notation*

In the following, vectors and matrices are denoted by lower case and upper case bold letters, respectively. Specifically $\mathbf{I}_\lambda$ and $\mathbf{0}_\lambda$ denote the identity and all zero matrix with dimension $\lambda \times \lambda$. The $i$th column of a Matrix $\mathbf{A}$ is described as $\mathbf{a}_i$, whereas an element will index as $[\mathbf{A}]_{i,j}$ with row $i$ and column

This work was supported in part by the German Research Foundation (DFG) under grant Ka841-18.

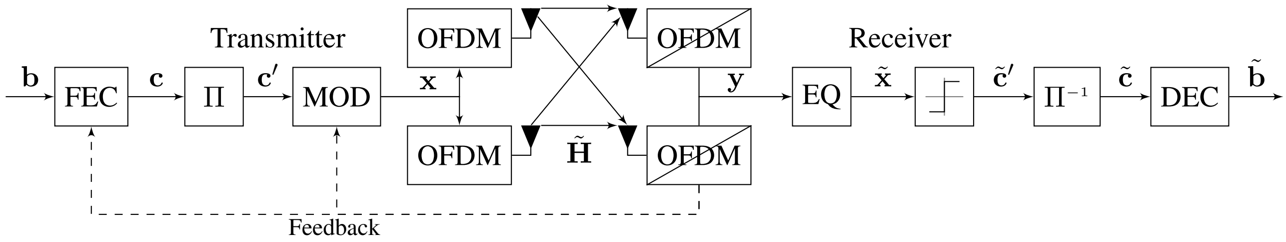


Fig. 1. MIMO-OFDM System Model

$j$. The $\mathrm{diag}(\cdot)$ operator either extracts the diagonal of a matrix or is used to create a diagonal matrix from a vector. Furthermore, probabilities are denoted as $\mathrm{P}$ and $\mathcal{N}_C(\mu, \sigma)$ describes a complex Gaussian distribution with mean $\mu$ and variance $\sigma$.

## II. MIMO-OFDM System

The frequency domain system model of a perfectly synchronized and ISI free MIMO-OFDM system as shown in Fig. 1 at subcarrier $k = 1, \ldots, N_C$ can be described as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{P}_k^{1/2} \mathbf{x}_k + \mathbf{n}_k \ , \tag{1}$$

where $\mathbf{y}_k \in \mathbb{C}^{N_R}$, $\mathbf{x}_k \in \mathbb{C}^{N_T}$ and $\mathbf{n}_k \sim \mathcal{N}_C(\mathbf{0}_{N_R}, \mathbf{I}_{N_R})$ are the receive signal vector, the transmit symbol vector and the AWGN noise vector, respectively. The frequency domain channel matrix $\mathbf{H}_k \in \mathbb{C}^{N_R \times N_T}$ is defined as the FFT of the time domain Rayleigh channel $\tilde{\mathbf{H}}$ with $L_F$ equal power taps and $\mathbf{P}_k = \mathrm{diag}\left([p_{1,k}, \ldots, p_{N_T,k}]\right)$ denoting the power allocation matrix. The total power over all subcarriers and antennas is then $\mathcal{P} = \sum_{i=1}^{N_T} \sum_{k=1}^{N_C} p_{i,k}$.

### A. Channel State Information

Availability of CSI at the transmitter enables adaptive transmission schemes. The assumption of perfect CSI, however, is hardly realistic, even if some form of channel reciprocity can be assumed. In this contribution transmit CSI is constrained to the feedback of the used modulation either on each subcarrier or a group of subcarriers called chunk.

In such a scenario the well known singular value decomposition (SVD) scheme cannot be used. Therefore, different linear and non-linear equalizers at the receiver are considered to "diagonalize" the MIMO channels into a set of $N_T \cdot N_C$ "parallel" channels, which can be used for bit (and power) loading. Henceforth, these "parallel" channels will be called *subchannels* to differentiate from the $N_C$ subcarriers and the $N_T$ layers of the spatial multiplexing MIMO system.

### B. Modulation and Coding

Throughout this paper transmit symbols stemming from $M$-QAM modulation alphabets $\mathcal{A}$ are considered. To each subcarrier $k$ an individual alphabet of cardinality $M_k = |\mathcal{A}_k|$ may be assigned. Soft-Demapping via a-posteriori-probability (APP) detection is used to supply soft information to the decoder.

Fig. 1 shows the general system model including channel coding and interleaving. The applied coding scheme uses a single forward error correction (FEC), which encodes the information bits of one OFDM symbol. Non-systematic non-recursive convolutional encoders of rates $R_C \in \{1/4, 1/3, 1/2, 2/3, 3/4\}$ with constraint lengths $L_C = 3$ are considered. In all cases the code word length is fixed to the number of bits in one OFDM symbol, leading to longer code words for higher data rates.

## III. Equalizers

The basics of MIMO equalization are a well known topic in MIMO communications, therefore Zero Forcing (ZF) and MMSE linear and Successive Interference Cancellation (SIC) receivers and their implementation will only be reviewed briefly. In general, the system after equalization can be described as $N_T$ (assuming $N_T \leq N_R$) equivalent single input single output (SISO) channels, termed layers, for each subcarrier $k$

$$z_{i,k} = \phi_{i,k} \cdot \sqrt{p_{i,k}} x_{i,k} + \tilde{n}_{i,k} \quad i = 1, \ldots, N_T \ , \tag{2}$$

where $\phi_{i,k}$ denotes the effective channel and $\tilde{n}_{i,k}$ the equivalent noise, characterized by some noise power $\sigma_{i,k}^2$. The equivalent channel-to-interference-and-noise-ratio (CINR) $\gamma_{i,k}$ is then defined as

$$\gamma_{i,k} = \frac{|\phi_{i,k}|^2}{\sigma_{i,k}^2} \ . \tag{3}$$

In the following, the $\phi_{i,k}$ and $n_{i,k}$ will be characterized for different types of equalizers.

### A. Linear Equalizers

The simplest considered equalizers are the two well known linear Zero Forcing and Minimum Mean Square Error filters. To incorporate the loading scheme, specifically the power loading, into the equalizers, we consider the power weighted channel matrix $\hat{\mathbf{H}} = \mathbf{H}_k \mathbf{P}_k^{1/2}$ on a subcarrier $k$, neglecting the subcarrier index for ease of nomenclature.

*1) Zero Forcing:* The well known ZF equalizer can be calculated as

$$\mathbf{G}_{\text{ZF}} = (\hat{\mathbf{H}}^H \hat{\mathbf{H}})^{-1} \hat{\mathbf{H}}^H \,, \tag{4}$$

which results in

$$\boldsymbol{\phi}_k = [\phi_{1,k}, \ldots, \phi_{N_T,k}]^T = \mathbf{1}_{N_T} \tag{5}$$
$$\boldsymbol{\sigma}_k^2 = [\sigma_{1,k}^2, \ldots, \sigma_{N_T,k}^2]^T = \text{diag}(\mathbf{G}_{\text{ZF}} \mathbf{G}_{\text{ZF}}^H) \,. \tag{6}$$

*2) Minimum Mean Square Error:* Accordingly, the mean square error minimizing equalizer is defined as

$$\mathbf{G}_{\text{MMSE}} = (\hat{\mathbf{H}}^H \hat{\mathbf{H}} + \mathbf{I}_{N_T})^{-1} \hat{\mathbf{H}}^H \,, \tag{7}$$

which results in the slightly more complicated effective system

$$\boldsymbol{\phi}_k = \text{diag}(\mathbf{G}_{\text{MMSE}} \hat{\mathbf{H}}) \tag{8}$$
$$\sigma_{i,k}^2 = \frac{(1 - \phi_{i,k})}{\phi_{i,k}} \,. \tag{9}$$

### B. Successive Interference Cancellation

The SIC equalizer can be efficiently realized via $\mathbf{QL}$ decomposition of the channel matrix $\hat{\mathbf{H}}$ or an extended channel matrix $\left[\hat{\mathbf{H}}^T \, \mathbf{I}\right]^T$ to achieve a ZF or MMSE solution, respectively. Filtering with the unitary matrix $\mathbf{Q}^H$ results in a new system characterized by the lower triangular matrix $\mathbf{L}$, which enables a simple implementation of the SIC. For further details, see, e.g., [3].

*1) ZF:* The resulting parameters for the ZF-SIC are then

$$\phi_{i,k} = 1 \tag{10}$$
$$\sigma_{i,k}^2 = \frac{1}{|L_{i,i}|^2} \,. \tag{11}$$

*2) MMSE:* In contrast to the ZF-SIC, interference is not perfectly suppressed, resulting in interference plus colored noise

$$\phi_{i,k} = L_{i,i} - \frac{1}{L_{i,i}} \tag{12}$$
$$\sigma_{i,k}^2 = \sum_{\ell=i+1}^{N_T} \left| \left[\mathbf{L}^{-H}\right]_{i,\ell} \right|^2 + \mathbf{q}_i^H \mathbf{q}_i \,. \tag{13}$$

## IV. Bit and Power Loading

In order to adapt the MIMO-OFDM system to the current channel condition, the $\gamma_{i,k}$ can be considered as "parallel" and independent, neglecting correlations and coloring, to apply arbitrary bit and power loading schemes. Throughout this paper, the link adaptation schemes presented in [4] will be employed to adapt the code rate (see II-B), modulation and power on each parallel channel aiming at a minimum power at a given target frame error rate $\mathrm{P}_{\text{Target}} = 10^{-2}$.

### A. Influence of the Equalizers

As can be seen from Section III, the actual equalizer and therefore the conditions of the subchannels depend on the power loading, due to incorporation of $\mathbf{P}_k^{1/2}$ into the filtering. Strictly speaking, an iterative procedure should be used repeating the loading and accordingly adapted equalization until a global power minimum is reached. Unfortunately, convergence cannot be guaranteed and especially in the MMSE case the interdependence of post-equalization CINRs and the power distribution is non-trivial. As a first attempt, we will only consider results, where the loading is at first based on equalization with an equal power distribution $\mathbf{P}_k^{1/2} = \mathcal{P}/(N_T \cdot N_C)\mathbf{I}_{N_T}$ for all $k = 1, \ldots, N_C$. The actual equalization will then be based on the calculated bit and power distribution, which ensures correct/meaningful equalization even in the MMSE case, but is not optimal.

### B. Feedback

As previously stated, a limited form of feedback from the receiver to the transmitter is assumed. Due to this system design, it is not feasible to feed back the used modulation and power on each subcarrier with unlimited accuracy. Signaling the modulation can be achieved with a limited number of bits, e.g., with 4 bits per subchannel if a maximum of 256-QAM is allowed, the power, however, has to be quantized. To further simplify the system design, only a coarse power allocation will be considered. If a subchannel is switched off, its power will be redistributed to the other subchannels. Therefore, $p_{i,k} = \mathcal{P}/|\mathcal{S}| \,, (i,k) \in \mathcal{S}$, where $\mathcal{S}$ is the set of active subchannels and no further signaling besides the modulation is needed. Additionally, 3 bits will be required to identify the code rate, but compared to the amount of feedback needed for the modulation this is negligible.

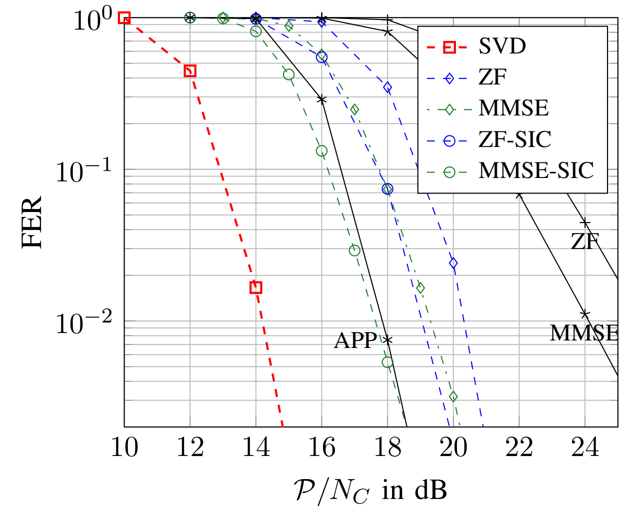


Fig. 2. Frame Error Rate vs. average power per subcarrier $\mathcal{P}/N_C$ for non-adaptive (black lines) and adaptive systems, $\eta = 8$ bit/s/Hz, full CSI at the transmitter and convolutional codes of constraint length $L_C = 3$; $N_C = 1024$, $N_T = N_R = 4$, $L_F = 10$.

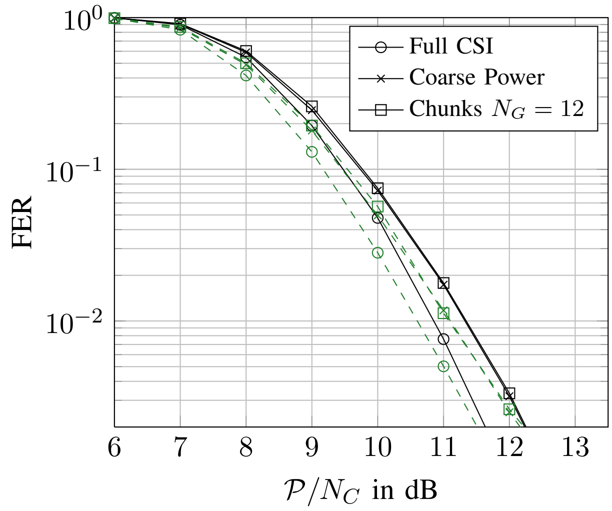


Fig. 3. Frame Error Rate vs. average power per subcarrier $\mathcal{P}/N_C$ for MMSE —⊖— and MMSE-SIC - ⊖ - at $\eta = 4$ bit/s/Hz and convolutional codes of constraint length $L_C = 3$; $N_C = 1024$, $N_T = N_R = 4$, $L_F = 10$.

Depending on the frequency selectivity of the channel given by $L_F$ and the number of subcarriers $N_C$ it may not be necessary to adapt the modulation on each subcarrier, but a group of subcarriers instead. These groups, called chunks, may consist of $N_G$ subcarriers, assuming that the channel quality is nearly the same over the group. Note, that the effective channels on a subcarrier after equalization may be very different, meaning, that we can group subchannels from one MIMO layer over $N_G$ subcarriers in frequency direction, but should not group in spatial direction. To apply the loading algorithm the well known mutual information based CINR averaging will be used [5] to calculate the average CINR

$$\bar{\gamma}_{i,j} = \exp\left(\sum_{\ell=N_G\cdot(j-1)+1}^{N_G\cdot j} \log\left(1+\gamma_{i,\ell}\right)\right) - 1 \quad (14)$$

for the $j = 1, \ldots, N_G$ chunks.

## V. Results

Fig. 2 shows Frame Error Rate (FER) results for various receivers assuming perfect CSI at the transmitter. The curves marked with "ZF", "MMSE" and "APP" (A-Posterior Probability, here a soft sphere detection) are results without any adaptation on the transmitter side and a fixed code rate $R_C = 1/2$. At the shown spectral efficiency of $\eta = 8$ bit/s/Hz the performance loss of the best adaptive system, naturally the MMSE-SIC, in comparison to a SVD approach is roughly 3dB. This clearly points out the suboptimality of the applied receivers if full CSI can be used.

Due to the complexity of APP detection for higher modulations and large number of antennas, the APP results can only be regarded as a benchmark, which is almost achieved by the simpler MMSE-SIC system with adaptive modulation, power and code rate. However, due to the full CSI assumption this is only possible at the cost of either reciprocity or a vast amount of feedback. At even higher spectral efficiencies APP detection is outperformed by the adaptive system, which is not shown here due to space constraints. In general it can be seen, that even though channel dependencies are neglected, significant performance gains can be achieved compared to non-adaptive systems. Fig. 3 to 5 show results for linear MMSE equalization and MMSE-SIC as the best considered linear/non-linear adaptive system. To analyze the performance loss incurred by a) the coarse power control and b) the additional grouping of subcarriers to chunks spectral efficiencies from $\eta = 4$ bit/s/Hz to $\eta = 12$ bit/s/Hz have been analyzed.

The system with linear MMSE equalization shows a very limited performance loss of approximately 0.5 to 0.8 dB at FER $= 10^{-2}$ (the target FER for the link adaptation) due to the coarse power scheme, which is consistent with results from literature regarding the importance of power loading, e.g. [6]. Compared to that, the MMSE-SIC performance loss is marginally smaller, the overall performance gain

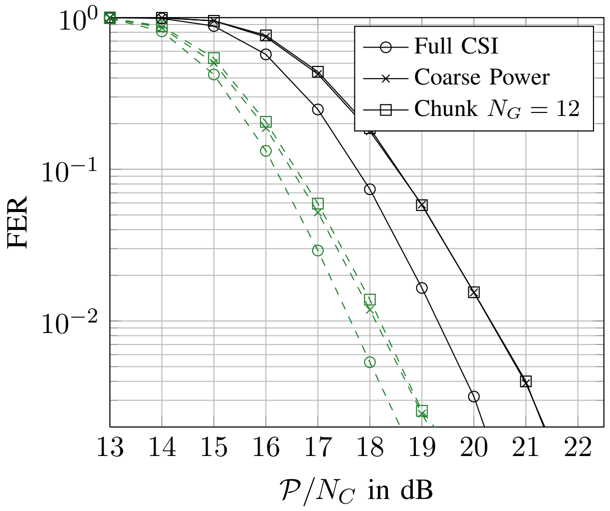


Fig. 4. Frame Error Rate vs. average power per subcarrier $\mathcal{P}/N_C$ for MMSE and MMSE-SIC at $\eta = 8$ bit/s/Hz and convolutional codes of constraint length $L_C = 3$; $N_C = 1024$, $N_T = N_R = 4$, $L_F = 10$.

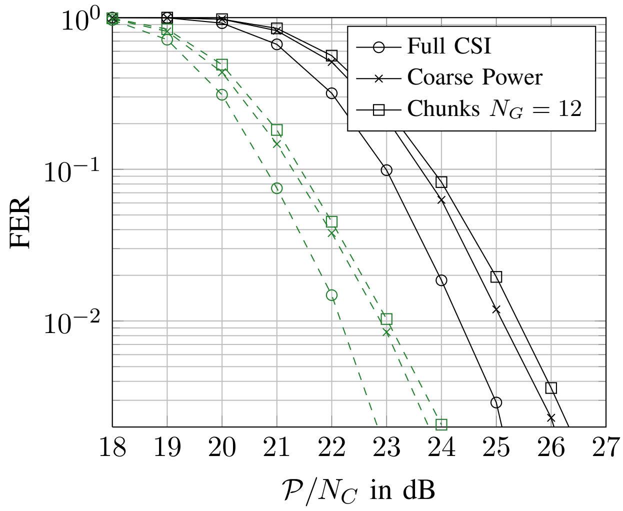


Fig. 5. Frame Error Rate vs. average power per subcarrier $\mathcal{P}/N_C$ for MMSE and MMSE-SIC at $\eta = 12$ bit/s/Hz and convolutional codes of constraint length $L_C = 3$; $N_C = 1024$, $N_T = N_R = 4$, $L_F = 10$.

by SIC, however, increases with spectral efficiency. Note, that "chunking" in addition to the coarse power adaptation does not decrease the performance noticeable at small to medium spectral efficiency for the given system parameters, which is to be expected due to the limited frequency selectivity. At higher efficiencies, though, a loss is noticeable due to the increased constellation sizes.

## VI. Conclusion

The presented results clearly show that adaptive systems employing linear oder SIC receivers can vastly improve their performance by "coded" bit and power loading even if the amount of feedback information is limited and power loading is simplified.

Considering a chunk size of $N_G = 12$, 4 bit signaling for each chunk and 3 bit signaling for the code rate, 1379 bits would be needed to adapt the system, costing roughly 1.35 bits/s/Hz loss in spectral efficiency in the reverse link due to the overhead. Depending on the system parameters and the setup of forward and reverse link (symmetric/asymmetric) this may be a viable approach to achieve good (near ML or better) performance or to increase the throughput, e.g., using, [7] at reasonable complexity.

However, further improvements could be achieved by a common optimization of loading and equalization. If the power is only adapted coarsely with equal power over active subchannels, the overall behavior of the MMSE solutions and the loading algorithm seems to be much more smooth, possibly enabling an iterative solution. Furthermore, sorted SICs and a layer-wise coding scheme may be topics for further research.

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